

LINEAR OPTIMIZATION HW 1

“The textbook” refers to the Kolman–Beck book. Let me know if you don’t have access to a copy.

1. Solve *any three* of Problems 2, 4, 6, 8, and 10 from Section 1.1 of the textbook.
2. For each of the following sets, determine whether it is a convex polyhedron.
 - (a) The empty set.
 - (b) The 22-dimensional Euclidean space \mathbb{R}^{22} .
 - (c) The set of all $(x, y) \in \mathbb{R}^2$ satisfying $x \cos \theta + y \sin \theta \geq 0$ for all $\theta \in [0, \pi/2]$.
 - (d) The set of all $x \in \mathbb{R}$ satisfying $x^2 - 10x + 12 \leq 0$.
 - (e) The set of all $(x, y) \in \mathbb{R}^2$ satisfying $2x^2 + y^2 + xy - 6x + 8y + 10 \leq 0$.
3. Solve Problem 6 in Section 1.5 of the textbook.
4. Solve Problem 8 in Section 1.5 of the textbook.
5. A function of the form $\max_{i=1, \dots, k} (\mathbf{c}_i^\top \mathbf{x} + d_i)$ is called a *piecewise linear convex function*. An example of such a function is $|x + 2| = \max\{x + 2, -(x + 2)\}$. Show that the problem of *minimizing* a piecewise linear convex function subject to linear constraints can be formulated as a linear programming problem. That is, show an LP equivalent to

$$\begin{array}{ll} \min & \max_{i=1, \dots, k} (\mathbf{c}_i^\top \mathbf{x} + d_i) \\ \text{s.t.} & \mathbf{Ax} \leq \mathbf{b} \end{array}$$

(*Hint: the scheduling problem we solved is a special case of this problem.*)

Extra credit: could we also introduce constraints involving piecewise linear convex functions?

Extra credit: how about *maximizing* piecewise linear convex functions subject to linear constraints? Can we turn this into an equivalent LP?

6. (For extra credit) We defined a convex polyhedron as the intersection of finitely many halfspaces, and we called a polyhedron a polytope if it was bounded. Another intuitive way could be to define a convex polytope as the set of convex combinations of finitely many points. Prove that the two definitions are equivalent. (*It is actually easier to show a little more: that a convex polytope is the convex hull of its extreme points.*)

Due on February 7, Thursday, in the beginning of the class.