Solutions to selected Midterm 2 problems
Problems 1–4 and 8 were discussed in class.

5. (a) ... because both methods is based on solving a sequence of LPs, where each LP is obtained from another, already solved, one by adding an extra constraint. The dual simplex method is a tool designed to handle such LPs.

(b) ... because in that case the integrality constraints are redundant, so the optimal solutions of the two problems are the same.

(c) i. No, we might even obtain a matrix with entries other than 0, 1, and -1;

ii. Yes, this is true by definition, the subdeterminants of the matrix don't change;

iii. No, trivial 2×2 counterexamples exist, see class notes;

iv. Yes, and one can argue many ways. Probably the easiest is by definition: the submatrices of the new matrix are a subset of those of the old matrix, so the subdeterminants of the new matrix must be the same or the negatives of the subdeterminants of the old matrix.

v. Yes, and one can argue many ways. This is also true by definition, the subdeterminants of the matrix either don't change or became the negatives of the old ones. Also, permuting rows and columns corresponds reordering the constraints and renaming the variables. Trivially none of these change the basic feasible solutions.

vi. No, but this is a bit trickier. It is obviously true for 2×2 matrices, so one has to find a 3×3 counterexample. I'll leave it as an exercise.

6. (a) Let us name the nodes from top to bottom, and from left to right, by $A, \ldots, G$. We started at A, branched, examined B and C, branched at C, examined D and E (infeasible), branched at D, examined F (integer solution, the best found so far) and G. At this point we can prune all the leaves except the integer solution, see part (c).

(b) The branches at A are labeled $y \leq 1$ and $y \geq 2$. The branches at C are labeled $x \leq 8$ and $x \geq 9$. The branches at D are labeled $y \leq 2$ and $y \geq 3$.

(c) As soon as we found an integer solution at node F, we can prune all other leaves with objective function value $z \leq 44$. This means we can prune all the other leaves.

(d) We pruned everything, no more branching is possible. The only integer solution we found must be optimal.

7. (a) It has to be in canonical form. I'll leave this as an exercise.

(b) We are not done, because the LP relaxation is feasible, but its optimal solution is not integer. We proceed by adding a new constraint (cut) to the problem.

(c) See the book or the class notes for details. There are two fractional constraints, we can use either of them. Moreover, there are two ways to derive a cut from both. Suppose we use the second constraint. The updated tableau will be either

s_1	0	0	1	$-1/3$	$-1/3$	0	$2/3$
x_2	0	1	0	$1/3$	$-2/3$	0	$7/3$
x_1	1	0	0	0	1	0	4
s_4	0	0	0	0	$-1/3$	1	$-1/3$
	0	0	0	$-1/3$	$-5/3$	0	$-31/3$

(this is better, ready for the next dual simplex step), or

s_1	0	0	1	$-1/3$	$-1/3$	0	$2/3$
x_2	0	1	0	$1/3$	$-2/3$	0	$7/3$
x_1	1	0	0	0	1	0	4
s_4	0	1	0	0	-1	1	2
	0	0	0	$-1/3$	$-5/3$	0	$-31/3$