

COMMENTS ON THE LINEAR PROGRAMMING FORMULATION OF THE FOOTBALLS PROBLEM

Decision variables: we surely need X_1, X_2, \dots, X_6 , where the meaning of X_m is the number of footballs produced in month m . It is helpful to introduce the additional variables I_m , $m = 1, \dots, 6$ to denote the inventory at the end of month m . These are, technically, also *decision variables*, even though they don't really represent business decisions independent from the X_m 's – the inventory at the end of each month is *determined by* X_1, \dots, X_6 . The only reason why we introduce them is that it will serve as an excellent shorthand, making our formulae (and hence our lives) easier.

In the hope that we don't get overwhelmed by notation, I introduce some more: let D_m be the demand in month m . Of course, these are *not* decision variables, as their values are known; for example, $D_1 = 100$, $D_2 = 150$, and so on. Why do we need letters then? To save even more writing, see below.

Without further ado, here is the formulation.

$$\begin{array}{ll}
 \max & 12.50X_1 + 12.55X_2 + 12.70X_3 + 12.80X_4 + 12.85X_5 + 12.95X_6 + \\
 & + 0.05(12.50I_1 + 12.55I_2 + 12.70I_3 + 12.80I_4 + 12.85I_5 + 12.95I_6) \\
 \text{s.t.} & X_m \leq 300 \qquad \qquad \qquad m = 1, \dots, 6 \\
 & I_m \leq 100 \qquad \qquad \qquad m = 1, \dots, 6 \\
 & I_m = I_{m-1} + X_m - D_m \quad m = 1, \dots, 6 \\
 & I_m \geq 0 \qquad \qquad \qquad m = 1, \dots, 6 \\
 & X_m \geq 0 \qquad \qquad \qquad m = 1, \dots, 6
 \end{array}$$

A couple of comments:

- The third constraint describes how the inventory changes from month to month. We used the D_m notation here: without it we would have to write out six constraints separately, which would be sad.
- The last constraint is just the “usual” nonnegativity constraint. *But the one before is not!!* The inventory “variable” is just a function of our production, so we can't just claim it to be nonnegative by common sense. Its nonnegativity expresses the constraint that *all demand is satisfied, in each month*. How? The third constraint shows that we compute the inventory by deducting the full demand from our stock. If the full demand is satisfied in each month, then the leftover in the inventory is nonnegative, and vice versa. This may not be totally obvious, then spend some time trying to convince yourself!