Solutions to the Second Midterm

Question 1

Disclaimer: there are several ways to formulate this as an integer program, the solution below is by no means the only right, or best, or even "most recommended" way.

My decision variables are:

- S_i: starting time of task i. (i = 1, …, 7, where the 7th task is the additional last step, with prerequisites 5 and 6.)
- $X_{i,j}$: is 1 if task i is shortened by exactly j weeks, and zero otherwise. Here i = 1, …, 6, and j = 1, 2, 3.
- D: is 1 if the project is delayed (takes more than 15 weeks), and zero otherwise.
- W: number of weeks the project is delayed by (can be zero, if not delayed).

The objective function is:

Minimize $4000D + 3000W + 1000X_{1,1} + 3000X_{1,2} + \ldots + 7000X_{6,3}$

The constraints then can be formulated as follows:

1. $S_3 \geq S_1 + 6 - (X_{1,1} + 2X_{1,2} + 3X_{1,3})$
   $S_3 \geq S_2 + 7 - (X_{2,1} + 2X_{2,2} + 3X_{2,3})$
   …
   $S_7 \geq S_6 + 6 - (X_{6,1} + 2X_{6,2} + 3X_{6,3})$
2. $X_{i,1} + X_{i,2} + X_{i,3} \leq 1$ for all i = 1, …, 6.
3. $X_{1,1} + X_{1,2} + \ldots + X_{6,3} \leq 3$
4. $W + 15 \geq S_7$
5. $D \leq W \leq 21D$ (21 is an upper bound on W, since the whole project can easily be finished in 36 weeks.)
6. $S_i \geq 0$ for all i = 1, …, 6.
7. $W \geq 0$
8. $X_{i,j}$ is binary for all i = 1, …, 6; j = 1, 2, 3
9. D is binary
10. W is an integer

Question 2

(a) The decision variables are: the number ingots to purchase (one variable for each type), the amount of alloys to purchase (again, one variable for each type), and the amount of steel scrap to purchase. The corresponding cells are: B25:E25, B30:D30, and E30, respectively.

(b) =SUM(B26:E26)+SUM(B30:E30)

(c) =B20*$B$34

(d) =SUMPRODUCT($B$26:$E$26;B5:E5)+SUMPRODUCT($B$30:$D$30;B11:D11)+$E$30*B16

(e) =SUMPRODUCT(B25:E25;B4:E4)+SUMPRODUCT(B30:D30;B10:D10)+E30*B15

(f) The target cell is B41; it should be minimized. The changing cells are $B$25:$E$25 and $B$30:$E$30. The constraints are:

1. B34 = D34
2. B38:B39 = D38:D39
3. B25:E25 <= 2

4. B25:E25 >= 0
5. B30:E30 >= 0
6. B25:E25 are integers

The purpose of the Tolerance option is to set up to what precision we want to approximate the optimal solution. Because of the integrality constraints, which we would like to enforce strictly, it should be set to zero, or very close to zero, like 0.01%.

(g) The optimal blend consists of 1 ingot of type one, 1 ingot of type two, 2 ingots of type three, 3.75 tons of alloy type one, 0.75 tons of alloy type three and 4.5 tons of steel scrap. No ingots of type four and no type two alloy should be purchased.

Question 3

(a) Geometric distribution.
(b) Normal distribution.
(c) Poisson distribution. Could be normal, too, but Poisson looks better. (I accepted both.)
(d) Normal distribution.
(e) Normal distribution.
(f) Poisson distribution.
(g) Normal distribution.
(h) Binomial distribution.