

LINEAR PROGRAMMING FORMULATION OF THE INVESTMENT PROBLEM

Decision variables: A, B, C , where A is the money (in US dollars) we invest in A, etc. I would also introduce the *convenience variables* M_0, \dots, M_3 , where M_y is the amount of money we have on our money market account in year y . (The current year is year 0, and so on.)

$$\begin{aligned} \max \quad & M_3 & (1) \\ \text{s.t.} \quad & A \leq 0.30(A + B + C) & (2) \\ & B \geq 0.25(A + B + C) & (3) \\ & C \leq 0.40(A + B + C) & (4) \\ & M_0 = 1000 - A - B & (5) \\ & M_1 = 1.08M_0 + 0.2A + 0.1B - C & (6) \\ & M_2 = 1.08M_1 + 1.4B & (7) \\ & M_3 = 1.08M_2 + 1.25A + 1.6C & (8) \\ & M_y \geq 0 \quad y = 0, 1, 2, 3 & (9) \\ & A, B, C \geq 0 & (10) \end{aligned}$$

The first four lines are obvious, and so are (9)–(10). Lines (5)–(8) are the *cash flow balance equations*.

One can do without the convenience variables, but the formulae will become cumbersome:

$$\begin{aligned} \max \quad & 1.08 \left(1.08 \left(1.08(1000 - A - B) + 0.2A + 0.1B - C \right) + 1.4B \right) + 1.25A + 1.6C \\ \text{s.t.} \quad & A \leq 0.30(A + B + C) \\ & B \geq 0.25(A + B + C) \\ & C \leq 0.40(A + B + C) \\ & 1000 - A - B \geq 0 \\ & 1.08(1000 - A - B) + 0.2A + 0.1B - C \geq 0 \\ & 1.08(1.08(1000 - A - B) + 0.2A + 0.1B - C) + 1.4B \geq 0 \\ & 1.08 \left(1.08 \left(1.08(1000 - A - B) + 0.2A + 0.1B - C \right) + 1.4B \right) + 1.25A + 1.6C \geq 0 \\ & A, B, C \geq 0 \end{aligned}$$

You can see the trick easily: whenever we saw a pattern repeated several times (like $1000 - A - B$), we introduced a “shorthand” for it. It is usually the case that the best shorthands have intuitive meanings associated to them. (In this case this is our cash balance.)