Planning Electricity Portfolios

Approximate Models

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Stochastic Modeling and Optimization
dedicated to the 80th birthday of
Professor András Prékopa
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Outline

Introduction

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2. Case study of an electricity distribution company where the ideas result in some remarkable improvements.

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  - B&B or B&C algorithms for mixed integer programming.
- Users want solution instantly.

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Intuition is also required, however, it **can be supported by** some **paradigms**.

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- Electricity distribution company (**EDC**) **supplies electricity** to an entire region (e.g. Southern England). It has the **legal obligation to provide electricity** to all users in the region **in every half-hour period of the year** estimated to be $b_i,\ i = 1, \ldots, 17520$.

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- Electricity comes from generating plants (**GPs**) (domestic or foreign) on the basis of yearly contracts.
- During the planning period (once a year) GPs put forward **contract offers**.

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3. Throughout every timeslot of the year: 0.40 units (from a nuclear power station).

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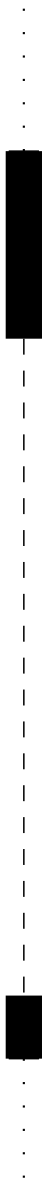

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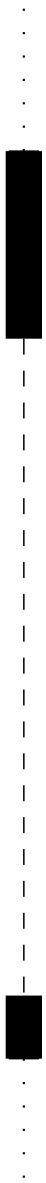

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$\mathbf{Ax} \approx \mathbf{b}$, or componentwise, $\sum_{j=1}^{n} a_{ij}x_j \approx b_i, \; i = 1, \ldots, m$,

while minimizing $\mathbf{c}^T\mathbf{x}$, where x_j is the level of contract offer j.

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In general:

$$\boxed{\text{minimize} \quad \mathbf{c}^T\mathbf{x} + \lambda\|\mathbf{Ax} - \mathbf{b}\|, \quad \mathbf{x} \geq \mathbf{0},} \tag{1}$$

where $\|.\|$ denotes some vector norm, usually ℓ_1, ℓ_2 or ℓ_∞, and $\lambda \geq 0$ is a scaling (balancing) factor.

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If ℓ_2 **norm** (or rather its square) is taken in (1) then a **quadratic function** is to be **minimized**:

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Disadvantages:

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2. all violations are uniformly penalized.

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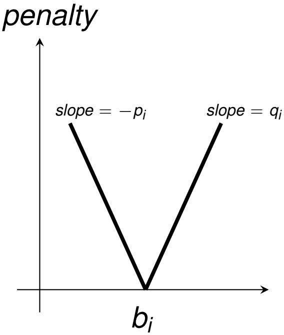

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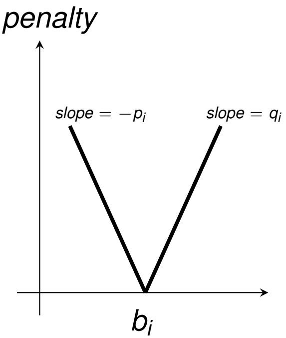

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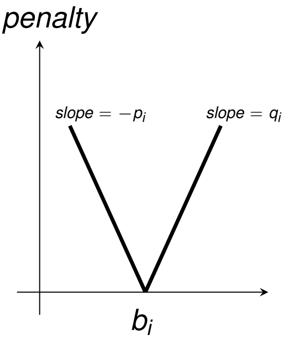

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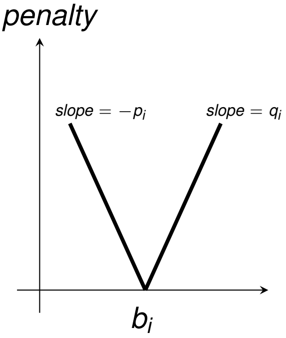

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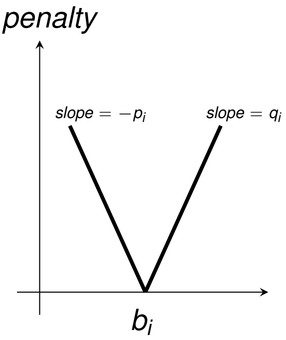

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- **uniform** if $p_i = p$ and $q_i = q, \ \forall i$,
- **uniform symmetric** if uniform and symmetric: $p = q$, $\approx$ the case of Model #1.

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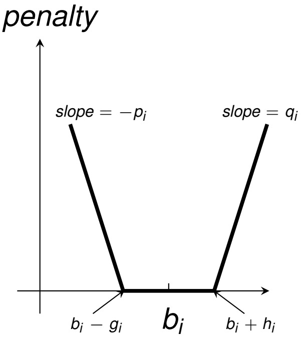

If $p_i = q_i$ and $g_i = h_i$ then it is the case of **symmetric** flat-bottomed penalty.

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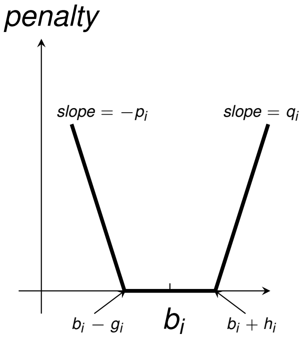

Now the **goal**:

$$b_i - g_i \leq \sum_{j=1}^{n} a_{ij}x_j \leq b_i + h_i$$

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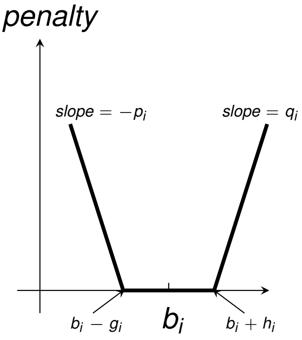

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Now the **goal**:

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which is a **range constraint.** It can be converted to

$$\sum_{j=1}^{n} a_{ij}x_j + y_i = b_i + h_i,$$

$$0 \leq y_i \leq g_i + h_i.$$

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This **goal** is approximated as well as possible if we solve

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\min \quad & \sum_{j=1}^{n} c_j x_j + \lambda \sum_{i=1}^{m} (p_i u_i + q_i v_i) \\
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& 0 \le y_i \le g_i + h_i, \quad i = 1, \dots, m, \\
& x_j \ge 0, \; j = 1, \dots, n, \quad \text{and} \quad u_i, v_i \ge 0, \quad i = 1, \dots, m.
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Note: y_i does not appear in the objective function.

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There can be **side constraints** in each model.

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2. Computational Efficiency
3. Electricity Portfolio
   - Approximate Models
   - Exact Solution

Solution of Model-2a

$$
\begin{array}{lllllll}
(P1) & \min & \mathbf{c}_1\mathbf{x}_1 & + & \mathbf{c}_2\mathbf{x}_2 & + & \mathbf{c}_3\mathbf{x}_3 & & \\
 & & \mathbf{A}_1\mathbf{x}_1 & + & \mathbf{I}\mathbf{x}_2 & - & \mathbf{I}\mathbf{x}_3 & = & \mathbf{b}_1 \\
 & & \mathbf{A}_2\mathbf{x}_1 & & & & & = & \mathbf{b}_2 \\
 & \text{s.t.} & \mathbf{x}_1 \geq \mathbf{0}, & & \mathbf{x}_2 \geq \mathbf{0}, & & \mathbf{x}_3 \geq \mathbf{0}. & &
\end{array}
$$

$\mathbf{b}_1,\ \mathbf{c}_2,\ \mathbf{c}_3,\ \mathbf{x}_2,\ \mathbf{x}_3 : m_1;$
$\mathbf{c}_1,\ \mathbf{x}_1 : n_1;$
$\mathbf{A}_1 : m_1 \times n_1;$
$\mathbf{A}_2 : m_2 \times n_1;\ \ \mathbf{b}_2 : m_2;$

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\end{array}
$$

$\mathbf{b}_1,\ \mathbf{c}_2,\ \mathbf{c}_3,\ \mathbf{x}_2,\ \mathbf{x}_3 : m_1;$

$\mathbf{c}_1,\ \mathbf{x}_1 : n_1;$

$\mathbf{A}_1 : m_1 \times n_1;$

$\mathbf{A}_2 : m_2 \times n_1;\ \ \mathbf{b}_2 : m_2;$

$\mathbf{c}_2 = \mathbf{c}_3 = \mathbf{e}$

$\mathbf{c}_1 = \mathbf{0}$

Solution of Model-2a

$$
\begin{array}{llllllll}
(P1) & \min & \mathbf{c}_1\mathbf{x}_1 & + & \mathbf{c}_2\mathbf{x}_2 & + & \mathbf{c}_3\mathbf{x}_3 & & \\
 & & \mathbf{A}_1\mathbf{x}_1 & + & \mathbf{I}\mathbf{x}_2 & - & \mathbf{I}\mathbf{x}_3 & = & \mathbf{b}_1 \\
 & & \mathbf{A}_2\mathbf{x}_1 & & & & & = & \mathbf{b}_2 \\
 & \text{s.t.} & \mathbf{x}_1 \geq \mathbf{0}, & & \mathbf{x}_2 \geq \mathbf{0}, & & \mathbf{x}_3 \geq \mathbf{0}. & &
\end{array}
$$

$\mathbf{b}_1,\ \mathbf{c}_2,\ \mathbf{c}_3,\ \mathbf{x}_2,\ \mathbf{x}_3 : m_1;$ $\qquad \mathbf{c}_2 = \mathbf{c}_3 = \mathbf{e}$

$\mathbf{c}_1,\ \mathbf{x}_1 : n_1;$ $\qquad \mathbf{c}_1 = \mathbf{0}$

$\mathbf{A}_1 : m_1 \times n_1;$ $\qquad \boxed{m_1 \gg n_1}$

$\mathbf{A}_2 : m_2 \times n_1;\ \ \mathbf{b}_2 : m_2;$ $\qquad m_2 \ll m_1$

Solution of Model-2a

$$
\begin{array}{llcccccccc}
(P1) & \min & \mathbf{c}_1\mathbf{x}_1 & + & \mathbf{c}_2\mathbf{x}_2 & + & \mathbf{c}_3\mathbf{x}_3 & & \\
 & & \mathbf{A}_1\mathbf{x}_1 & + & \mathbf{I}\mathbf{x}_2 & - & \mathbf{I}\mathbf{x}_3 & = & \mathbf{b}_1 \\
 & & \mathbf{A}_2\mathbf{x}_1 & & & & & = & \mathbf{b}_2 \\
 & \text{s.t.} & \mathbf{x}_1 \geq \mathbf{0}, & & \mathbf{x}_2 \geq \mathbf{0}, & & \mathbf{x}_3 \geq \mathbf{0}. & &
\end{array}
$$

$\mathbf{b}_1,\ \mathbf{c}_2,\ \mathbf{c}_3,\ \mathbf{x}_2,\ \mathbf{x}_3 : m_1;$

$\mathbf{c}_1,\ \mathbf{x}_1 : n_1;$

$\mathbf{A}_1 : m_1 \times n_1;$

$\mathbf{A}_2 : m_2 \times n_1;\ \ \mathbf{b}_2 : m_2;$

$\mathbf{c}_2 = \mathbf{c}_3 = \mathbf{e}$

$\mathbf{c}_1 = \mathbf{0}$

$\boxed{m_1 \gg n_1}$

$m_2 \ll m_1$

Electricity Portfolio:

Actual sizes:	$m_1 =$	17,520	$n_1 =$	381
	$m_2 =$	1,218		
$m =$	$m_1 + m_2 =$	18,738		
$n =$	$2m_1 + n_1 =$	35,421		

Very useful: **visualization of structure**

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$\mathbf{c}_1^T$	$\mathbf{c}_2^T$	$\mathbf{c}_3^T$
$\mathbf{A}_1$	$\mathbf{I}$	$-\mathbf{I}$
$\mathbf{A}_2$	$\mathbf{0}$	$\mathbf{0}$

Very useful: **visualization of structure**

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$\mathbf{A}_2$	$\mathbf{0}$	$\mathbf{0}$

If $\mathbf{A}_2 = \mathbf{0}$

$$\left[\begin{array}{ccc} \mathbf{A}_1 & \mathbf{I} & -\mathbf{I} \end{array}\right] \left[\begin{array}{c} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{array}\right] = \mathbf{b}_1$$

Very useful: **visualization of structure**

$\mathbf{c}_1^T$	$\mathbf{c}_2^T$	$\mathbf{c}_3^T$
$\mathbf{A}_1$	$\mathbf{I}$	$-\mathbf{I}$
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If $\mathbf{A}_2 = \mathbf{0}$

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{I} & -\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} = \mathbf{b}_1$$

A starting feasible basis:

$$x_{Bi} = \begin{cases} x_{2i} & \text{if } b_{1i} \geq 0, \\ x_{3i} & \text{if } b_{1i} < 0. \end{cases}$$

Dual of (P1) if $\mathbf{A}_2 = \mathbf{0}$

The **idea**: consider **the dual** of (P1).

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If $\mathbf{A}_2 = \mathbf{0}$

$$
\begin{array}{llll}
\textbf{(D1)} & \max & \mathbf{b}_1^T \mathbf{y}_1 & \\
& \text{s.t.} & \mathbf{A}_1^T \mathbf{y}_1 & \leq \mathbf{0} \\
& & \mathbf{y}_1 & \leq \mathbf{e} \\
& & -\mathbf{y}_1 & \leq \mathbf{e}
\end{array}
$$

Dual of (P1) if $\mathbf{A}_2 = \mathbf{0}$

The **idea**: consider **the dual** of (P1).

If $\mathbf{A}_2 = \mathbf{0}$ reduces to:

$$
\begin{array}{llrl}
\textbf{(D1)} & \max & \mathbf{b}_1^T \mathbf{y}_1 & \\
& \text{s.t.} & \mathbf{A}_1^T \mathbf{y}_1 & \leq \mathbf{0} \\
& & \mathbf{y}_1 & \leq \mathbf{e} \\
& & -\mathbf{y}_1 & \leq \mathbf{e}
\end{array}
$$

$$
\begin{array}{lrcl}
\max & & \mathbf{b}_1^T \mathbf{y}_1 & \\
\text{s.t.} & & \mathbf{A}_1^T \mathbf{y}_1 & \leq \mathbf{0} \\
& -\mathbf{e} \leq & \mathbf{y}_1 & \leq \mathbf{e}
\end{array}
$$

$\mathbf{A}_2 = \mathbf{0}$

$$\begin{bmatrix} \mathbf{A}_1^T \\ \mathbf{I} \\ -\mathbf{I} \end{bmatrix}$$

Size: $35,421 \times 17,520$

$\mathbf{A}_2 = \mathbf{0}$

$$\begin{bmatrix} \mathbf{A}_1^T \\ \mathbf{I} \\ -\mathbf{I} \end{bmatrix}$$

Size: $35,421 \times 17,520$

$$\begin{bmatrix} \mathbf{A}_1^T \end{bmatrix}$$

Size: $381 \times 17,520$
plus individual upper bounds

$\mathbf{A}_2 = \mathbf{0}$

$$\begin{bmatrix} \mathbf{A}_1^T \\ \mathbf{I} \\ -\mathbf{I} \end{bmatrix}$$

Size: $35,421 \times 17,520$

$$\begin{bmatrix} \mathbf{A}_1^T \end{bmatrix}$$

Size: $381 \times 17,520$
plus individual upper bounds
$\Rightarrow$ upper bounded simplex

If $\mathbf{A}_2 \neq \mathbf{0}$

Primal:

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{I} & -\mathbf{I} \\ \mathbf{A}_2 & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix}$$

$$\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \geq \mathbf{0}$$

$$\min \ \mathbf{c}_1\mathbf{x}_1 + \mathbf{c}_2\mathbf{x}_2 + \mathbf{c}_3\mathbf{x}_3$$

If $\mathbf{A}_2 \neq \mathbf{0}$

Primal:

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{I} & -\mathbf{I} \\ \mathbf{A}_2 & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix}$$

$$\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \geq \mathbf{0}$$

$$\min \ \mathbf{c}_1\mathbf{x}_1 + \mathbf{c}_2\mathbf{x}_2 + \mathbf{c}_3\mathbf{x}_3$$

Dual:

$$\begin{bmatrix} \mathbf{A}_1^T & \mathbf{A}_2^T \\ \mathbf{I} & \mathbf{0} \\ -\mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} \leq \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \mathbf{c}_3 \end{bmatrix}$$

If $\mathbf{A}_2 \neq \mathbf{0}$

Primal:

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{I} & -\mathbf{I} \\ \mathbf{A}_2 & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix}$$

$$\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \geq \mathbf{0}$$

$$\min \ \mathbf{c}_1\mathbf{x}_1 + \mathbf{c}_2\mathbf{x}_2 + \mathbf{c}_3\mathbf{x}_3$$

Dual:

$$\begin{bmatrix} \mathbf{A}_1^T & \mathbf{A}_2^T \\ \mathbf{I} & \mathbf{0} \\ -\mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} \leq \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \mathbf{c}_3 \end{bmatrix}$$

$$\begin{array}{lll} \max & \mathbf{b}_1^T\mathbf{y}_1 + \mathbf{b}_2^T\mathbf{y}_2 & \\ \text{s.t.} & \mathbf{A}_1^T\mathbf{y}_1 + \mathbf{A}_2^T\mathbf{y}_2 & \leq \mathbf{c}_1 \\ & \mathbf{I}\mathbf{y}_1 & \leq \mathbf{c}_2 \\ & -\mathbf{I}\mathbf{y}_1 & \leq \mathbf{c}_3 \end{array}$$

If $\mathbf{A}_2 \neq \mathbf{0}$

Primal:

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{I} & -\mathbf{I} \\ \mathbf{A}_2 & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix}$$

$$\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \geq \mathbf{0}$$

$$\min \; \mathbf{c}_1\mathbf{x}_1 + \mathbf{c}_2\mathbf{x}_2 + \mathbf{c}_3\mathbf{x}_3$$

Dual:

$$\begin{bmatrix} \mathbf{A}_1^T & \mathbf{A}_2^T \\ \mathbf{I} & \mathbf{0} \\ -\mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} \leq \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \mathbf{c}_3 \end{bmatrix}$$

$$\begin{aligned} \max \quad & \mathbf{b}_1^T\mathbf{y}_1 + \mathbf{b}_2^T\mathbf{y}_2 \\ \text{s.t.} \quad & \mathbf{A}_1^T\mathbf{y}_1 + \mathbf{A}_2^T\mathbf{y}_2 \leq \mathbf{c}_1 \end{aligned}$$

$$-\mathbf{c}_3 \leq \mathbf{y}_1 \leq \mathbf{c}_2$$

Comparison with presolve

Primal of **S3PF**

	Before presolve	After presolve
$m =$	18,738	17,981
$n =$	35,421	35,223
$nz =$	148,318	141,256
Long col	17,520	17,520

Comparison with presolve

Primal of **S3PF**

	Before presolve	After presolve
$m =$	18,738	17,981
$n =$	35,421	35,223
$nz =$	148,318	141,256
Long col	17,520	17,520

Dual of **S3PF**

	Before reduction	After reduction
$m =$	35,421	381
$n =$	18,738	18,738
$nz =$	148,318	113,278
Long col	176	176

Lesson/Morale

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1. Large scale LP problems have to be analyzed *prior to* solution.

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1. Large scale LP problems have to be analyzed *prior to* solution.
2. Challenge for presolve procedures.
3. Modelling systems could contribute.
4. Knowledge of theoretical background of optimization can be vital.

T H E E N D