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Solution of a Product Substitution Problem Using Stochastic Programming

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Abstract. A stochastic programming model of optical fiber manufacturing is created. The purpose is to set the best fiber manufacturing goals while accounting for the uncertainty primarily in the yield and secondly in the demand. The model is solved for the case when the data follows a multivariate discrete distribution. The model is also solved for the case when the distribution is approximated by a multivariate normal distribution.

1 Manufacturing Process

The process of manufacturing optical fibers can be divided into two major parts. The first is preform manufacturing. One process to make preforms is called modified chemical vapor deposition (MCVD). In the MCVD process, glass is deposited on the inside of a quartz tube. When the deposition is complete, the tube is collapsed into a solid rod called a preform (Flegal, Haney, Elliott, Kamino, and Ernst [7]).

The second part is fiber draw. In this process the end of the preform is heated in a furnace and fiber is drawn from it. The fiber has the same cross-sectional structure as the preform except that the fiber is much thinner and much longer (Jablonowski, Paek, and Watkins [10]).

The fiber draw process produces a variety of lengths of fiber. The fiber lengths that are produced depend on the size of the preform and on the capacity of the fiber spool. When the length reaches the spool capacity, the fiber is cut and a new spool is started. Naturally the lengths produced also depend on the rate of unplanned fiber breakage.

We assume that the preforms are used to produce the longest fibers, that is, fibers are not cut in the course of the drawing process. They may break, though. The fibers obtained are the primary products. After some cutting has been done to satisfy demands, remnants are produced. Some of these are just thrown away because their lengths are very short but some of them are used to satisfy future demands. These can be called secondary products. It is not necessary, however, to distinguish between the two kinds of products in the model.

In addition to length, fibers have other characteristics, too. For the sake of simplicity we will speak about one additional characteristic that we term "performance" but note that performance is in reality determined by more than one additional measurement.

2 Requirements of A Mathematical Model

Our goal is to calculate a recommended production level to meet the demand. Typical results of the calculation are expected to be, for example, 80%, 90%, or 100% of the manufacturing capacity. To reach a sufficient level of accuracy, the model needs to include several features outlined below. Leaving out any of these features will lead to inaccuracies amounting to at least 10% of the manufacturing capacity and will reduce the usefulness of the results of the calculation. The necessary features include:

- account for substitution of longer length fibers to meet shorter length demand,
- account for substitution of higher performance fibers to meet adequate performance demand,
- account for the inventory on hand at each length and performance level,
- account for the expected production at each length and performance level,
- account for the expected demand at each length and performance level, and

- account for the opportunity to produce fiber in the current period to meet a surge in demand in a later period.

Leaving out any of the following features will lead to inaccuracies amounting to at least 5% of the manufacturing capacity. These additional necessary features are:

- account for the various possible outcomes (randomness) of the production at each length and performance level, and
- account for the various possible outcomes (randomness) of the demand at each length and performance level.

3 Mathematical Model

Let r be the number of performance levels and assume that these obey a linear ordering, performance level number 1 being the best. Thus, to have a performance level i product satisfy the requirements imposed on a performance level j product, it is necessary to have $i < j$.

The model presented here is a multi-period stochastic programming model. The following notation is used:

n:	number of different lengths of fibers
l_k:	length of fibers of type k
$m_{hk} = \left\lfloor \frac{l_h}{l_k} \right\rfloor$:	the number of fibers of length k that can be obtained by cutting one fiber of length h.
T:	number of time periods
y^t:	overall intended production level in period t, $t = 1, 2, \ldots, T$
a^t_{ih}:	expected number of performance level i fibers of length h produced in period t per unit of production, $i = 1, \ldots, r$, $h = 1, \ldots, n$, $t = 1, 2, \ldots, T$
$f^t_{ih}(y^t) = a^t_{ih} y^t + \xi^t_{ih}$:	number of performance level i fibers of length h produced in period t, $i = 1, \ldots, r$, $k = 1, \ldots, n$, $t = 1, 2, \ldots, T$. Note that the random components ξ^t_{ih} have an expected value of 0.
ζ^t_{ih}:	number of performance level i fibers of length h available at the beginning of period t, $i = 1, \ldots, r$, $h = 1, \ldots, n$, $t = 1, 2, \ldots, T$
c^t_{ih}:	cost (per fiber) to produce each performance level i fiber of length h in period t, $i = 1, \ldots, r$, $h = 1, \ldots, n$, $t = 1, 2, \ldots, T$

z_{ih}^t: number of performance level i fibers of length h carried in inventory between period t and period $t+1$, $i = 1, \ldots, r$, $h = 1, \ldots, n$, $t = 1, 2, \ldots, T$

d_{jk}^t: demand for performance level j fibers of length k in period t, $j = 1, \ldots, r$, $k = 1, \ldots, n$, $t = 1, 2, \ldots, T$

x_{ijhk}^t: number of performance level i fibers of length h used to meet demand for performance level j fibers of length k in period t, $1 \leq i \leq j \leq r$, $1 \leq h \leq k \leq n$, $t = 1, 2, \ldots, T$

X_{ijhk}^t: upper bound for x_{ijhk}^t

Summary of notation

Constants:	n, l_k, m_{hk}, a_{ih}^t, c_{ih}^t, X_{ijhk}^t
Random variables:	ξ_{ik}^t, ζ_{ik}^t, d_{ik}^t
Decision variables:	x_{ijhk}^t, y^t, z_{ih}^t

Holding the random variables fixed for now, our downgrading model is the following network flow model:

Constraints

We need for the inventory of fibers of performance level i and length h at the beginning of the first period plus the number of fibers of performance level i and length h produced during the first period to equal or exceed the number of fibers of performance level i and length h assigned to meet demand during the first period, for each performance level i, $i = 1, \ldots, r$ and each length h, $h = 1, \ldots, n$:

$$\zeta_{ih}^1 \ + \ a_{ih}^1 y^1 \ + \ \xi_{ih}^1 \ - \ z_{ih}^1 \geq \sum_{k=h}^{n} \ \sum_{j=i}^{r} x_{ijhk}^1. \tag{1}$$

We need for the number of fibers of performance level i and length h carried from the first period to the second period plus the number of fibers of performance level i and length h produced during the second period to equal or exceed the number of fibers of performance level i and length h assigned to meet demand during the second period, for each performance level i, $i = 1, \ldots, r$ and each length h, $h = 1, \ldots, n$:

$$z_{ih}^1 \ + \ a_{ih}^2 y^2 \ + \ \xi_{ih}^2 - z_{ih}^2 \geq \sum_{k=h}^{n} \ \sum_{j=i}^{r} x_{ijhk}^2. \tag{2}$$

In general, for each time period t, $t = 2, \ldots, T$, we need for the number of fibers of performance level i and length h carried from period $t-1$ to period t plus the number of fibers of performance level i and length h produced during period t to equal or exceed the number of fibers of performance level i and length h assigned to meet demand during period t, for each performance level i, $i = 1, \ldots, r$ and each length h, $h = 1, \ldots, n$:

$$z_{ih}^{t-1} \ + \ a_{ih}^{t} y^{t} \ + \ \xi_{ih}^{t} - z_{ih}^{t} \geq \sum_{k=h}^{n} \sum_{j=i}^{r} x_{ijhk}^{t}. \tag{3}$$

We need for the sum of the number of fibers assigned to meet the demand to equal or exceed the demand, in each performance level j, $j = 1, \ldots, r$, in each length category k, $k = 1, \ldots, n$, and in each period t, $t = 1, \ldots, T$. The number of fibers assigned needs to be multiplied by the appropriate factor m_{hk} if the length of fibers in length category h is at least twice the length of fibers in category k:

$$\sum_{h=1}^{k} \sum_{i=1}^{j} x_{ijhk}^{t} m_{hk} \geq d_{jk}^{t}. \tag{4}$$

We may want to limit the amount of downgrading and cutting, using upper bounds, for each i and j, $1 \leq i \leq j \leq r$; for each h and k, $1 \leq h \leq k \leq n$; and in each period t, $t = 1, 2, \ldots, T$:

$$0 \leq x_{ijhk}^{t} \leq X_{ijhk}^{t}. \tag{5}$$

4 Objective Function

The objective function of the underlying deterministic problem is the total production cost equal to

$$\sum_{t=1}^{T} \sum_{h=1}^{n} \sum_{i=1}^{r} c_{ih} f_{ih}^{t}(y) \ = \ \sum_{t=1}^{T} \sum_{h=1}^{n} \sum_{i=1}^{r} c_{ih}(a_{ih}^{t} y^{t} \ + \ \xi_{ih}^{t}), \tag{6}$$

where the c_{ih} are some positive constants.

The underlying deterministic problem consists of minimizing the objective function (6) subject to the constraints (1) - (5).

5 The Stochastic Programming Problem

The formulation of a multi-period stochastic programming problem, based on the underlying deterministic problem presented above, would lead us to extremely large sizes that we want to avoid. We would like to, however,

- capture the dynamics of the production control process, and
- impose a probabilistic constraint regarding demand satisfiability.

We can take into account the above aspects in a rolling horizon model system where each model encompasses the present and a few future periods. We choose only one period from the future and thus, altogether two periods are included in any model that we formulate and solve. The problem that we have of periods t, $t+1$ contains ζ_{ih}^t that we assume to be a known value. In principle the problem contains ζ_{ih}^{t+1} too. However, if the first stochastic constraint in (7) and (8) holds, then $\zeta_{ih}^{t+1} = z_{ih}^t$ and therefore we enter z_{ih}^t in the second stochastic constraint instead of ζ_{ih}^{t+1}.

We want to ensure that the constraints (1) - (3) are satisfied for periods t, $t+1$ by a prescribed large probability p. Under this condition and the constraints (4), we want to minimize the production cost.

We distinguish two cases:

Case 1: ξ_{ih}^t are random and d_{jk}^t are known. This is a case where we know the future demand.

Case 2: ξ_{ih}^t and d_{jk}^t are random. We may wish to allow for randomness in the future demand.

5.1 Case 1

The optimization problem is the following:

Minimize

$$\sum_{h=1}^{n} \sum_{i=1}^{r} \left[c_{ih}^t a_{ih}^t y^t \; + \; c_{ih}^{t+1} a_{ih}^{t+1} y^{t+1} \right]$$

subject to the probabilistic constraint

$$P \left(\begin{array}{lcll} \zeta_{ih}^t + a_{ih}^t y^t + \xi_{ih}^t - z_{ih}^t & \geq & \displaystyle\sum_{k=h}^{n} \sum_{j=i}^{r} x_{ijhk}^t & \text{all } i, h \\ & & & \\ z_{ih}^t + a_{ih}^{t+1} y^{t+1} + \xi_{ih}^{t+1} & \geq & \displaystyle\sum_{k=h}^{n} \sum_{j=i}^{r} x_{ijhk}^{t+1} & \text{all } i, h \end{array} \right) \geq p \tag{7}$$

and the other constraints

$$\begin{array}{rcll} \displaystyle\sum_{h=1}^{k} \sum_{i=1}^{j} x_{ijhk}^t m_{hk} & \geq & d_{jk}^t & \text{all } j,\ k \\ \displaystyle\sum_{h=1}^{k} \sum_{i=1}^{j} x_{ijhk}^{t+1} m_{hk} & \geq & d_{jk}^{t+1} & \text{all } j,\ k \\ 0 \leq x_{ijhk}^t & \leq & X_{ijhk}^t & \text{all } i,\ j,\ h,\ k,\ t. \end{array}$$

5.2 Case 2

In this case, the optimization problem is the following:

Minimize

$$\sum_{h=1}^{n} \sum_{i=1}^{r} \left[c_{ih}^{t} a_{ih}^{t} y^{t} \ + \ c_{ih}^{t+1} a_{ih}^{t+1} y^{t+1} \right]$$

subject to the probabilistic constaint

$$P \left(\begin{array}{lll} \zeta_{ih}^{t} \ + \ a_{ih}^{t} y^{t} \ + \ \xi_{ih}^{t} \ - \ z_{ih}^{t} \ \geq \ \sum_{k=h}^{n} \sum_{j=i}^{r} x_{ijhk}^{t} & & \text{all } i, \ h \\ z_{ih}^{t} \ + \ a_{ih}^{t+1} y^{t+1} \ + \ \xi_{ih}^{t+1} \ \geq \ \sum_{k=h}^{n} \sum_{j=i}^{r} x_{ijhk}^{t+1} & & \text{all } i, \ h \\ \sum_{h=1}^{k} \sum_{i=1}^{j} x_{ijhk}^{t} m_{hk} \ \geq \ d_{jk}^{t} & & \text{all } j, \ k \\ \sum_{h=1}^{k} \sum_{i=1}^{j} x_{ijhk}^{t+1} m_{hk} \ \geq \ d_{jk}^{t+1} & & \text{all } j, \ k \end{array} \right) \geq p \qquad (8)$$

and the bounds

$$0 \ <= \ x_{ijhk}^{t} \ <= \ X_{ijhk}^{t} \qquad \text{all } i, \ j, \ h, \ k, \ t.$$

The solution of Problem (7) (or Problem (8)) yields optimal x_{ijhk}^{t}, y^{t} and x_{ijhk}^{t+1}, y^{t+1} but we accept as final only the x_{ijhk}^{t}, y^{t} whereas the x_{ijhk}^{t+1}, y^{t+1} will be finalized only after the solution of the next problem.

A joint probabilistic constraint is generally given in the form $Tx \geq \xi$, where T is a matrix and x and ξ are vectors. Below is the matrix T for Problem (8). The column headings are the components of the vector x. The right hand side (RHS) is the vector ξ.

x^1_{1111}	x^1_{1112}	x^1_{1122}	x^1_{1211}	x^1_{1212}	x^1_{1222}	x^1_{2211}	x^1_{2212}	x^1_{2222}	y^1	z_{11}	z_{21}	z_{12}	z_{22}	x^2_{1111}	x^2_{1112}	x^2_{1122}	x^2_{1211}	x^2_{1212}	x^2_{1222}	x^2_{2211}	x^2_{2212}	x^2_{2222}	y^2	*RHS*
-1	-1		-1	-1					a^1_{11}	-1														$-\zeta^1_{11}-\xi^1_{11}$
						-1	-1		a^1_{21}		-1													$-\zeta^1_{21}-\xi^1_{21}$
		-1			-1				a^1_{12}			-1												$-\zeta^1_{12}-\xi^1_{12}$
								-1	a^1_{22}				-1											$-\zeta^1_{22}-\xi^1_{22}$
1																								d^1_{11}
			1			1																		d^1_{21}
	2	1																						d^1_{12}
				2	1		2	1																d^1_{22}
										1				-1	-1		-1	-1					a^2_{11}	$-\xi^2_{11}$
											1									-1	-1		a^2_{21}	$-\xi^2_{21}$
												1				-1			-1				a^2_{12}	$-\xi^2_{12}$
													1									-1	a^2_{22}	$-\xi^2_{22}$
														1										d^2_{11}
																	1			1				d^2_{21}
															2	1								d^2_{12}
																		2	1		2	1		d^2_{22}

6 Properties and Solutions of Models (7) and (8)

Models (7) and (8) contain a probabilistic constraint in a joint constraint form and penalize the violations of the stochastic constraints. Without the probabilistic constraint it is the "simple recourse problem" first introduced and studied by Dantzig (1955) and Beale (1955). The probabilistic constraints applied individually for the stochastic constraints were used first by Charnes, Cooper, and Symonds (1958). Joint constraints involving independent random variables were used by Miller and Wagner (1965). General joint probabilistic constraints including stochastically dependent random variables were introduced by Prékopa (1970) who also obtained convexity results of the problem and also solution methods (1971, 1973, 1980). For a detailed description of programming under probabilistic constraints, see Prékopa (1995). For the case of Problems (7) and (8) we have, as a special case, the following convexity theorem.

Theorem 1 *If the random variables ξ^t_{ih}, ξ^{t+1}_{ih}, $i = 1, \ldots, r$, $h = 1, \ldots, n$ have continuous joint probability distribution and logconcave joint probability density function then the probability on the left hand side in the probabilistic constraint is a logconcave function of the variables x^t_{ijhk}, x^{t+1}_{ijhk}, (all i, j, h, k) and y^t, y^{t+1}.*

For our problem the data are essentially discrete. The continuous case is an approximation. For the case when the random variables are discrete, Prékopa (1990) proposed a dual type method, Prékopa and Li (1995) presented a more general version of it, and

Prékopa, Vizvári, and Badics (1996) proposed a cutting plane method. All these methods allow for the solution of problems which are combinations of probabilistic constrained and simple recourse models and thus, contain as special cases both model constructions. The above methods require the generation of all p-level efficient points (pLEPs). Methods for this have been developed by Murr (1992) and in the above cited paper by Prékopa, Vizvári, and Badics.

T. Szántai (1988) has developed a method for the solution of a probabilistic constrained stochastic programming problem involving multivariate normal, gamma, or Dirichlet distributions. In his method the deterministic constraints as well as the objective function are linear. Successful uses of this method and code are reported in Prékopa and Szántai(1978), Dupaçová, Gaivoronski, Kos, and Szántai (1991), and Murr (1992). Numerical examples for both the discrete case and the continuous approximation will be presented in sections below.

7 Modeling the Variability of the Production

Let us examine more closely our model for the number of performance level i fibers of length h produced. We are saying that

$$f_{ih}^t(y^t) \;=\; a_{ih}^t y^t \;+\; \xi_{ih}^t.$$

In other words, the number produced is the decision variable y^t (the intended production level) multiplied by a_{ih}^t (the expected, i. e. mean, capability of the process to produce performance level i, length h fibers) plus a random variable depending on i and h.

We have a problem in that a more valid model would be to say that

$$f_{ih}^t(y^t) \;=\; (a_{ih}^t \;+\; \xi_{ih}^t)y^t.$$

In other words, the amount of randomness that we anticipate is proportional to y^t rather than independent of y^t.

To modify the model to account for this would introduce random variables on the left hand side of the probabilistic constraint. This would make solution of the model using existing codes difficult or impossible.

Nonetheless we can do almost as well with the model as originally written and with existing codes. Let us consider a model

$$f_{ih}^t(y^t) \;=\; a_{ih}^t y^t \;+\; \xi_{ih}^t b^t,$$

where b^t is a multiplier of the random variable in time period t.

We recommend solving the model for several values of b^t. A solution is valid only when y^t and b^t are approximately equal.

8 Data Common to Continuous Case and Discrete Case

The expected numbers of fibers produced in each category are the following:

a_{11}^t	a_{21}^t	a_{12}^t	a_{22}^t
84	658	126	679

The production cost coefficients for the first time period are:

c_{11}^t	c_{21}^t	c_{12}^t	c_{22}^t
720	720	300	300

The production cost coefficients for the second time period are 5% less:

c_{11}^{t+1}	c_{21}^{t+1}	c_{12}^{t+1}	c_{22}^{t+1}
684	684	285	285

The deterministic demands in the first period are:

d_{11}^t	d_{21}^t	d_{12}^t	d_{22}^t
10	300	30	1000

The deterministic demands in the second period are:

d_{11}^{t+1}	d_{21}^{t+1}	d_{12}^{t+1}	d_{22}^{t+1}
10	300	100	1000

The starting inventories are:

ζ_{11}^t	ζ_{21}^t	ζ_{12}^t	ζ_{22}^t
5	200	10	400

The multipliers for fiber length substitution are:

m_{11}	m_{12}	m_{22}
1	2	1

The upper bounds for the use or downgrading of fiber to meet demand are:

X_{1111}^t	X_{1112}^t	X_{1122}^t	X_{1211}^t	X_{1212}^t	X_{1222}^t	X_{2211}^t	X_{2212}^t	X_{2222}^t
1000	100	1000	40	20	40	1000	1000	1000

The probability level is:

p
0.95

9 Discrete Case

9.1 Case 1

The production variable ξ_{11}^t has 50 possible values, each with probability .02. They are $-25, -24, \ldots, -2, -1, 1, 2, \ldots, 24, 25$.

The production variable ξ_{21}^t also has 50 possible values, each with probability .02. They are $-125, -120, \ldots, -10, -5, 5, 10, \ldots, 120, 125$.

The production variable ξ_{12}^t has 100 possible values, each with probability .01. They are $-50, -49, \ldots, -2, -1, 1, 2, 3, \ldots, 49, 50$.

The production variable ξ_{22}^t has 100 possible values, each with probability .01. They are $-150, -147, -144, \ldots, -6, -3, 3, 6, 9, \ldots, 147, 150$.

The discrete distribution of the production in period $t+1$ is the same as the discrete distribution in period t and is independent of the distribution in period t.

9.2 Case 2

The demand variable d_{11}^t has 50 possible values, each with probability .02. They are $0, 1, 2, \ldots, 48, 49$.

The demand variable d_{21}^t has 100 possible values, each with probability .01. They are $251, 252, 253, \ldots, 349, 350$.

The demand variable d_{12}^t has 100 possible values, each with probability .01. They are $21, 22, 23, \ldots, 119, 120$.

The demand variable d_{22}^t has 100 possible values, each with probability .01. They are $902, 904, 906, \ldots, 1098, 1100$.

The discrete distribution of the demand in period $t+1$ is the same as the discrete distribution in period t and is independent of the distribution in period t.

The remainder of the data for Case 2 is identical to the data for Case 1.

10 Continuous Case

Here we assume that the random variables each have a normal distribution. As stated in the description of the model, the production variables ξ_{11}^t, ξ_{21}^t, ξ_{12}^t, and ξ_{22}^t have mean 0. Their standard deviations are:

ξ_{11}^t	ξ_{21}^t	ξ_{12}^t	ξ_{22}^t
10	45	15	50

The distributions of the production variables within a given time period are correlated.

The following correlation matrix will be used:

	ξ_{11}^t	ξ_{21}^t	ξ_{12}^t	ξ_{22}^t
ξ_{11}^t	1			
ξ_{21}^t	0	1		
ξ_{12}^t	0.7	0	1	
ξ_{22}^t	0	0.7	0	1

The correlation of production variables between the two different time periods is assumed to be zero.

The demands are also assumed to have a normal distribution. Their means and standard deviations are:

	d_{11}^t	d_{21}^t	d_{12}^t	d_{22}^t
mean	20	300	70	1000
std. dev.	10	25	25	50

The demands, within and between time periods, are assumed to be uncorrelated.

11 Solution Method

An approach to solving discrete probabilistic constrained stochastic programming problems has been developed by Prékopa, Vizvári, and Badics (1996). Let the pLEPs be represented by $z^{(1)}$, $z^{(2)}, \ldots, z^{(N)}$. Recall that the matrix version of the stochastic constraint may be written as:

$$Tx \geq \xi.$$

The probabilistic constraint $P(Tx \geq \xi) \geq p$ can be written in the form: $Tx \geq z^{(i)}$ holds for at least one $i = 1, \ldots, N$. If we also have deterministic constraints $Ax = b$, then the problem to be solved becomes:

Minimize

$$c^T x \tag{9}$$

subject to

$$\begin{aligned} Ax &= b \\ Tx &\geq z^{(i)}, \text{for at least one } i = 1, \ldots, N \\ x &\geq 0. \end{aligned}$$

In the above cited paper the second constraint of problem (9) is approximated by the constraint:

$$Tx \geq \sum_{i=1}^{N} \lambda_i z^{(i)},$$

where

$$\sum_{i=1}^{N} \lambda_i = 1$$
$$\lambda_i \geq 0, \quad i = 1, \ldots, N.$$

Then the approximate problem to be solved becomes:

Minimize

$$c^T x \tag{10}$$

subject to

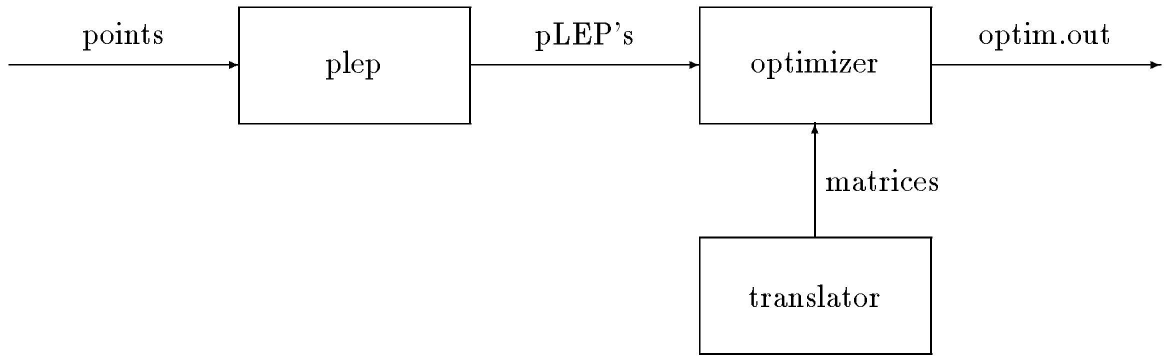
$$\begin{aligned} Ax &= b \\ Tx - u - \sum_{i=1}^{N} \lambda_i z^{(i)} &= 0 \\ \sum_{i=1}^{N} \lambda_i &= 1 \\ \lambda_i &\geq 0, \quad i = 1, \ldots, N \\ x &\geq 0 \\ u &\geq 0. \end{aligned}$$

The solution method works in such a way that first we drop the constraint involving the pLEP's and then subsequently build them up, by the use of a cutting plane method.

If the number of pLEP's is small or the sizes of the matrix T are small, then problem (9) can be solved exactly by the solution of N linear programming problems, where the ith one has the constraint $Tx \geq z^{(i)}$. If $x^{(i)}$ is the optimal solution of the ith problem, and $c^T x = \min_{1 \leq i \leq N} c^T x^{(i)}$, then $x^{(i)}$ is the optimal solution of problem (9).

Systems to solve discrete probabilistic constrained stochastic programming problems this way have been developed by Maros and Prékopa (1990) and Murr (1992). The former one is based on the linear programming system MILP, developed by Maros (1990), the second one is based on the linear programming subroutine DLPRS of IMSL (1987) available on the Convex C220 machine.

Here we report about results of the use of the second system. It consists of the programs *translator*, *plep*, and *optimizer*. The relationships among them are diagrammed below. Names in boxes represent executable programs. Names associated with arrows represent input and/or output files.



Translator: Contains the matrices and vectors in a human (or, more accurately, programmer) readable format. Writes out these data in a format readable by the stochastic optimization programs.

Plep: Computes p-Level Efficient Points for a multivariate discrete distribution given the discrete distribution of each variable.

Optimizer: It has two input files. One is the output of translator, giving the matrices, vectors, and other data about the optimization problem. The second input file is the list of pLEPs. The optimal solution of the discretized stochastic programming problem is at the end of its output.

On a Convex C220 it took us 6 seconds to run *plep*, and it took us 202 seconds to run *optimizer* to solve the linear program 3532 times and obtain the optimal solution.

For the continuous case, we used *pcsp*, which is described in Szántai (1988). On a Convex C220, the optimal solution was obtained in 408 minutes.

12 Computational Results for Production and Demand Both Random (Case 2)

12.1 Decision Variables

For the first period, the intended production level computed in each case is:

	Discrete Case	Continuous Case
y_1	0.997	1.013

The plan for assigning/downgrading the inventory and production to meet the demand in the first period is:

		Discrete Case	Continuous Case
x^1_{1111}	Use high performance, long length for high performance, long length	48.0	44.6
x^1_{1112}	Cut high performance, long length to high performance, short length	15.7	15.2
x^1_{1122}	Use high performance, short length for high performance, short length	88.6	100.0
x^1_{1211}	Downgrade high performance, long length to adequate performance, long length	0.	0.
x^1_{1212}	Downgrade and cut high performance, long length to adequate performance, short length	0.	0.
x^1_{1222}	Downgrade high performance, short length to adequate performance, short length	0.	0.
x^1_{2211}	Use adequate performance, long length for adequate performance, long length	350.	390.8
x^1_{2212}	Cut adequate performance, long length to adequate performance, short length	86.7	133.4
x^1_{2222}	Use adequate performance, short length for adequate performance, short length	926.7	905.4

12.2 Other Information about the Solution

The worst case production levels (values of the random variables) that the model is planning for are as follows. The value p is the (univariate) probability of the variable taking a value equal to or worse than the value shown.

	Discrete Case		Continuous Case	
	ξ	p	ξ	p
ξ^1_{11}	$-25.$	.00	-24.2	.008
ξ^1_{21}	$-125.$	.00	-144.1	.001
ξ^1_{12}	$-47.$	.03	-36.0	.006
ξ^1_{22}	$-150.$	.00	-182.4	.000

The maximum demand levels in the first period that the model guarantees to satisfy are as follows. Here the value p is the (univariate) probability of the random variable taking a value equal to or greater than the value shown.

	Discrete Case		Continuous Case	
	d	p	d	p
d_{11}^1	48.	.02	44.6	.007
d_{21}^1	350.	.00	390.8	.000
d_{12}^1	120.	.00	130.4	.008
d_{22}^1	1100.	.00	1172.2	.000

13 Conclusion

13.1 Summary

Both discrete and continuous probability distributions can be used successfully for this problem. The probability distributions are not difficult to create for a person who understands a probability distribution.

The discrete case and the continuous case differed by less than 2% in the recommended production level for the first period.

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