1. C&G Imports, Inc., imports two brands of white wine, one from Germany and the other one from Italy. It has been estimated that if the German wine retails at p dollars/bottle and the Italian wine is sold for q dollars/bottle, then

$$2360 - 150p + 100q$$

bottles of German wine and

$$960 + 80p - 120q$$

bottles of Italian wine will be sold per week. Determine the unit price for each brand that will allow C&G to realize the largest possible weekly revenue.

Solution

$$R(p, q) = (2360 - 150p + 100q) \cdot p + (960 + 80p - 120q) \cdot q = -150p^2 - 120q^2 + 180pq + 2360p + 960q$$

$$\frac{\partial R(p, q)}{\partial p} = -300p + 180q + 2360 = 0$$

$$\frac{\partial R(p, q)}{\partial q} = -240q + 180p + 960 = 0$$

$$p = \frac{4}{3}q - \frac{16}{3}$$

$$30\left(\frac{4}{3}q - \frac{16}{3}\right) - 18q = 236$$

$$p = 18.67$$
$$q = 18$$

$$H = \begin{pmatrix} -300 & 180 \\ 180 & -120 \end{pmatrix}, \quad H_1 = -300 < 0, \quad H_2 = (-300)(-240) - 180^2 = 39600 > 0.$$

So $p = \$18.87$, $q = \$18$ realize the maximum profit.

2. A consulting firm is trying to determine how to minimize the annual costs associated with purchasing computer paper. Each time an order is placed, an ordering cost of $20 is incurred. The annual holding cost is 20% of the dollar value of inventory. During each month, the consulting firm uses 80 boxes of computer paper. The price per box of computer paper depends on q, the number of boxes ordered, as shown in the table below.

Number of Boxes Ordered	Price per Box
$q < 300$	$10.00
$300 \leq q < 500$	$9.80
$q \geq 500$	$9.70

Determine the optimal order quantity and the number of orders placed each year.

Solution

$p_3 = 9.7$, $EOQ = \left(\dfrac{2 \cdot 20 \cdot 960}{(0.2)(9.7)}\right)^{1/2} = 140.69 < 500$ not admissible, so $q_3^* = 500$.

$$TC(500) = \frac{20 \cdot 960}{500} + 960(9.7) + \frac{(0.2)(9.7)500}{2} = 9835.4$$

$p_2 = 9.8$, $EOQ = \left(\dfrac{2 \cdot 20 \cdot 960}{(0.2)(9.8)}\right)^{1/2} = 139.97 < 300$ not admissible, so $q_2^* = 300$.

$$TC(300) = \frac{20 \cdot 960}{500} + 960(9.8) + \frac{(0.2)(9.8)300}{2} = \mathbf{9766 \text{ min cost}}$$

$p_1 = 10$, $EOQ = \left(\dfrac{2 \cdot 20 \cdot 960}{(0.2)(10)}\right)^{1/2} = 138.56 < 300$ admissible, so $q_1^* = 138.56$.

$$TC(138.56) = \frac{20 \cdot 960}{500} + 960(10) + \frac{(0.2)(10)(138.56)}{2} = 9877.13$$

Order 300 boxes 3.2 times a year.

3. Furnco sells secretarial chairs. Annual demand is normally distributed with mean 1040 chairs and standard deviation 130 chairs. Furnco orders its chairs from a flagship store. It costs \$100 to place an order and the lead time is 2 weeks. Furnco estimates that the stockout cost (future goodwill loss cost) is \$50 per chair. Furnco pays \$60 for each chair and sells it for \$100. The annual cost of holding a chair in inventory is 30% of its purchase cost.

a) Assuming that all demand is backlogged, what are the optimal reorder point and the safety stock level?

b) If all stockouts result in lost sales, what are the optimal reorder point and the safety stock level?

Solution

$$E(X) = 1040 \cdot \frac{1}{26} = 40, \quad \sigma_X = \frac{130}{\sqrt{26}} = 25.5, \quad EOQ = \left(\frac{2 \cdot 10 \cdot 1040}{(0.3)(60)}\right)^{1/2} = 107.5$$

a. $P(X \geq r) = \dfrac{(107.5)(0.3)(60)}{(50)(1040)} = 0.03721, \quad P(X \leq r) = 1 - 0.03721 = 0.96279$

$$P\left(Z \leq \frac{r-40}{25.5}\right) = 0.96279, \quad \frac{r-40}{25.5} = 1.78, \quad r = 85.39, \quad r - E(X) = 45.39$$

b. $P(X \geq r) = \dfrac{(107.5)(0.3)(60)}{(107.5)(0.3)(60) + (50)(1040)} = 0.02025, \quad P(X \leq r) = 1 - 0.02025 = 0.97975$

$$P\left(Z \leq \frac{r-40}{25.5}\right) = 0.97975, \quad \frac{r-40}{25.5} = 2.05, \quad r = 92.28, \quad r - E(X) = 52.28$$

4. A department store is trying to decide how many JP Desksquirt II printers to order. Since JP is about to come out with a new model in a few months, the store will order only a limited number of model IIs. The cost per printer is \$200, and each printer is sold for \$230. If any model IIs are still in stock when the next model comes out, they will be sold for \$150 apiece. The store estimates that the number of model IIs that will be demanded during the next few months (before the next model comes out) is normally distributed with a mean of 40 and a standard deviation of 20.

a) How many JP Descsquirt II printers should the department store order?

b) The store management is determined to satisfy all demand. If a customer wants a model II and there are none left, the store will special order the printer at an axtra cost (to the store) of \$25. How many JP Descsquirt II printers should the department store order in this case?

Solution

a. $c_u = 230 - 200 = 30, \quad c_o = 200 - 150 = 50, \quad \dfrac{c_u}{c_o + c_u} = \dfrac{3}{8}$

$P(D \leq q) = \dfrac{3}{8}, \quad P\left(Z \leq \dfrac{q-40}{20}\right) = \dfrac{3}{8}, \quad \dfrac{q-40}{20} = -0.32, \quad q = 33.6$

b. $c_u = 225 - 200 = 25, \quad c_o = 200 - 150 = 50, \quad \dfrac{c_u}{c_o + c_u} = \dfrac{1}{3}$

$P(D \leq q) = \dfrac{1}{3}, \quad P\left(Z \leq \dfrac{q-40}{20}\right) = \dfrac{1}{3}, \quad \dfrac{q-40}{20} = -0.43, \quad q = 31.4$

5. A hospital must order a certain drug from Daisy Drug Company. It costs \$500 to place an order. Annual demand for the drug is normally distributed with mean 10,000 and standard deviation 3000, and it costs \$5 to hold one unit in inventory for 1 year. Orders arrive 1 month after being placed. Assume that all shortages are backlogged.

a) Find the reorder policy for which 95% of all customer demand is met on time.

b) What is the probability that stockouts occur?

c) What is the average number of cycles per year during which a shortage occurs?

Solution

$$E(X) = 10000 \cdot \frac{1}{12} = 833.3, \quad \sigma_X = \frac{3000}{\sqrt{12}} = 866.03, \quad EOQ = \left(\frac{2(500)(10000)}{5}\right)^{1/2} = 1414.21$$

a. $NL\left(\frac{r - 833.3}{866.03}\right) = \frac{1414.21(1 - .95)}{866.03} = 0.081527, \quad \frac{r - 833.3}{866.03} = 1.01, \quad r = 1708.02$

b. $P(X \geq r) = P\left(\frac{X - 833.3}{866.03} \geq 1.01\right) = 1 - P\left(\frac{X - 833.3}{866.03} \leq 1.01\right) = 1 - 0.8438 = 0.1562$

c. $\frac{E(D)}{q} \cdot P(X \geq r) = \frac{10000}{141421}(0.1562) = 1.1045$