

**FACILITY
LOCATION
PROBLEM**

MATHEMATICAL MODEL

Description:

A firm has some possible locations to build facilities in. At each location the firm has a particular number of employees. Given a budget to build the facilities and a fixed construction cost (Which depends on size of facility), the problem is to minimize the total travelling distance by the employees.

Input :

loc = number of locations
 $nsiz$ = number of possible sizes of facility
 $cust$ = number of employees at each location
 $dist$ = distances between locations
 $cost$ = cost to build a facility of a particular size
 $size$ = possible sizes of facilities
 $budget$ = total allocated budget

Decision variables:

x array of $(loc) \times (nsiz)$

$$x(i, j) = \begin{cases} 1 & \text{if a facility of size } j \text{ is build at location } i \\ 0 & \text{otherwise} \end{cases}$$

D : array of size $(loc) \times (loc)$

$$D(i, j) = \begin{cases} 1 & \text{If employees at loction } i \text{ work at location } j \\ 0 & \text{otherwise} \end{cases}$$

Constraints:

$$totdist = \sum_{i=1}^{loc} \sum_{j=1}^{loc} dist(i, j) \times cost(i) \times D(i, j)$$

Gives the total distance travelled by all the employees.

$$\text{Forall } (i \text{ in } loc) \sum_{j=1}^{loc} D(i, j) = 1$$

Enforce the fact that employees at every location have to work at only 1 place.

$$\text{Forall } (i, j \text{ in } loc) D(i, j) \leq \sum_{l=1}^{nsiz} x(j, l)$$

Makes sure that if an employee is working at a location a facility is present there.

$$\text{Forall } (i \text{ in } loc) p(i) = \sum_{j=1}^{loc} D(j, i) \times cust(j)$$

$$\text{Forall } (i \text{ in } loc) \sum_{l=1}^{nsiz} x(i, l) \times size(l) \geq p(i)$$

A facility of sufficient size is build.

$$\sum_{i=1}^{loc} \sum_{j=1}^{nsiz} x(i, j) \times Cost(j) \leq budget$$

Budget constraint

$$\text{Forall } (i \text{ in } loc) \sum_{j \text{ in } nsiz} x(i, j) \leq 1$$

At every loction a maximum of only one facility is build.

Forall $(i \text{ in } loc)$ Forall $(j \text{ in } nsiz)$ $x(i, j)$ is binary

Forall $(i, j \text{ in } loc)$ $D(i, j)$ is binary.

x, D are binary variables.

Minimize($totdist$)

Minimizes the objective function.

CODE:

```
model loc
uses "mmxprs"; !gain access to the Xpress-Optimizer solver

parameters
locations=4
budget=50
numsize=4

end-parameters

declarations
loc=1..locations           !Number of locations
nsize=1..numsize          !Number of possible sizes of facilities
cust:array(loc) of integer !Number of employees at each location
dist:array(loc,loc) of integer !Distances between locations
cost:array(nsize) of integer !Cost of a facility of a particular size
size:array(nsize) of integer !Possible sizes of facilities

x: array(loc,nsize) of mpvar !Decision to build a facility of a particular size
D:array(loc,loc) of mpvar    !Dependency matrix

end-declarations

totdist:=sum(i,j in loc)dist(i,j)*cust(i)*D(i,j) !Total distance travelled by all the employees

forall(i in loc) sum(j in loc) D(i,j)=1

forall(i,j in loc) D(i,j)<= sum(l in nsize)x(j,l)

forall(i in loc)
p(i):=sum(j in loc) D(j,i)*cust(j)

forall(i in loc)
sum(l in nsize)x(i,l)*size(l)>=p(i)

sum(i in loc)sum(j in nsize) x(i,j)*cost(j)<=budget !Budget constraint
bu:=sum(i in loc)sum(j in nsize) x(i,j)*cost(j)
bup:=bu/budget*100

forall(i in loc)
sum(j in nsize)x(i,j)<= 1
```

```
forall(i in loc)forall(j in nsize) x(i,j) is_binary
forall(i,j in loc) D(i,j) is_binary
```

```
minimize(totdist)
```

```
!compute minimum total distance for a given budget
```

```
writeln("total travelling distance: ",getobjval)
writeln("\nloc No.of employees")
forall( i in loc) writeln(i," ", getsol(p(i)))
writeln("\nBudget used: ",getsol(bup),"%\n")
writeln(" Decision matrix")
forall( i in loc)
forall(j in nsize) writeln(getsol(x(i,j)))
writeln("\n Dependency matrix")
forall(i,j in loc) writeln(getsol(D(i,j)))
```

```
end-model
```

RESULT:

Total travelling distance: 62

loc	No.of employees
1	30
2	0
3	26
4	143

Budget used: 94%

Decision matrix

- 1
- 0
- 0
- 0
- 0
- 0
- 0
- 0
- 0
- 1
- 0
- 0
- 0
- 0
- 0
- 1
- 0

Dependency matrix

- 1
- 0
- 0
- 0
- 0
- 0
- 0
- 0
- 1
- 0
- 0
- 1
- 0
- 0
- 0
- 0
- 1