Probability of a Feasible Flow in a Stochastic Transportation Network

Santino Fanelli
Advisor: Dr. András Prékopa

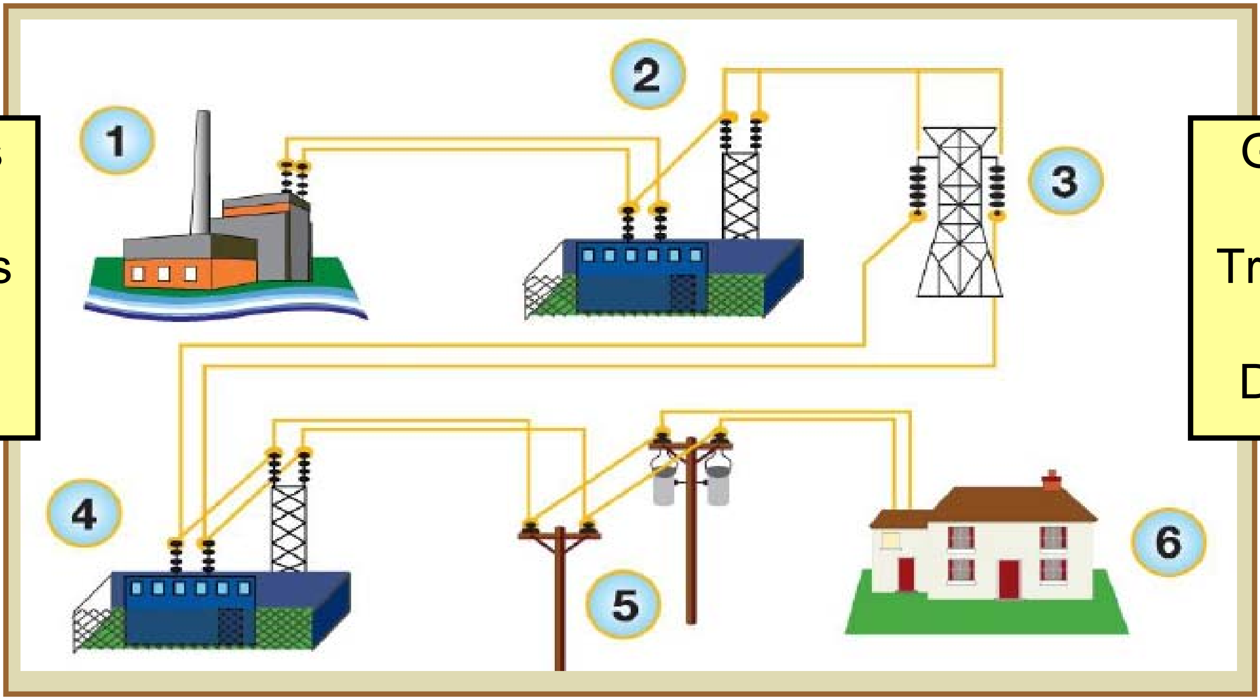
October 12, 2011

Outline

- Electric Utility Industry
- Problem Formulation
- Methods to Solve
- Numerical Examples

Electric Utility Industry
General Overview

Power Plants
↓
Poles & Wires
↓
Customers



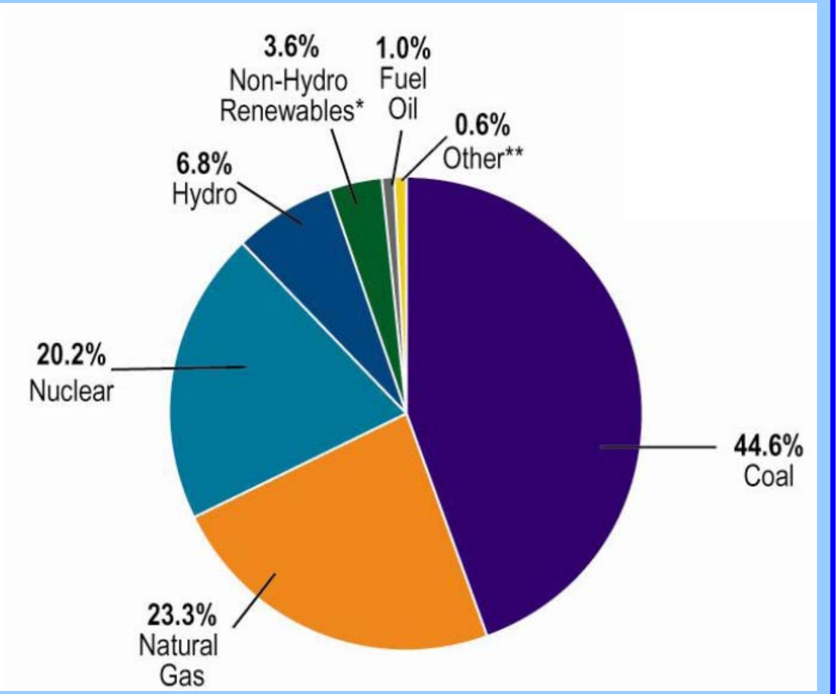
Generation
↓
Transmission
↓
Distribution

Principal objective: Maintain sufficient Generation and Transmission capacity to satisfy Customer Demand

Electric Utility Industry

Industry Statistics*

Generation



Transmission

Estimated Transmission Capacity Additions (Miles)						
2011	2012	2013	2014	2015	2016	Average
2,230	3,831	5,503	3,616	3,970	3,451	3,767

Distribution

Number of End Use Customers				
Year	Residential	Commercial	Industrial	Total
2003	117,280,481	16,550,646	713,221	134,544,348
2004	118,763,768	16,607,808	747,600	136,119,176
2005	120,760,839	16,872,458	733,862	138,367,159
2006	122,471,071	17,173,290	759,604	140,403,965
2007	123,949,916	17,377,969	793,767	142,121,652
2008	124,937,469	17,563,453	774,713	143,275,635
2009	125,177,175	17,562,366	757,519	143,497,060

U.S. Peak Demand (MW)				
2005	2006	2007	2008	2009
758,876	789,475	782,227	752,470	725,958

* Generation fuel mix from 2009. All data courtesy of Edison Electric Institute (EEI) and the U.S. Energy Information Administration (EIA)

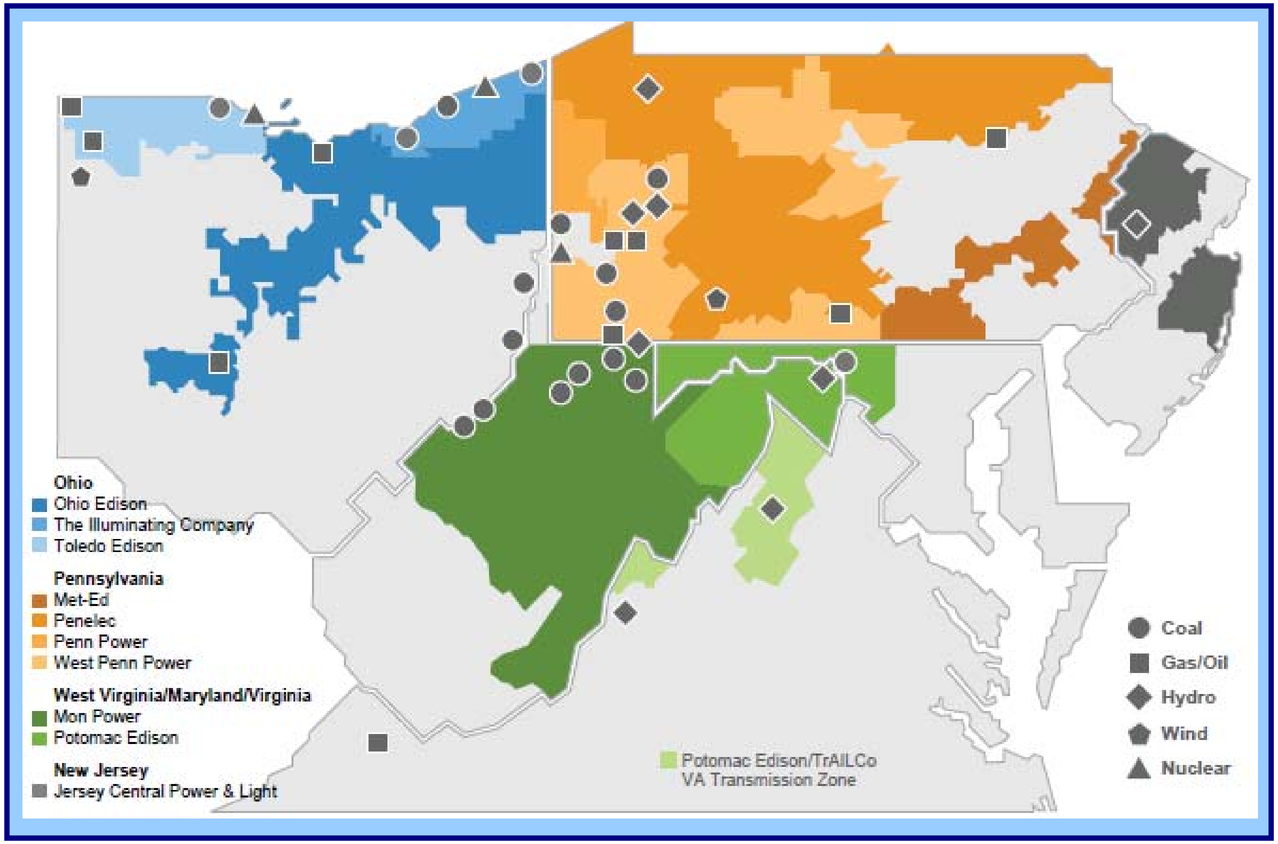
Electric Utility Industry
FirstEnergy Corp. – Company Overview

Generating Capacity

Fuel Type	Capacity (MW)	% Total
Coal	14,678	63.1%
Gas/Oil	2,195	9.4%
Hydro	1,832	7.9%
Wind	564	2.4%
Nuclear	3,991	17.2%
	23,260	100.0%

Transmission

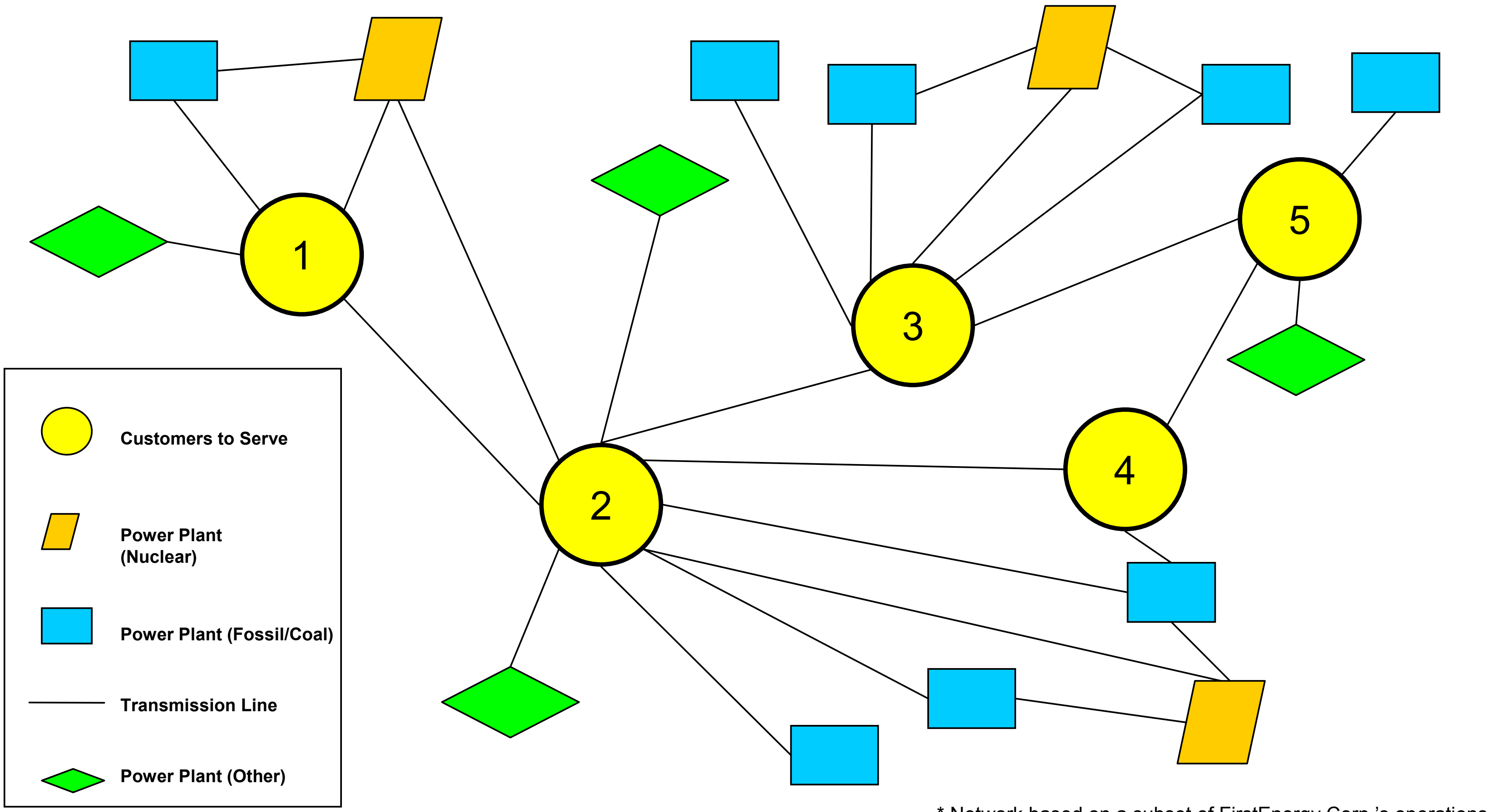
Over 20,000 miles



* Data courtesy of FirstEnergy Corp. www.firstenergycorp.com

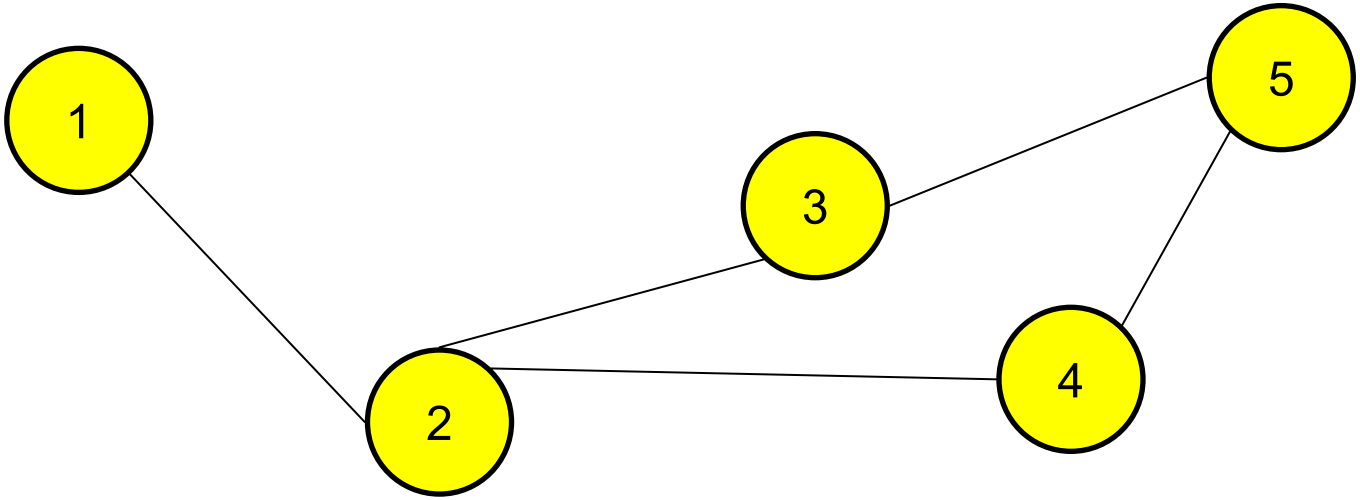
Problem Formulation

Stochastic Network *



* Network based on a subset of FirstEnergy Corp.'s operations

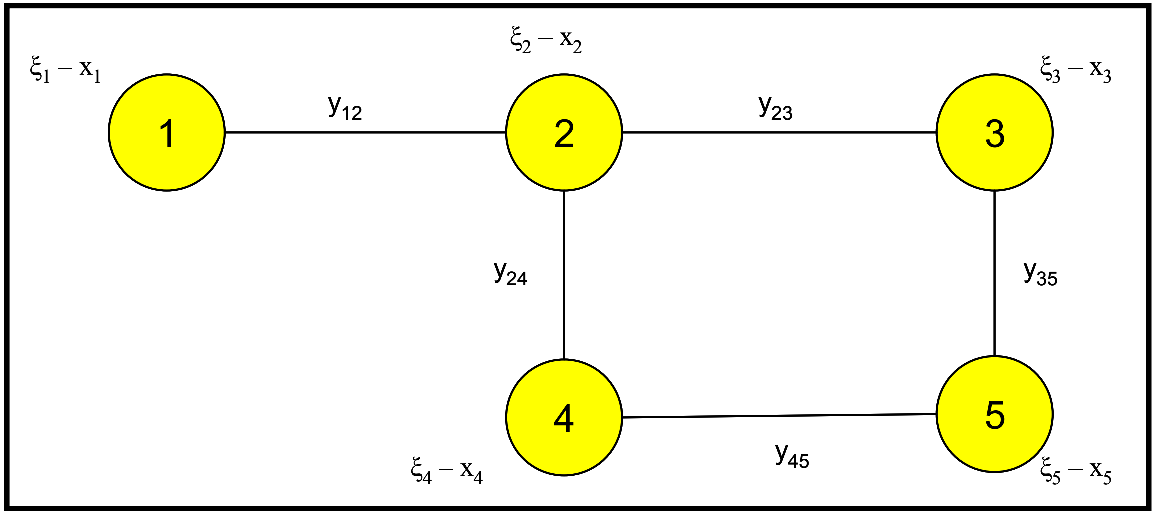
Problem Formulation
Stochastic Network (continued)



Each node in the condensed network above reflects net customer demand less available generating capacity

Problem Formulation

Stochastic Network (continued)



<u>Definitions</u>

y_{ij} = Transmission Capacity Between Node i and Node j

x_i = Random Generating Capacity at Node i

ξ_i = Random Local Demand at Node i

$d(i) = \xi_i - x_i$ = Net Demand (Supply) at Node i

$d(i) < 0$ → Surplus of Generation at Node i (excess power)

$d(i) > 0$ → Shortage of Generation at Node i (need power)

Problem Formulation

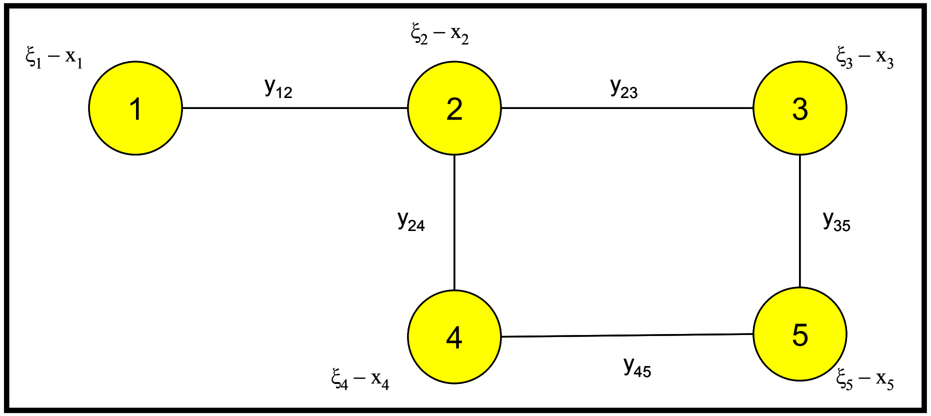
Gale-Hoffman Theorem *

Theorem – We are given a directed transportation network G = (N,E), a demand function d(i) for each node $i \in N$, and non-negative edge capacities y_{ij} for all $(i,j) \in E$. We also assume that the lower bound on the flow value along any edge in G is zero and that the total demand across all nodes is zero, i.e., $\sum_{i \in N} d(i) = 0$.

The network G admits a feasible flow, or equivalently, the demand function d(i) is feasible, if and only if, for every subset of demand nodes $S \subseteq N$, we have the inequality

$$d(S) \leq \sum_{(i,j) \in (S^C, S)} y_{ij}$$

where $\sum_{(i,j) \in (S^C, S)} y_{ij}$ is the total available capacity into S.

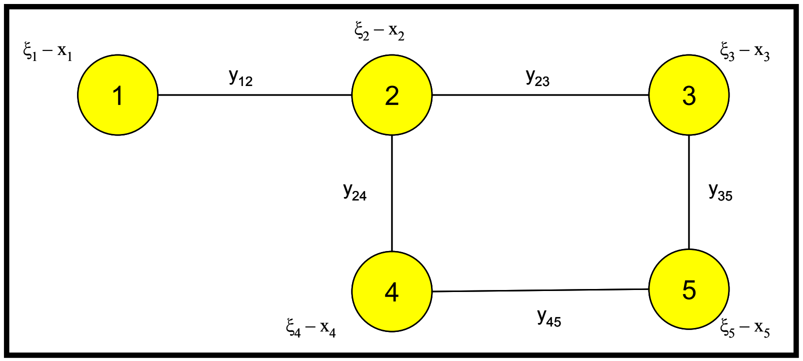


Thus, we can set up an inequality $d(S) \leq \sum_{(i,j) \in (S^C, S)} y_{ij}$ for all $S \subseteq N$.

* A directed network can be transformed to an undirected one without loss of generality. *Network Flows* (Ahuja, Magnanti, Orlin).

Problem Formulation

Gale-Hoffman Inequalities



The problem formulation begins with $2^n - 1$ inequalities, where n = the number of nodes in the network

(1)	$\xi_1 - x_1$					$\leq$	y_{12}				
(2)		$\xi_2 - x_2$				$\leq$	y_{12}	$+ y_{23}$	$+ y_{24}$		
(3)			$\xi_3 - x_3$			$\leq$		y_{23}		$+ y_{35}$	
(4)				$\xi_4 - x_4$		$\leq$			y_{24}		$+ y_{45}$
(5)					$\xi_5 - x_5$	$\leq$				y_{35}	$+ y_{45}$
(6)	$\xi_1 - x_1$	$+ \xi_2 - x_2$				$\leq$		y_{23}	$+ y_{24}$		
(7)	$\xi_1 - x_1$		$+ \xi_3 - x_3$			$\leq$	y_{12}	$+ y_{23}$		$+ y_{35}$	
(8)	$\xi_1 - x_1$			$+ \xi_4 - x_4$		$\leq$	y_{12}		$+ y_{24}$		$+ y_{45}$
(9)	$\xi_1 - x_1$				$+ \xi_5 - x_5$	$\leq$	y_{12}			y_{35}	$+ y_{45}$
(10)		$\xi_2 - x_2$	$+ \xi_3 - x_3$			$\leq$	y_{12}		$+ y_{24}$	$+ y_{35}$	
(11)		$\xi_2 - x_2$		$+ \xi_4 - x_4$		$\leq$	y_{12}	$+ y_{23}$			$+ y_{45}$
(12)		$\xi_2 - x_2$			$+ \xi_5 - x_5$	$\leq$	y_{12}	$+ y_{23}$	$+ y_{24}$	$+ y_{35}$	$+ y_{45}$
(13)			$\xi_3 - x_3$	$+ \xi_4 - x_4$		$\leq$		y_{23}	$+ y_{24}$	$+ y_{35}$	$+ y_{45}$
(14)			$\xi_3 - x_3$		$+ \xi_5 - x_5$	$\leq$		y_{23}			$+ y_{45}$
(15)				$\xi_4 - x_4$	$+ \xi_5 - x_5$	$\leq$			y_{24}	$+ y_{35}$	
(16)	$\xi_1 - x_1$	$+ \xi_2 - x_2$	$+ \xi_3 - x_3$			$\leq$			y_{24}	$+ y_{35}$	
(17)	$\xi_1 - x_1$	$+ \xi_2 - x_2$		$+ \xi_4 - x_4$		$\leq$		y_{23}			$+ y_{45}$
(18)	$\xi_1 - x_1$	$+ \xi_2 - x_2$			$+ \xi_5 - x_5$	$\leq$		y_{23}	$+ y_{24}$	$+ y_{35}$	$+ y_{45}$
(19)	$\xi_1 - x_1$		$+ \xi_3 - x_3$	$+ \xi_4 - x_4$		$\leq$	y_{12}	$+ y_{23}$	$+ y_{24}$	$+ y_{35}$	$+ y_{45}$
(20)	$\xi_1 - x_1$		$+ \xi_3 - x_3$		$+ \xi_5 - x_5$	$\leq$	y_{12}	$+ y_{23}$			$+ y_{45}$
(21)	$\xi_1 - x_1$			$+ \xi_4 - x_4$	$+ \xi_5 - x_5$	$\leq$	y_{12}		$+ y_{24}$	$+ y_{35}$	
(22)		$\xi_2 - x_2$	$+ \xi_3 - x_3$	$+ \xi_4 - x_4$		$\leq$	y_{12}			$+ y_{35}$	$+ y_{45}$
(23)		$\xi_2 - x_2$	$+ \xi_3 - x_3$		$+ \xi_5 - x_5$	$\leq$	y_{12}		$+ y_{24}$		$+ y_{45}$
(24)		$\xi_2 - x_2$		$+ \xi_4 - x_4$	$+ \xi_5 - x_5$	$\leq$	y_{12}	$+ y_{23}$		$+ y_{35}$	
(25)			$\xi_3 - x_3$	$+ \xi_4 - x_4$	$+ \xi_5 - x_5$	$\leq$		y_{23}	$+ y_{24}$		
(26)	$\xi_1 - x_1$	$+ \xi_2 - x_2$	$+ \xi_3 - x_3$	$+ \xi_4 - x_4$		$\leq$				y_{35}	$+ y_{45}$
(27)	$\xi_1 - x_1$		$+ \xi_3 - x_3$	$+ \xi_4 - x_4$	$+ \xi_5 - x_5$	$\leq$	y_{12}	$+ y_{23}$	$+ y_{24}$		
(28)	$\xi_1 - x_1$	$+ \xi_2 - x_2$		$+ \xi_4 - x_4$	$+ \xi_5 - x_5$	$\leq$		y_{23}		$+ y_{35}$	
(29)	$\xi_1 - x_1$	$+ \xi_2 - x_2$	$+ \xi_3 - x_3$		$+ \xi_5 - x_5$	$\leq$			y_{24}		$+ y_{45}$
(30)		$\xi_2 - x_2$	$+ \xi_3 - x_3$	$+ \xi_4 - x_4$	$+ \xi_5 - x_5$	$\leq$	y_{12}				
(31)	$\xi_1 - x_1$	$+ \xi_2 - x_2$	$+ \xi_3 - x_3$	$+ \xi_4 - x_4$	$+ \xi_5 - x_5$	$\leq$	0				

Problem Formulation

Elimination of Redundant Inequalities

Algorithm*

0. Let b(H) = 1 and e(H) = 0 for all $H \subseteq N$, where H is non-empty
1. Choose a non-empty subset $H \subseteq N$ such that b(H) = 1 and e(H) = 0. (If no such subset exists, then STOP).
2. Let $T \subseteq N \setminus H$ be a maximal subset such that there is no arc between T and H
3. Let b(V) = 0 for all $V \subseteq H \cup T$, where $V \cap T$ and $V \cap H$ are both non-empty.
4. Let e(H) = 1 and return to Step 1.

As an example, consider initial inequalities (1), (3), and (7). Since (7) = (1) + (3), then (7) is redundant and can be eliminated.

(1)	$\xi_1 - x_1$		$\leq$	y_{12}
(3)		$\xi_3 - x_3$	$\leq$	$y_{23} + y_{35}$
(7)	$\xi_1 - x_1$	$+ \ \xi_3 - x_3$	$\leq$	$y_{12} + y_{23} + y_{35}$

Inequality	Set H	b(H)	e(H)
(1)	{1}	1	0
(2)	{2}	1	0
(3)	{3}	1	0
(4)	{4}	1	0
(5)	{5}	1	0
(6)	{1,2}	1	0
(7)	{1,3}	1	0
(8)	{1,4}	1	0
(9)	{1,5}	1	0
(10)	{2,3}	1	0
(11)	{2,4}	1	0
(12)	{2,5}	1	0
(13)	{3,4}	1	0
(14)	{3,5}	1	0
(15)	{4,5}	1	0
(16)	{1,2,3}	1	0
(17)	{1,2,4}	1	0
(18)	{1,2,5}	1	0
(19)	{1,3,4}	1	0
(20)	{1,3,5}	1	0
(21)	{1,4,5}	1	0
(22)	{2,3,4}	1	0
(23)	{2,3,5}	1	0
(24)	{2,4,5}	1	0
(25)	{3,4,5}	1	0
(26)	{1,2,3,4}	1	0
(27)	{1,3,4,5}	1	0
(28)	{1,2,4,5}	1	0
(29)	{1,2,3,5}	1	0
(30)	{2,3,4,5}	1	0
(31)	{1,2,3,4,5}	1	0

* Algorithm courtesy of Prékopa and Boros (*On the Existence of A Feasible Flow in A Stochastic Transportation Network*)

Problem Formulation

Elimination of Redundant Inequalities - Results

(1) $\xi_1 - x_1 \le y_{12}$

(2) $\xi_2 - x_2 \le y_{12} + y_{23} + y_{24}$

(3) $\xi_3 - x_3 \le y_{23} + y_{35}$

(4) $\xi_4 - x_4 \le y_{24} + y_{45}$

(5) $\xi_5 - x_5 \le y_{35} + y_{45}$

(6) $\xi_1 - x_1 + \xi_2 - x_2 \le y_{23} + y_{24}$

(7) $\xi_2 - x_2 + \xi_3 - x_3 \le y_{12} + y_{24} + y_{35}$

(8) $\xi_2 - x_2 + \xi_4 - x_4 \le y_{12} + y_{23} + y_{45}$

(9) $\xi_3 - x_3 + \xi_5 - x_5 \le y_{23} + y_{45}$

(10) $\xi_4 - x_4 + \xi_5 - x_5 \le y_{24} + y_{35}$

(11) $\xi_1 - x_1 + \xi_2 - x_2 + \xi_3 - x_3 \le y_{24} + y_{35}$

(12) $\xi_1 - x_1 + \xi_2 - x_2 + \xi_4 - x_4 \le y_{23} + y_{45}$

(13) $\xi_2 - x_2 + \xi_3 - x_3 + \xi_4 - x_4 \le y_{12} + y_{35} + y_{45}$

(14) $\xi_2 - x_2 + \xi_3 - x_3 + \xi_5 - x_5 \le y_{12} + y_{24} + y_{45}$

(15) $\xi_2 - x_2 + \xi_4 - x_4 + \xi_5 - x_5 \le y_{12} + y_{23} + y_{35}$

(16) $\xi_3 - x_3 + \xi_4 - x_4 + \xi_5 - x_5 \le y_{23} + y_{24}$

(17) $\xi_1 - x_1 + \xi_2 - x_2 + \xi_3 - x_3 + \xi_4 - x_4 \le y_{35} + y_{45}$

(18) $\xi_1 - x_1 + \xi_2 - x_2 + \xi_4 - x_4 + \xi_5 - x_5 \le y_{23} + y_{35}$

(19) $\xi_1 - x_1 + \xi_2 - x_2 + \xi_3 - x_3 + \xi_5 - x_5 \le y_{24} + y_{45}$

(20) $\xi_2 - x_2 + \xi_3 - x_3 + \xi_4 - x_4 + \xi_5 - x_5 \le y_{12}$

(21) $\xi_1 - x_1 + \xi_2 - x_2 + \xi_3 - x_3 + \xi_4 - x_4 + \xi_5 - x_5 \le 0$

Inequality	Set H	FINAL	
		b(H)	e(H)
(1)	{1}	1	1
(2)	{2}	1	1
(3)	{3}	1	1
(4)	{4}	1	1
(5)	{5}	1	1
(6)	{1,2}	1	1
(7)	{1,3}	0	0
(8)	{1,4}	0	0
(9)	{1,5}	0	0
(10)	{2,3}	1	1
(11)	{2,4}	1	1
(12)	{2,5}	0	0
(13)	{3,4}	0	0
(14)	{3,5}	1	1
(15)	{4,5}	1	1
(16)	{1,2,3}	1	1
(17)	{1,2,4}	1	1
(18)	{1,2,5}	0	0
(19)	{1,3,4}	0	0
(20)	{1,3,5}	0	0
(21)	{1,4,5}	0	0
(22)	{2,3,4}	1	1
(23)	{2,3,5}	1	1
(24)	{2,4,5}	1	1
(25)	{3,4,5}	1	1
(26)	{1,2,3,4}	1	1
(27)	{1,3,4,5}	0	0
(28)	{1,2,4,5}	1	1
(29)	{1,2,3,5}	1	1
(30)	{2,3,4,5}	1	1
(31)	{1,2,3,4,5}	1	1

As a result of the elimination of redundant inequalities, the number of inequalities has been reduced from 31 to 21

Methods to Solve

Multivariate Normal Distribution

Assumption: The relevant data in this problem are normally distributed random variables

Argument #1: Net demand values $d(i) = \xi_i - x_i$ are random variables

- Available generating capacity at an individual power plant is not constant
- Individual customer demand is not constant

Argument #2: Net demand values $d(i) = \xi_i - x_i$ are identically distributed

- Network represents contiguous geographic area
- Nodes in the network represent similar socio-economic conditions, on average

Argument #3: Net demand values $d(i) = \xi_i - x_i$ are independent

- Power plants generally operate as stand-alone entities
- A customer's individual demand profile is unique

By the Central Limit Theorem, the average net demand values d(i) can be approximated by the normal distribution.

Methods to Solve
Multivariate Normal Distribution (continued)

General Case: Suppose we have m nodes in the network and k inequalities remaining after the elimination procedure.

- Each remaining inequality can be written as

$$\eta_j = \sum_{i=1}^{m} a_{ji} d(i) \leq b_j$$

for $j = 1, \ldots, k$
where $a_{ji} = 0$ or 1

- Standardizing the above, we have

$$\eta_j \leq b_j \rightarrow \frac{\eta_j - \mu_j}{\sigma_j} \leq \frac{b_j - \mu_j}{\sigma_j}$$

$$\eta_j^{'} = \frac{\eta_j - \mu_j}{\sigma_j} \rightarrow \boldsymbol{\eta}' = [\eta_1^{'}, \eta_2^{'}, \ldots, \eta_k^{'}]^T$$

$$b_j^{'} = \frac{b_j - \mu_j}{\sigma_j} \rightarrow \mathbf{b}' = [b_1^{'}, b_2^{'}, \ldots, b_k^{'}]^T$$

Our problem, then, is to find the multivariate normal probability $P(\boldsymbol{\eta}' \leq \mathbf{b}')$ that all k remaining inequalities are satisfied simultaneously.

Methods to Solve

Probability Bounding

Suppose we have k events, $A_1,\dots,A_k$.

$$P(A_1 \cap A_2 \cap \dots \cap A_k) \le P(A_1 \cup A_2 \cup \dots \cup A_k) \qquad (1)$$

$$P(A_1 \cup A_2 \cup \dots \cup A_k) \le \sum_{i=1}^{k} P(A_i) \qquad (2)$$

$$P(A_1 \cap A_2 \cap \dots \cap A_k) = 1 - P(A_1^C \cup A_2^C \cup \dots \cup A_k^C) \qquad (3)^*$$

Using (1), (2), and (3) above, we have

$$\sum_{i=1}^{k} P(A_i) - (k-1) \le P(A_1 \cap A_2 \cap \dots \cap A_k) \le P(A_1 \cup A_2 \cup \dots \cup A_k) \le \sum_{i=1}^{k} P(A_i) \qquad (4)$$

Inequality (4) provides bounds on the probability of the intersection and union of the k events.

* Inequality (3) is also known as DeMorgan's Law

Methods to Solve
Hunter's Upper Bound

$$P(A_1 \cup A_2 \cup \ldots \cup A_k) \leq \sum_{i=1}^{k} P(A_i) - \sum_{(i,j) \in T} P(A_i \cap A_j)$$

- T is a maximum spanning tree of the complete graph of k events
- The complete graph is depicted by k nodes, (one for each remaining inequality), and an undirected edge connecting each pair of nodes in the graph
- Edges of T are denoted by (i,j)
- The weight of the edge (i,j) is equal to the joint probability p_{ij} between events i and j
- Kruskal's Algorithm is used to find T

Hunter's Upper Bound on the probability of the union is tighter than Inequality (4). It utilizes only the individual and pairwise joint probabilities.

Methods to Solve

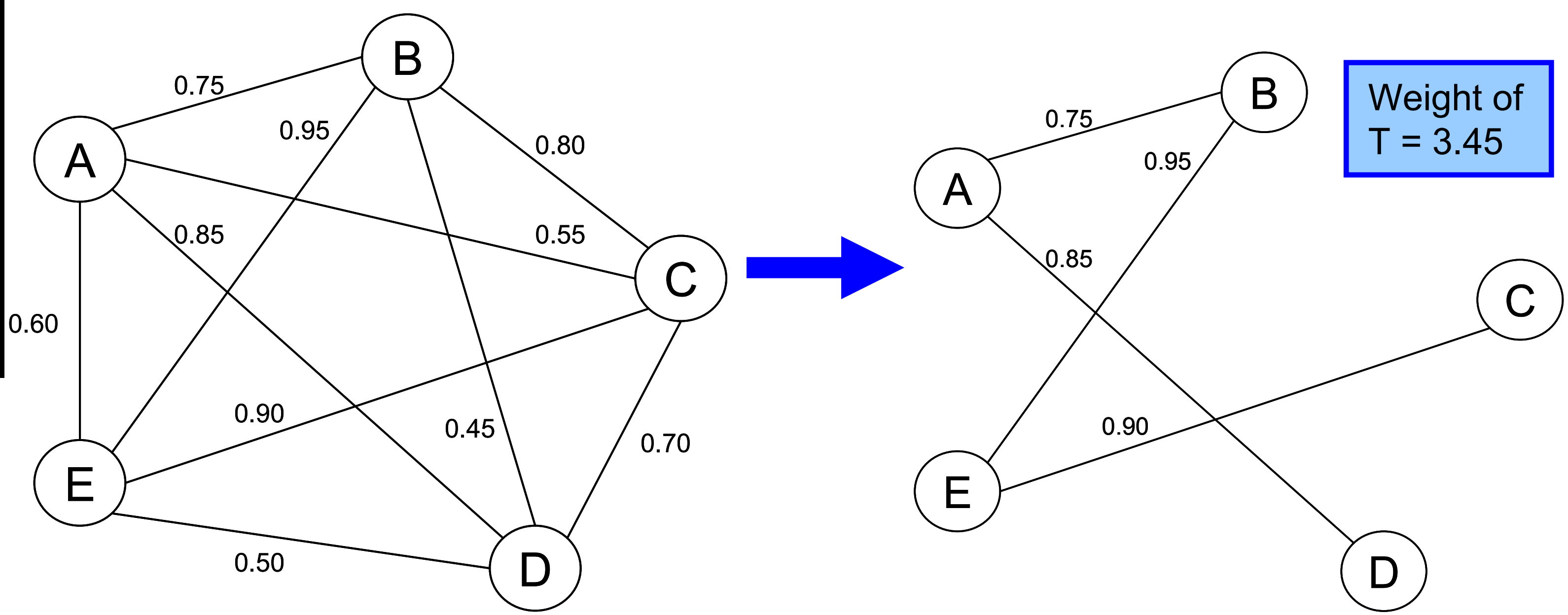
Hunter's Upper Bound (continued)

Kruskal's Algorithm

a. Select the heaviest edge in the graph and add it to the tree.

b. Find the next heaviest edge that has not yet been selected and does not create a cycle. Add it to the tree. Repeat step (b) until the tree has n-1 edges.

Edge	Weight	Rank
(A,B)	0.75	5
(A,C)	0.55	8
(A,D)	0.85	3
(A,E)	0.60	7
(B,C)	0.80	4
(B,D)	0.45	10
(B,E)	0.95	1
(C,D)	0.70	6
(C,E)	0.90	2
(D,E)	0.50	9



Methods to Solve

Boolean Probability Bounding

- Considers all 2^k possible combinations of the k remaining events
- Focuses on those combinations whose probabilities are known, e.g., individual and pairwise joint probabilities for the k events
- Utilizes Linear Programming to bound the probability of the union

Boolean Probability Bounding

For each combination I with a known probability and all combinations J, we establish the matrix A:

$$a_{IJ} = \begin{cases} 1 & \text{if } I \subset J \text{ for combinations I and J} \\ 0 & \text{otherwise.} \end{cases}$$

- $\mathbf{A}$ is $m \times n$
- Vector $\mathbf{b}_{m \times 1}$ = known probabilities
- Vector $\mathbf{x}_{n \times 1}$ = probability that each combination of events is true

→ System of equations $\mathbf{Ax} = \mathbf{b}$

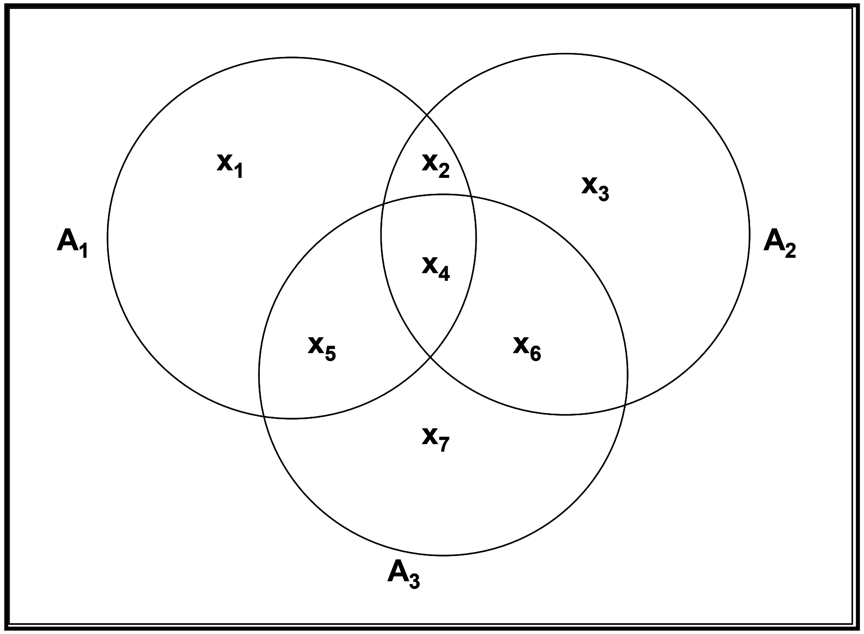
→

$$\begin{aligned} \text{max (min)} \quad & \sum_{i=1}^{n} x_i \\ \text{s.t.} \quad & \mathbf{Ax} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

Solving the above LP as minimization (maximization) provides a lower (upper) bound on the probability of the union of the k events

Methods to Solve

Boolean Probability Bounding – Example



$P(A_1) = 0.800$	$P(A_1 \cap A_2) = 0.725$
$P(A_2) = 0.850$	$P(A_1 \cap A_3) = 0.685$
$P(A_3) = 0.750$	$P(A_2 \cap A_3) = 0.645$

$$\max (\min) \quad \sum_{i=1}^{7} x_i$$

$$\begin{aligned}
\text{s.t.} \quad & x_1 + x_2 + x_4 + x_5 = 0.800 \\
& x_2 + x_3 + x_4 + x_6 = 0.850 \\
& x_4 + x_5 + x_6 + x_7 = 0.750 \\
& x_2 + x_4 = 0.725 \\
& x_4 + x_5 = 0.685 \\
& x_4 + x_6 = 0.645 \\
& x_i \geq 0 \text{ for all } i
\end{aligned}$$

Optimum Values *

Minimization: 0.955

Maximization: 0.990

$$0.955 \leq P(A_1 \cup A_2 \cup A_3) \leq 0.990$$

* Linear program solved using AMPL Cplex solver

Numerical Examples
General Assumptions *

<u>Customer Demand</u> – Based on actual hourly data from June – August 2010 (2,208 hours)

<u>Generating Capacity</u> – Randomly generated hourly values based on actual installed capacity values and assumed normal distribution (2,208 hours)

<u>Transmission Capacity</u> – Assumed to be constant based on actual current levels

Three numerical examples were studied. Only the hourly Generating Capacity changed between scenarios. The hourly Customer Demand and Transmission Capacity did not change.

* Customer Demand, Generating Capacity and Transmission Capacity courtesy of FirstEnergy Corp.

Numerical Examples

Generating Capacity Assumptions

I. Total Generating Fleet*

FirstEnergy Historical Annual Generating Capacity by Fuel Type							
Fuel Type	**2006**	**2007**	**2008**	**2009**	**2010**	**Average**	**MAX**
Nuclear	87%	89%	93%	84%	88%	88%	93%
Coal	89%	80%	84%	72%	76%	80%	89%
Other *	69%	71%	64%	31%	52%	57%	71%

* Other consists primarily of natural gas, oil, hydro, and wind

II. Portion of Generating Fleet Applicable to Network

Fuel Type	Available Capacity (MW)	Scenario #1		Scenario #2		Scenario #3	
		Average (MW)	Average Factor	Average (MW)	Average Factor	Average (MW)	Average Factor
Nuclear	4,200	3,502	83.4%	3,932	93.6%	3,932	93.6%
Coal	8,110	6,652	82.0%	7,217	89.0%	7,575	93.4%
Other	2,665	1,524	57.2%	1,752	65.7%	2,487	93.3%
Total	14,975	11,678	78.0%	12,901	86.2%	13,994	93.4%

Numerical Examples
Additional Elimination Procedure

Elimination by Upper Bounds

- Once the hourly Customer Demand and Generating Capacity values are obtained, we can determine the LHS of each remaining inequality. (RHS is assumed constant).
- Determine the maximum net demand values d(i) for each node i and plug them in to the LHS of each remaining inequality
- Those inequalities whose LHS still does not exceed the RHS can be eliminated because even in the extreme case where demand is maximized, the inequality is still satisfied

This additional inequality elimination procedure will reduce the number of remaining inequalities.

Numerical Example #1

Summary of Results *

Node	Demand ξ_i			Generating Capacity x_i			$d(i) = \xi_i - x_i$
	Min	Max	Average	Min	Max	Average	Max
1	849	2,006	1,353	0	1,663	1,348	2,006
2	1,827	5,024	3,249	0	7,921	6,157	5,024
3	1,503	4,043	2,562	0	3,280	2,744	4,043
4	319	964	590	0	895	735	964
5	1,028	3,298	1,868	0	1,085	694	3,298

Incorporating the maximum net demand values results in the elimination of 9 more inequalities, from 21 down to 12.

Multivariate Normal Probability Distribution:

$$P(\boldsymbol{\eta}' \leq \mathbf{b}') = 0.818375$$

Hunter's Bound:

$$0.755406 \leq P(A_1 \cap A_2 \cap \ldots \cap A_{12}) \leq 1$$

Boolean Probability Bounding:

$$P(A_1 \cup A_2 \cup \ldots \cup A_{12}) = 1$$

$$P(A_1 \cap A_2 \cap \ldots \cap A_{12}) \leq 1$$

Probability bounding techniques yield consistent, yet trivial, results. Result from Multivariate Normal Probability Distribution method is within the range offered by the probability bounding.

* Additional details are provided in the Appendix. Multivariate Normal Probability solved using NORTEST solver (Szantai, Budapest University).

Numerical Example #2

Summary of Results *

Node	Demand ξ_i			Generating Capacity x_i			$d(i) = \xi_i - x_i$
	Min	Max	Average	Min	Max	Average	Max
1	849	2,006	1,353	188	1,665	1,462	1,818
2	1,827	5,024	3,249	1,939	8,050	6,922	3,085
3	1,503	4,043	2,562	652	3,280	2,989	3,390
4	319	964	590	0	895	795	964
5	1,028	3,298	1,868	0	1,085	733	3,298

Incorporating the maximum net demand values results in the elimination of 11 more inequalities, from 21 down to 10.

Multivariate Normal Probability Distribution:

$$P(\boldsymbol{\eta}' \leq \mathbf{b}') = 0.930576$$

Hunter's Bound:

$$0.910527 \leq P(A_1 \cap A_2 \cap \ldots \cap A_{10}) \leq 1$$

Boolean Probability Bounding:

$$P(A_1 \cup A_2 \cup \ldots \cup A_{10}) = 1$$

$$P(A_1 \cap A_2 \cap \ldots \cap A_{10}) \leq 1$$

Probability bounding techniques yield a tighter range than Example #1. Result from Multivariate Normal Probability Distribution method is higher than Example #1.

* Additional details are provided in the Appendix. Multivariate Normal Probability solved using NORTEST solver (Szantai, Budapest University).

Numerical Example #3

Summary of Results *

Node	Demand ξ_i			Generating Capacity x_i			$d(i) = \xi_i - x_i$
	Min	Max	Average	Min	Max	Average	Max
1	849	2,006	1,353	365	1,665	1,561	1,641
2	1,827	5,024	3,249	3,475	8,050	7,516	1,549
3	1,503	4,043	2,562	650	3,280	3,074	3,392
4	319	964	590	0	895	830	964
5	1,028	3,298	1,868	304	1,085	1,013	2,994

Incorporating the maximum net demand values results in the elimination of 13 more inequalities, from 21 down to 8.

Multivariate Normal Probability Distribution:

$$P(\boldsymbol{\eta}' \leq \mathbf{b}') \leq 0.983261$$

Hunter's Bound:

$$0.981157 \leq P(A_1 \cap A_2 \cap \ldots \cap A_8) \leq 1$$

Boolean Probability Bounding:

$$P(A_1 \cup A_2 \cup \ldots \cup A_8 = 1$$

$$P(A_1 \cap A_2 \cap \ldots \cap A_8) \leq 1$$

Probability bounding techniques yield the tightest range of all the Examples. Similarly, the result from the Multivariate Normal Probability Distribution method is also the highest.

* Additional details are provided in the Appendix. Multivariate Normal Probability solved using NORTEST solver (Szantai, Budapest University).

Numerical Examples
Summary and Conclusions

Example	Avg. Gen. Capacity	Estimated Probability	Lower Bound	Upper Bound
1	78.0%	0.818375	0.755406	1.000000
2	86.2%	0.930576	0.910527	1.000000
3	93.4%	0.983261	0.981157	1.000000

- Probability of a feasible flow increases with more aggressive assumptions for average Generating Capacity
- In all Examples, the probability bounding techniques were consistent. Though, the upper bounds on the probability of the union of the remaining inequalities was trivial in all cases.
- In all Examples, the result from the Multivariate Normal Distribution method was within the range provided by the probability bounding methods

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Questions

Appendix

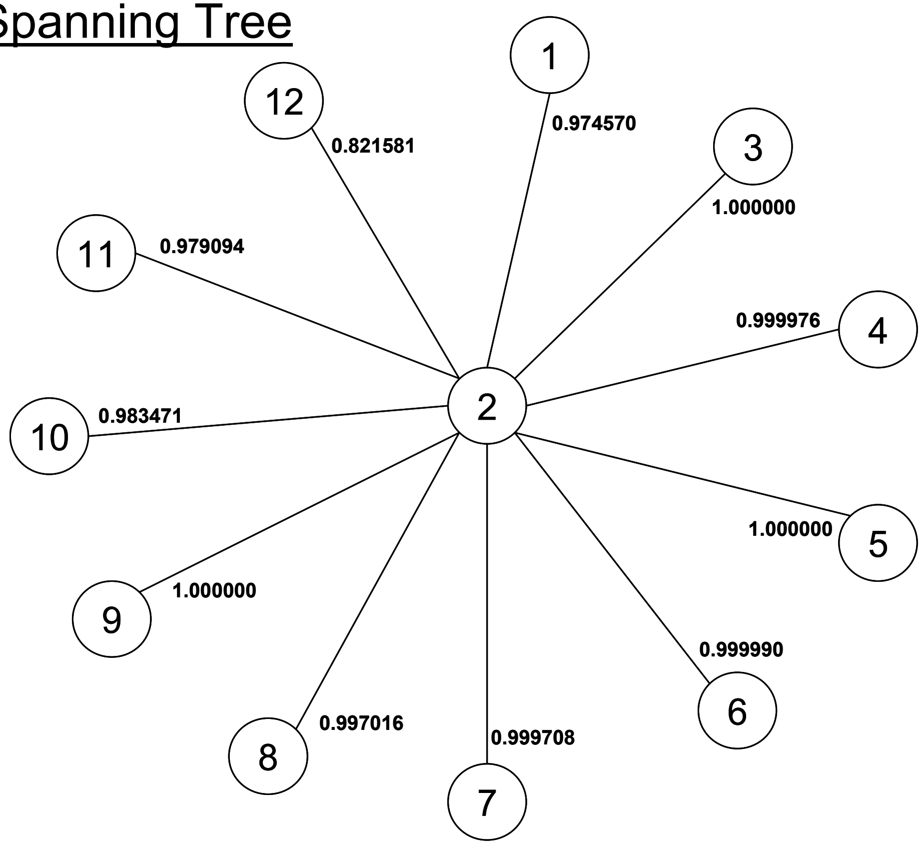
Numerical Example #1
Additional Details – Hunter's Upper Bound

Individual Probabilities

Inequality	p_i
1	0.974570
2	1.000000
3	1.000000
4	0.999976
5	1.000000
6	0.999990
7	0.999708
8	0.997016
9	1.000000
10	0.983471
11	0.979094
12	0.821581
	11.755406

Maximum Spanning Tree

No.	Edge	Weight
1	(2,3)	1.000000
2	(2,5)	1.000000
3	(2,9)	1.000000
4	(2,6)	0.999990
5	(2,4)	0.999976
6	(2,7)	0.999708
7	(2,8)	0.997016
8	(2,10)	0.983471
9	(2,11)	0.979094
10	(1,2)	0.974570
11	(2,12)	0.821581
		10.755406



$$P(A_1 \cap A_2 \cap \ldots \cap A_{12}) \geq \sum_{i=1}^{12} P(A_i) - (k-1)$$

$$P(A_1 \cap A_2 \cap \ldots \cap A_{12}) \geq 11.755406 - (12-1)$$

$$P(A_1 \cap A_2 \cap \ldots \cap A_{12}) \geq 0.755406$$

$$P(A_1 \cup A_2 \cup \ldots \cup A_k) \leq 11.755406 - 10.755406 = 1$$

Numerical Example #1

Additional Details – Boolean Probability Bounding

Excerpt from RHS vector **b**

$$\mathbf{b} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_{10} \\ p_{11} \\ p_{12} \\ p_{1,2} \\ p_{1,3} \\ p_{1,4} \\ \vdots \\ p_{10,11} \\ p_{10,12} \\ p_{11,12} \end{bmatrix} = \begin{bmatrix} 0.974570 \\ 1.000000 \\ 1.000000 \\ \vdots \\ 0.983471 \\ 0.979094 \\ 0.821581 \\ 0.974570 \\ 0.974570 \\ 0.974566 \\ \vdots \\ 0.977322 \\ 0.821581 \\ 0.821581 \end{bmatrix}$$

LP Formulation

m = 78 (12 individual and 66 pairs)

$n = 2^{12} - 1 = 4{,}095$

A is a 78 x 4,095 matrix

$$\text{max (min)} \quad \sum_{i=1}^{4,095} x_i$$

$$\text{s.t.} \quad \mathbf{Ax} = \mathbf{b}$$

$$\mathbf{x} \geq \mathbf{0}$$

LP solved using MOSEL

max = min = 1

Numerical Example #2

Additional Details – Hunter's Upper Bound

Individual Probabilities

Inequality	p_i
1	0.981389
2	1.000000
3	1.000000
4	1.000000
5	0.999984
6	0.999790
7	1.000000
8	0.997411
9	0.996190
10	0.935763
	9.910527

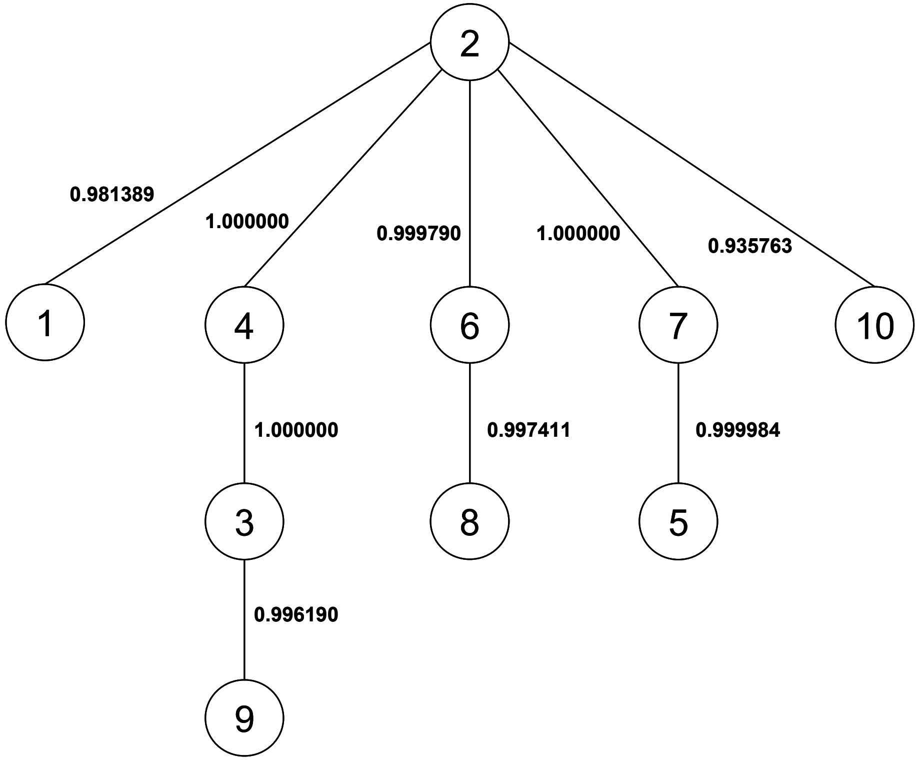
$$P(A_1 \cap A_2 \cap \ldots \cap A_{10}) \geq \sum_{i=1}^{10} P(A_i) - (k-1)$$

$$P(A_1 \cap A_2 \cap \ldots \cap A_{10}) \geq 9.910527 - (10-1)$$

$$P(A_1 \cap A_2 \cap \ldots \cap A_{10}) \geq 0.910527$$

Maximum Spanning Tree

No.	Edge	Weight
1	(2,4)	1.000000
2	(2,7)	1.000000
3	(3,4)	1.000000
4	(5,7)	0.999984
5	(2,6)	0.999790
6	(6,8)	0.997411
7	(3,9)	0.996190
8	(1,2)	0.981389
9	(2,10)	0.935763
		8.910527



$$P(A_1 \cup A_2 \cup \ldots \cup A_{10}) \leq 9.910527 - 8.910527 = 1$$

Numerical Example #2

Additional Details – Boolean Probability Bounding

Excerpt from RHS vector **b**

$$
\mathbf{b} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_8 \\ p_9 \\ p_{10} \\ p_{1,2} \\ p_{1,3} \\ p_{1,4} \\ \vdots \\ p_{8,9} \\ p_{8,10} \\ p_{9,10} \end{bmatrix} = \begin{bmatrix} 0.981389 \\ 1.000000 \\ 1.000000 \\ \vdots \\ 0.997411 \\ 0.996190 \\ 0.935763 \\ 0.981389 \\ 0.981389 \\ 0.981389 \\ \vdots \\ 0.995991 \\ 0.935763 \\ 0.935763 \end{bmatrix}
$$

LP Formulation

$m = 55$ (10 individual and 45 pairs)

$n = 2^{10} - 1 = 1{,}023$

A is a 55 x 1,023 matrix

$$
\begin{aligned}
\text{max (min)} \quad & \sum_{i=1}^{1{,}023} x_i \\
\text{s.t.} \quad & \mathbf{Ax} = \mathbf{b} \\
& \mathbf{x} \geq \mathbf{0}
\end{aligned}
$$

LP solved using MOSEL

max = min = 1

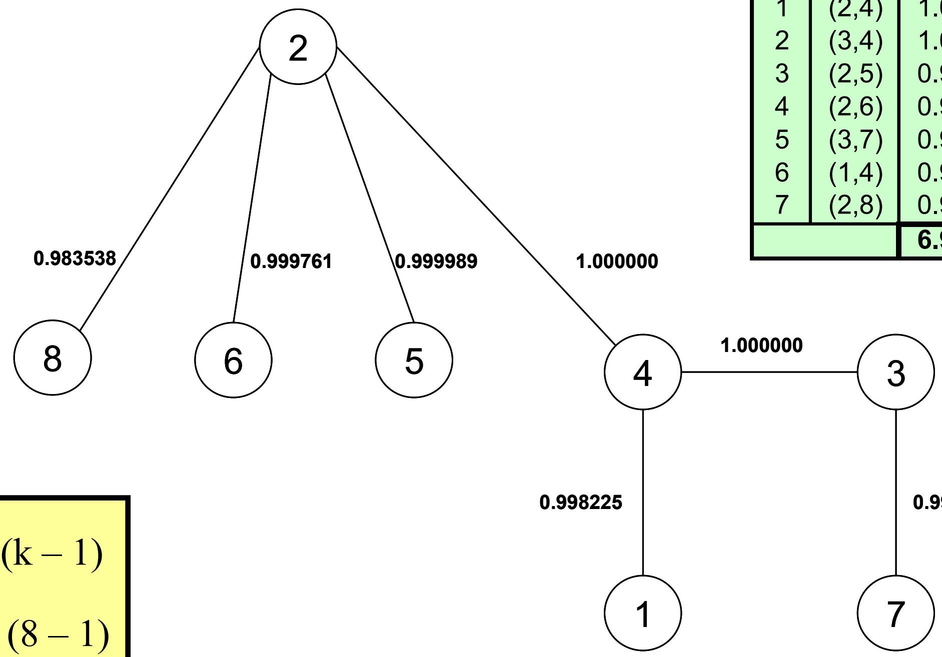
Numerical Example #3

Additional Details – Hunter's Upper Bound

Individual Probabilities

Inequality	p_i
1	0.998225
2	1.000000
3	1.000000
4	1.000000
5	0.999989
6	0.999761
7	0.999644
8	0.983538
	7.981157

Maximum Spanning Tree



No.	Edge	Weight
1	(2,4)	1.000000
2	(3,4)	1.000000
3	(2,5)	0.999989
4	(2,6)	0.999761
5	(3,7)	0.999644
6	(1,4)	0.998225
7	(2,8)	0.983538
		6.981157

$$P(A_1 \cap A_2 \cap \ldots \cap A_8) \geq \sum_{i=1}^{8} P(A_i) - (k-1)$$

$$P(A_1 \cap A_2 \cap \ldots \cap A_8) \geq 7.981157 - (8-1)$$

$$P(A_1 \cap A_2 \cap \ldots \cap A_8) \geq 0.981157$$

$$0.981157 \leq P(A_1 \cap A_2 \cap \ldots \cap A_8) \leq 1$$

Numerical Example #3

Additional Details – Boolean Probability Bounding

Excerpt from RHS vector b

$$\mathbf{b} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ \vdots \\ p_6 \\ p_7 \\ p_8 \\ p_{1,2} \\ p_{1,3} \\ p_{1,4} \\ \vdots \\ p_{6,7} \\ p_{6,8} \\ p_{7,8} \end{bmatrix} = \begin{bmatrix} 0.998225 \\ 1.000000 \\ 1.000000 \\ \vdots \\ 0.999761 \\ 0.999644 \\ 0.983538 \\ 0.998225 \\ 0.998225 \\ 0.998225 \\ \vdots \\ 0.999624 \\ 0.983538 \\ 0.983538 \end{bmatrix}$$

LP Formulation

$m = 36$ (8 individual and 28 pairs)

$n = 2^8 - 1 = 255$

A is a 36 x 255 matrix

$$\max (\min) \quad \sum_{i=1}^{255} x_i$$
$$\text{s.t.} \quad \mathbf{Ax} = \mathbf{b}$$
$$\mathbf{x} \geq \mathbf{0}$$

Due to the smaller size of matrix A, the LP was solved using AMPL

max = min = 1

Appendix

Electric Utility Industry
Upcoming Challenges

- Environmental legislation
- Energy efficiency mandates
- Smart metering
- State of the economy