The Negative Cycles Polyhedron and Hardness of Testing Polyhedral Properties

Khaled Elbassioni

Max-Planck-Institut für Informatik, Saarbrücken, Germany

Abstract

For a given graph $G = (V, E)$ and a weight function on the edges $w : E \mapsto \Re$, we consider the polyhedron $P(G, w)$ of negative-weight flows on G, and get a complete characterization of the vertices and extreme directions of $P(G, w)$. For any CNF formula ϕ, we give a construction mapping ϕ into a weighted graph $(G(\phi), w)$ such that the existence of a negative-weight cycle of length greater than 2 in G is equivalent to the satisfiability of ϕ.

We use this characterization and construction to show that the following problems are NP-hard:

(i) generating all vertices of a 0/1-polyhedron;

(ii) checking if a given integral polyhedron is 0/1, or if a given polyhedron is half-integral;

(iii) approximating the maximum support of a vertex in a polyhedron in $\Re^n$ to within a factor of $\Omega(1/n)$;

(iv) approximating the vertex centroid (which the average of the vertices) of a given polyhedron in $\Re^n$ to within a distance of $n^{1/2-\delta}$ for any fixed $\delta > 0$.

This is based on our last joint work with Leo Khachiyan.