DIMACS

Center for Discrete Mathematics & Theoretical Computer Science
Founded as a National Science Foundation Science and Technology Center

CCICADA

Command, Control, and Interoperability Center for Advanced Data Analysis
A Department of Homeland Security Center of Excellence

DIMACS/CCICADA Workshop on Stochastic Networks: Reliability, Resiliency, and Optimization

Heterogeneous models for nonlinear flows on networks

BENEDETTO PICCOLI

Joseph and Loretta Lopez Chair Professor of Mathematics
Department of Mathematical Sciences
and
Director of Graduate Program in Computational and Integrative Biology
Center for Computational and Integrative Biology
Rutgers University - Camden

RUTGERS
THE STATE UNIVERSITY OF NEW JERSEY

Vehicular Traffic

Lighthill-Witham-Richards model:

$$\rho_t + (\rho\, v(\rho))_x = 0, \quad \rho \in [0, \rho_{max}] \text{ density of cars}$$

Irrigation Channels

De Saint-Venant equation:

$$\begin{cases} H_t + (H v)_x = 0 \\ v_t + [\frac{1}{2} v^2 + g\, H]_x = 0 \end{cases}$$

H water height, v velocity, g gravity

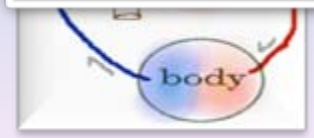
Gas pipelines

Isothermal Euler with friction:

$$\begin{cases} \rho_t + (\rho\, u)_x = 0 \\ (\rho\, u)_t + (\rho\, u^2 + a^2 \rho)_x = -f_g\, \frac{\rho\, u |\rho\, u|}{2 D \rho}. \end{cases}$$

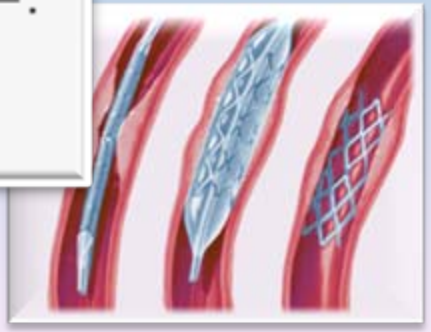
ρ density, u velocity

Real fluids



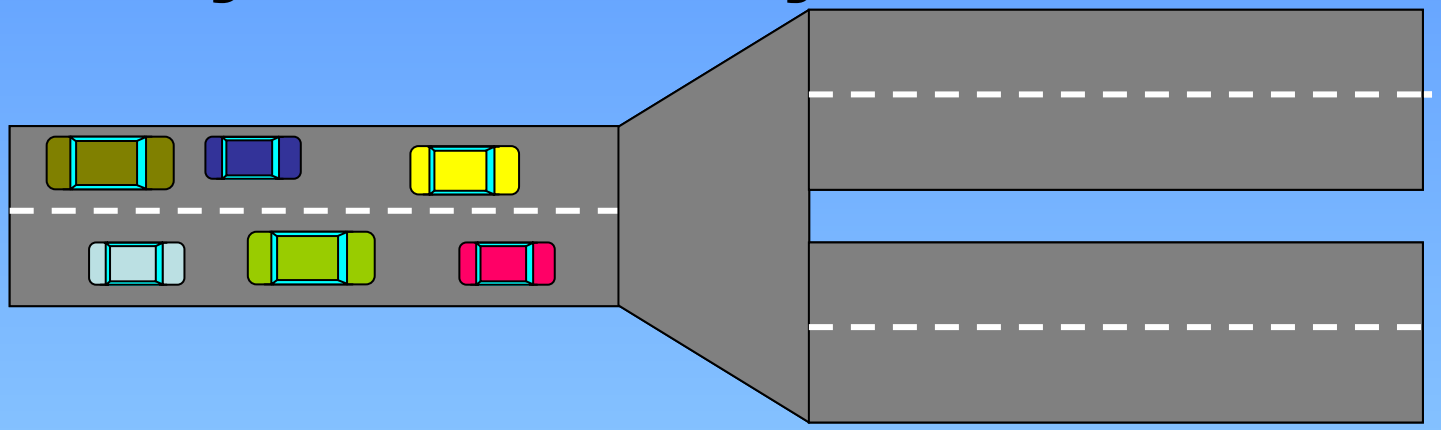
Blood circulation

Bio-Medical



Vascular stents

Dynamics at junctions



Rule (A) : Out. Fluxes Vector = $A \cdot$ Inc. Fluxes Vector

Traffic distribution matrix $A = (\alpha_{ji})$, $0<\alpha_{ji}<1$, $\Sigma_j \alpha_{ji} = 1$

Rule (B) : Max $\| \text{Inc. Fluxes Vector} \|$

Rule (B) is an "entropy" type rule : maximize velocity

Integration of models and scales

Car trajectories and moving bottlenecks

Goettlich-Herty-Klar supply chain model

Mixed ODE-PDE model

$$\rho_t + f(x, y(t), \rho)_x = 0$$

$$\partial_t \rho_j(x,t) + \partial_x \min\{\mu_j, \frac{L_j}{T_j}\rho_j(x,t)\} = 0 \quad \forall x \in [a_j, b_j],\ t \in \mathbb{R}^+$$

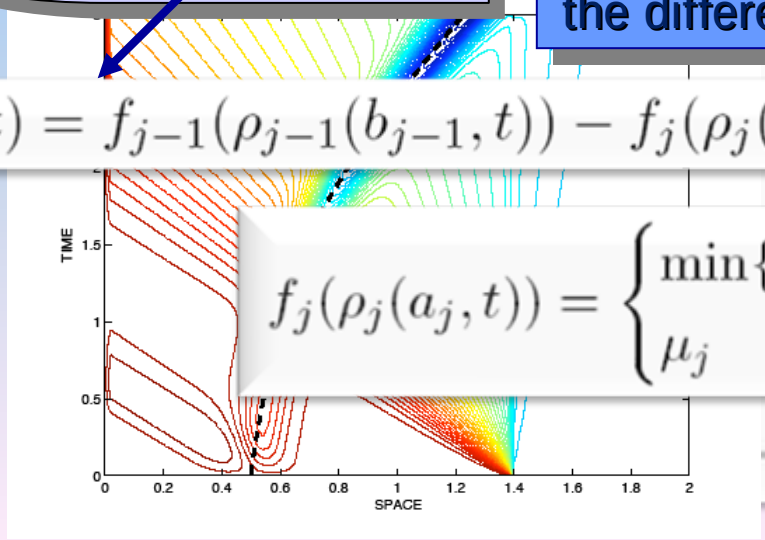
Queue

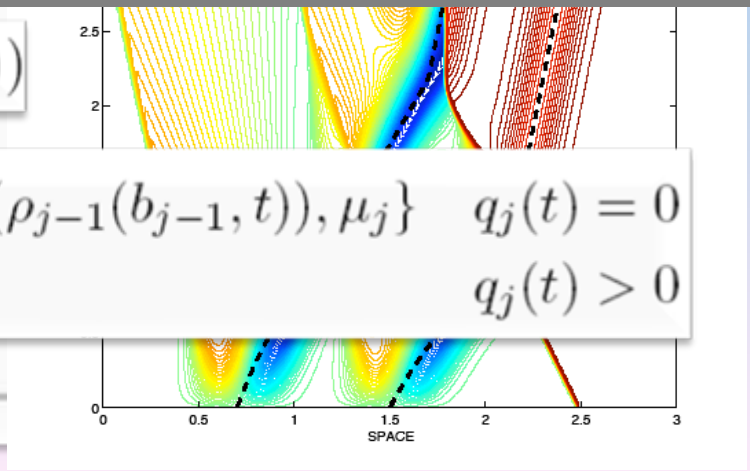
Processor j

Queue buffer occupancy change is given by the difference between incoming and outgoing flux

$$\partial_t q_j(t) = f_{j-1}(\rho_{j-1}(b_{j-1}, t)) - f_j(\rho_j(a_j, t))$$

$$f_j(\rho_j(a_j,t)) = \begin{cases} \min\{f_{j-1}(\rho_{j-1}(b_{j-1},t)), \mu_j\} & q_j(t) = 0 \\ \mu_j & q_j(t) > 0 \end{cases}$$



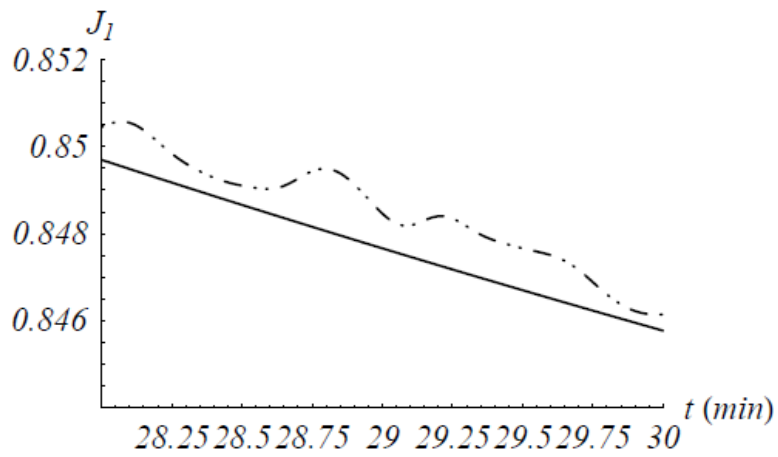


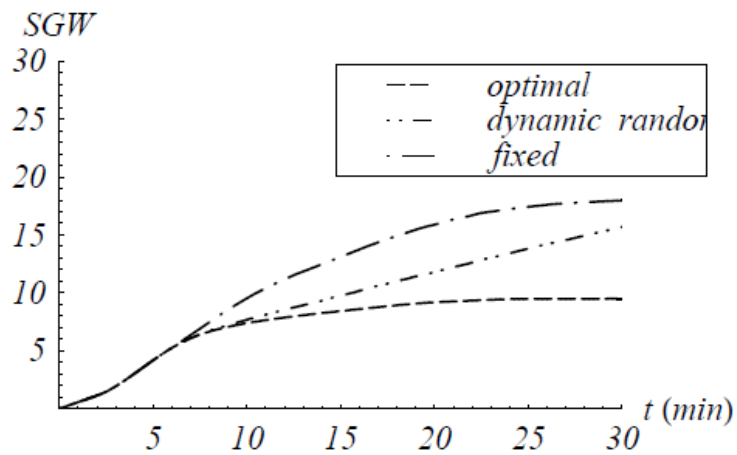
Optimization of vehicular traffic

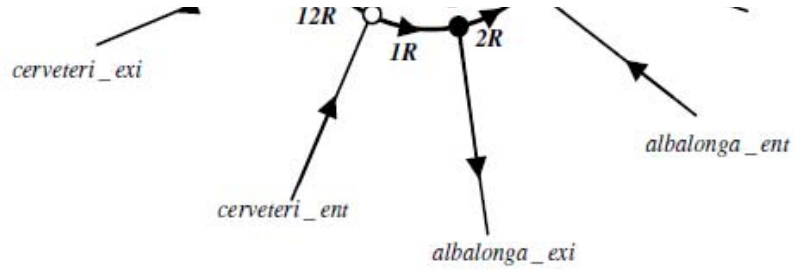
$$J_1(t) = \sum_i \int_{I_i} v(\rho_i(t,x))\, dx,$$

$$J_2(t) = \sum_i \int_{I_i} \frac{1}{v(\rho_i(t,x))}\, dx.$$

$$SGW = \int_0^T \int_{\cup I_i} |Dv(\rho)|\, dt dx.$$







Via Parmenide Via Parmenide

Optimal control for supply chains

$$\begin{cases} \partial_t \rho_j(x,t) + \partial_x \min\{\mu_j, \upsilon_j \rho_j(x,t)\} = 0 & j = 1, ..., N, \\ \dot{q}_j(t) = f_{j-1}(\rho_{j-1}(b_{j-1}, t)) - f_j^{inc} & j = 2, ..., N, \\ \rho_1(a_1, t) = u(t) & \\ \rho_j(x, 0) = \rho_{j,0}(x) & j = 1, ..., N, \\ q_j(x, 0) = q_{j,0} & j = 2, ..., N, \end{cases}$$

$$J(u) = \sum_{j=1}^{n} \int_0^T q_j(t)dt + \int_0^T [\upsilon_N \cdot \rho_N(b_N, t)) - \psi(t)]^2 \, dt \doteq J_1(u) + J_2(u),$$

Existence of solutions

Take minimizing sequence: compactness by Helly and Ascoli Arzela' Theorem.

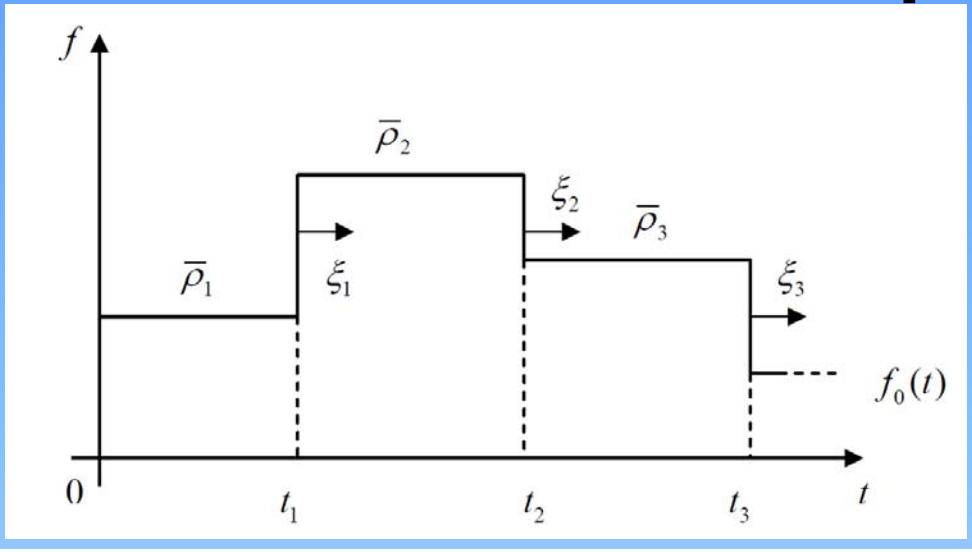
$$q_n \to q \text{ in } C^0, \text{ thus } J_1(u_n) \to J_1(u)$$

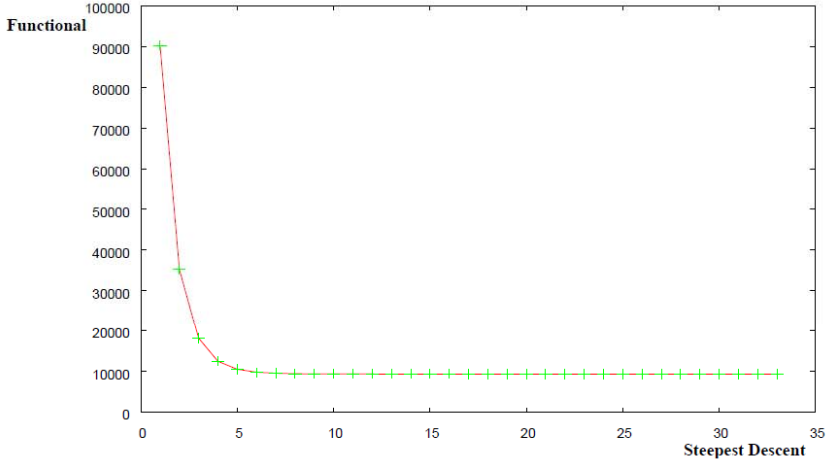
$$\int_0^T \left((\upsilon_N \cdot \rho_N^n(b_N, t)) - \psi(t))^2 - (\upsilon_N \cdot \rho_N(b_N, t)) - \psi(t))^2 \right) dt =$$

$$\int_0^T \left((\upsilon_N)^2 \left((\rho_N^n(b_N, t))^2 - (\rho_N(b_N, t))^2 \right) + 2\psi(t)\upsilon_N \left(\rho_N^n(b_N, t)) - \rho_N(b_N, t)\right) dt \leq$$

$$||2\psi(t) + \upsilon_N(\rho_N^n(b_N, t) + \rho_N(b_N, t))||_\infty \cdot ||\upsilon_N(\rho_N^n(b_N, t) - \rho_N(b_N, t))||_{L^1}.$$

Tangent vectors for numerical optimization





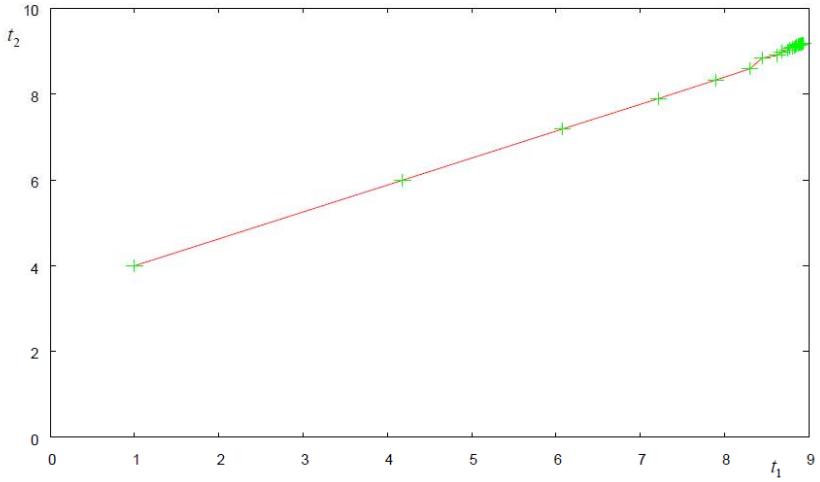


Figure 7: Supply chain with 11 arcs, case a. Left: J_1 versus iteration steps; right: "path" followed by the steepest descent algorithm in the plane (t_1, t_2).

Cyber-infrastructures for info-mobility

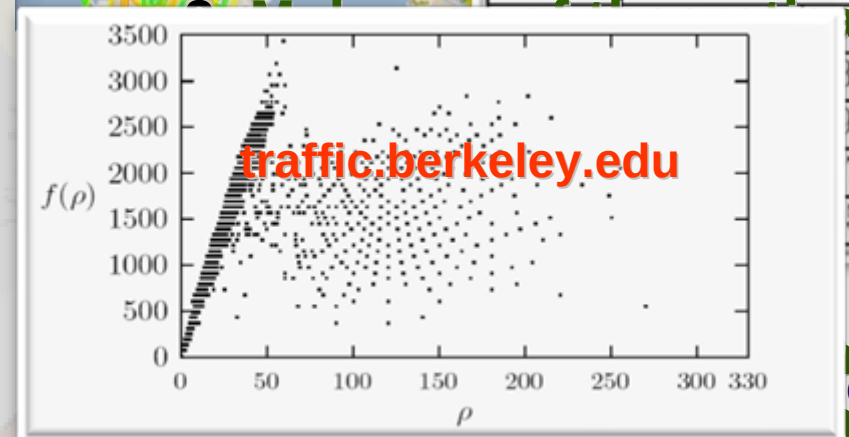
Cyber-integration for applications

Network with 5000 roads parametrized by [0,1],
h space mesh size, T real time

1. Use simplified flux function with two characteristic speeds

CPU time				
	$T = 10$			
h	G	FG	K3V	FSF
0.2	1.78 s	1.12 s	29.37 s	0.60 s
0.1	5.68 s	3.05 s	104.74 s	1.35 s
0.05	19.83 s	9.30 s	394.03 s	[illegible]
0.025	73.86 s	31.40 s	1515.32 s	[illegible]
	$T = 30$			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	85.71 s	[illegible]
[illegible]	[illegible]	[illegible]	309.55 s	[illegible]
[illegible]	[illegible]	[illegible]	1171.10 s	[illegible]
[illegible]	[illegible]	[illegible]	4527.80 s	[illegible]

2. [illegible] results to be [illegible] of



traffic.berkeley.edu

www.octotelematics.com

FG = Fast Godu[illegible]
[illegible] FSF = Fast [illegible]
[illegible]ch zone

CROWD DYNAMICS

VEHICULAR TRAFFIC

SUPPLY CHAINS

Andrea Tosin

Francesco Rossi

Paola Goatin

Mauro Garavello

Alessia Marigo

Simone Goett

Gabriella Bretti

Roberto Natalini

Dirk Helbing

Dan Work

Ciro D'Apice

Michael

Emiliano Cristiani

Alex Bayen

Corrado Lattanzio

Seb Blandin

Rosanna Manzo

Paolo Frasca

Yacine Chitour

Rinaldo Colombo

Giuseppe Coclite

Maur

Axel Klar

ANIMAL GROUPS

Thank you for your attention!

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Real data

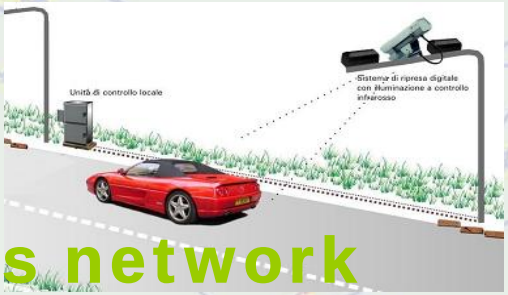
Problems :

1. Dimensionality: big networks
2. Data: measurements and elaboration

Manual counting

Satellite data

Videocameras



Plates reading

1500 arcs network

NETWORK of SALERNO

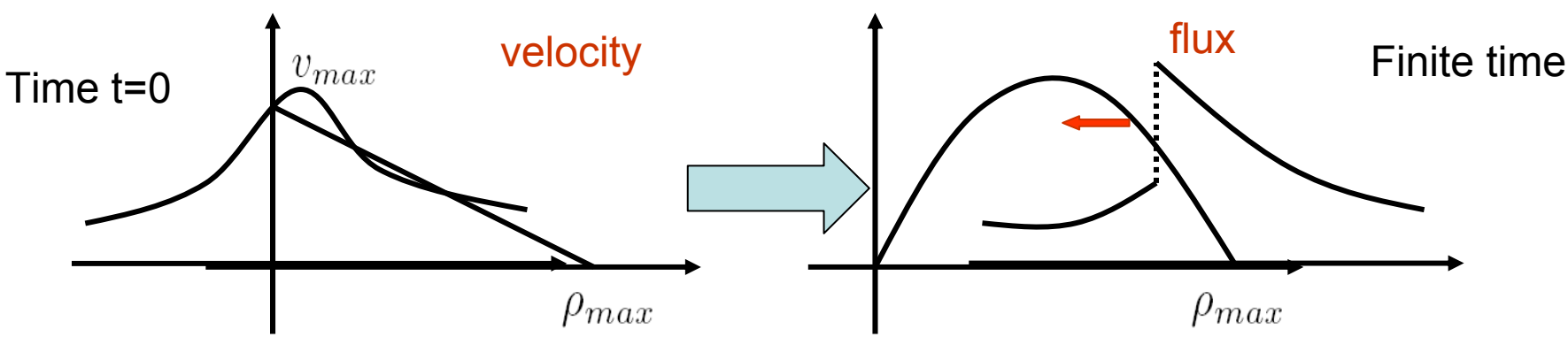
Lighthill-Whitham-Richards model

The flux is given by the density times the average velocity $f(t,x) = \rho(t,x) \cdot v(t,x)$

If we assume that the average velocity depends only on density $v(t,x) = v(\rho(t,x))$

Lighthill-Witham-Richards model:

$$\rho_t + (\rho\, v(\rho))_x = 0, \quad \rho \in [0, \rho_{max}]$$



Non unique (weak) solutions

Entropy (gas dynamics)

(disorder is increasing, stable shocks)

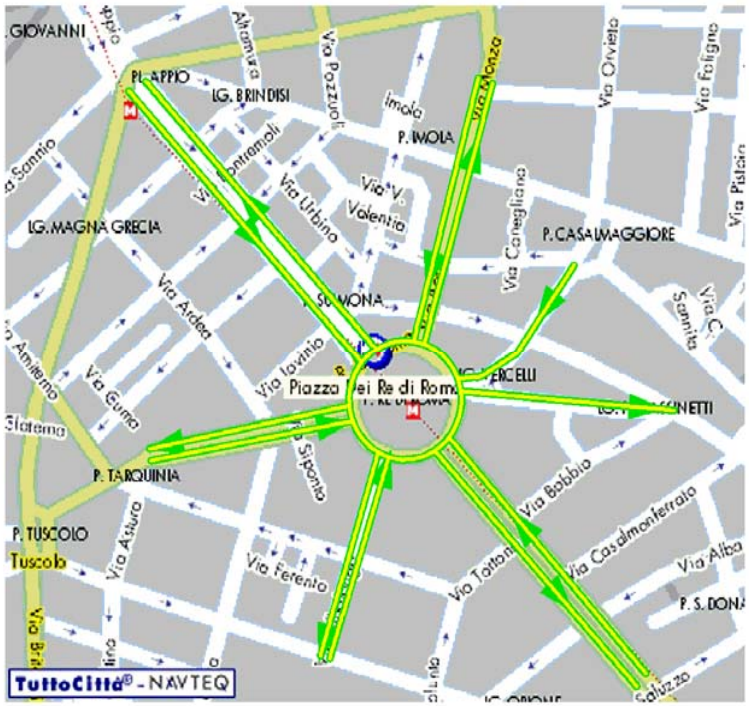
Networks and Re di Roma square

More incoming than exiting road → Priority parameters

Bifurcations, merging, complicate junctions, traffic circles

Theory : existence of solutions on networks for BV initial data.

Road flux total variation in x ~ Junctions flux total variation in t



MOVIE

ZOOM

Model for data networks

We essentially invert Rules (A) and (B), giving more importance to through flux than traffic distribution.

In the nth attempt $(1-\mathcal{P}(R))\mathcal{P}(R)^{n-1}$ packets are sent and $\mathcal{P}(R)^n$ are lost.

Maximal fluxes on each line γ_i^{max} and γ_j^{max}

The through flux is $\Gamma = \min\{\sum_i \gamma_i^{max}\ ,\ \sum_j \gamma_j^{max}\}$.