LINEAR OPTIMIZATION HW 2

"The textbook" refers to the Kolman–Beck book.

1. Solve Problem 8 in Section 2.1 of the textbook.

2. Solve Problem 2 in Section 2.2 of the textbook.

3. Solve the standard form LPs

$$\begin{aligned} \max \quad & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.} \quad & \mathbf{A}\mathbf{x} \le \mathbf{b} \\ & \mathbf{x} \ge \mathbf{0} \end{aligned}$$

(after converting them to canonical form) with the following parameters, using Bland's Rule to avoid cycling. *Note that these examples are those "easy" ones where you can skip Phase I.*

(a) $\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & -2 & 3 \\ -1 & 2 & 3 & 0 \end{pmatrix}$, $\mathbf{b} = (6, 5, 4)^\top$, $\mathbf{c} = (2, -1, 0, 0)^\top$;

(b) $\mathbf{A} = \begin{pmatrix} 1 & -2 & 0 & -1 \\ 0 & -1 & -2 & 3 \\ -1 & 2 & 3 & 0 \end{pmatrix}$, $\mathbf{b} = (6, 5, 4)^\top$, $\mathbf{c} = (2, -1, 0, 0)^\top$.

4. Carry out "Phase I" of the simplex method for the following canonical form problems to find an initial basic feasible solution, if there is any. Don't do Phase II for these problems.

(a) $\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & -2 & 3 \\ -1 & 2 & 3 & 0 \end{pmatrix}$, $\mathbf{b} = (6, -5, -4)^\top$, $\mathbf{c} = (2, -1, 0, 0)^\top$;

(b) $\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & -2 & 3 \\ -1 & 2 & 3 & 0 \end{pmatrix}$, $\mathbf{b} = (6, -5, 4)^\top$, $\mathbf{c} = (2, -1, 0, 0)^\top$;

5. Solve Problems 10, and 22 from Section 2.3 of the textbook using the two-phase method.

6. Solve Problems 2, 4, and 6 from Section 3.1 of the textbook. (Find duals of given primal problems.)

7. Decide whether the following statements are true or false. Justify your answers *briefly* by examples, counterexamples, or short proofs.

(a) The set of optimal solutions of a linear programming problem is a convex polyhedron.

(b) The set of optimal solutions of a linear programming problem is a convex polytope.

(c) In an iteration of the simplex method, if the variable exiting the basis is zero prior to the iteration, then the new basic feasible solution is degenerate.

(d) In an iteration of the simplex method, if the basic feasible solution is degenerate prior to the iteration, then the new basic feasible solution is also degenerate.

(e) While running the simplex method, once the reduced cost of some variable becomes nonpositive, it remains nonpositive throughout the algorithm.

(f) While solving an LP using the simplex method, if the basic feasible solution in the beginning of an iteration is degenerate, then the objective function value is not changed by the iteration.

(g) If a tie occurs while choosing the basic variable to leave the basis (i.e., the row of the tableau) during an iteration of the simplex method, then the next basic feasible solution will be degenerate.

8. ([Easy, but] extra credit.) Prove that if all non-basic variables of a basic feasible solution have nonpositive reduced costs, then it is an optimal solution. (We stated this when we claimed that the simplex algorithm works, but we haven't proven it in class.)

Due on February 21, Thursday, in the beginning of the class.