Parameterized approximation scheme for the multiple knapsack problem

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Abstract

The multiple knapsack problem (MKP) is a well-known generalization of the classical knapsack problem. We are given a set A of n items and set B of m bins (knapsacks) such that each item $a \in A$ has a size $size(a)$ and a profit value $profit(a)$, and each bin $b \in B$ has a capacity $c(b)$. The goal is to find a subset $U \subset A$ of maximum total profit such that U can be packed into B without exceeding the capacities. The decision version of MKP is strongly NP-complete, since it is a generalization of the classical knapsack and bin packing problem. Furthermore, MKP does not admit an FPTAS even if the number m of bins is two. Kellerer gave a PTAS for MKP with identical capacities and Chekuri and Khanna presented a PTAS for MKP with general capacities with running time $n^{O(\log(1/\epsilon)/\epsilon^8)}$. In this talk we propose an EPTAS with parameterized running time $2^{O(\log(1/\epsilon)/\epsilon^5)} \cdot poly(n) + O(m)$ for MKP. This solves also an open question by Chekuri and Khanna.

In contrast to previous approaches we use several new and different techniques. For MKP with many bins for each capacity value, we propose an interesting linear program relaxation of MKP that is a variant of the configuration LP for fractional bin or strip packing. The main idea is to select fractional pieces of the items and to distribute them among different bins. The linear program relaxation has an exponential number of variables and some negative covering constraints that can be solved approximately via techniques for the max-min resource sharing problem by Grigoriadis, Khachiyan, Porkolab and Villavicenco. Interestingly, the LP solution with fractional selected items can be interpreted as rectangles to be placed into rectangular blocks of different widths and large heights. Then we set up another linear program to modify the distribution of the selected fractional items among the blocks. To do this we use some techniques by Kenyon and Remila for 2D strip packing (i.e. the rounding of the wide rectangles via a

stack and counting the area for the narrow rectangles) and by Lenstra, Shmoys and Tardos for scheduling on unrelated machines (i.e. rounding the LP solution for the scheduling problem). The obtained solution gives us a unique assignment of almost all selected fractional items to blocks.

Since we have selected in general only fractional items, in a second phase we have to produce a solution with completely selected items. This can be done via fractional knapsack instances for the corresponding selected narrow and wide rectangles. Finally, we show that the selected items can be packed into the rectangular blocks or corresponding bins (via 2D strip packing) and that the profit of the solution (via the shifting technique) is close to the optimum profit. In the general case we modify the structure of the bins to obtain many bins for almost each capacity value. This is done via an interesting rounding step (linear rounding of all bins except a constant number of large ones). We end up with a constant number of large capacity values where the number of bins is small. For all other capacity values we obtain many bins and this makes it possible to use a modification of the techniques above. In the next step, we modify the structure of the high profit items to reduce the number of such items. Then we guess a constant number of high profit items to be placed into the constant number of large bins. For each such guess, we solve a modified linear program relaxation approximately and round the generated solution as described above.