

*Solutions to Final Exam Practice Questions*

**Insurance Advertising**

*Decision Variables*

$x_1$  – amount spent on TV ads (in thousands of dollars)

$x_2$  – letters sent

*Constraints*

$$(25 + 6)x_1 + (0.004 + 0.003)x_2 \geq 3000 \quad (\text{number of new insurance contracts})$$

$$6x_1 + 0.003x_2 \geq 0.30[(25 + 6)x_1 + (0.004 + 0.003)x_2] \quad (30\% \text{ for house insurance})$$

$$x_1, x_2 \geq 0$$

*Objective function*

$$\min 1000x_1 + 0.25x_2$$

**Cable TV**

*Variables*

$s_A$  – start of activity A

$s_B$  – start of activity B

$s_C$  – start of activity C

$s_D$  – start of activity D

$s_E$  – start of activity E

$s_F$  – start of activity F

$t_A$  – end of activity A

$t_B$  – end of activity B

$t_C$  – end of activity C

$t_D$  – end of activity D

$t_E$  – end of activity E

$t_F$  – end of activity F

$T$  – end of the whole project

*Constraints*

Relation of starting and end times:

$$t_A = s_A + 2$$

$$t_B = s_B + 4$$

$$t_C = s_C + 3$$

$$\begin{aligned}t_D &= s_D + 2 \\t_E &= s_E + 10 \\t_F &= s_F + 4\end{aligned}$$

Predecessors:

$$\begin{aligned}s_A &\geq 0 \\s_B &\geq t_A \\s_C &\geq t_B \\s_D &\geq t_B \\s_E &\geq t_C, \quad s_E \geq t_D \\s_F &\geq t_B\end{aligned}$$

Final time:

$$T \geq t_A, \quad T \geq t_B, \quad T \geq t_C, \quad T \geq t_D, \quad T \geq t_E, \quad T \geq t_F$$

*Objective*

$$\min T$$

### Issuing Bonds

(a) SUMPRODUCT(\$B13:\$B15,B4:B6)

(b) Target cell E24 (maximize). Changing cells: B13:B15, B18:E18. Constraints: B13:B15 <= E13:E15, B24:E24 >= B3:E3, Assume nonnegative, assume linear model.

Below is an algebraic formulation of the problem. It was not required by the question, but is included here as additional study material.

#### Decision variables:

$x_1$  - amount of bonds A issued

$x_2$  - amount of bonds B issued

$x_3$  - amount of bonds C issued

$y_i$  - amount of short term loans taken in year  $i$ ,  $i = 1, 2, 3$ .

$z_i$  - cash balance at the end of year  $i$ ,  $i = 1, 2, 3, 4$

All variables express amounts in millions of dollars.

#### Objective function:

$$\max z_4$$

#### Constraints:

Cash balance:

$$z_1 = 55 + y_1 + x_1 + x_2 + x_3 - 120$$

$$z_2 = 1.045z_1 + y_2 - 1.1y_1 - 0.085x_1 - 0.080x_2 - 0.040x_3 + 20$$

$$z_3 = 1.045z_2 + y_3 - 1.1y_2 - 0.085x_1 - 0.080x_2 - 0.040x_3 + 20$$

$$z_4 = 1.045z_3 - 1.1y_3 - 0.085x_1 - 0.040x_3 + 110$$

Cash on hand

$$z_1 \geq 25, \quad z_2 \geq 25, \quad z_3 \geq 25, \quad z_4 \geq 25,$$

Bond limits

$$0 \leq x_1 \leq 60, \quad 0 \leq x_2 \leq 60, \quad 0 \leq x_3 \leq 60$$

## Computer Assembly

*Decision Variables*

$x_t$  – number of computers assembled in Factory 1 in month  $t$ ,  $t = 1, 2, 3, 4$

$y_t$  – number of computers assembled in Factory 2 in month  $t$ ,  $t = 1, 2, 3, 4$

$s_t$  – number of computers held in inventory at the end of month  $t$ ,  $t = 1, 2, 3, 4$

*Constraints*

$$200 + x_1 + y_1 = s_1 + 500, \quad (\text{inventory balance for month 1})$$

$$s_1 + x_2 + y_2 = s_2 + 600, \quad (\text{inventory balance for month 2})$$

$$s_2 + x_3 + y_3 = s_3 + 700, \quad (\text{inventory balance for month 3})$$

$$s_3 + x_4 + y_4 = s_4 + 800, \quad (\text{inventory balance for month 4})$$

$$2x_t \leq 800, \quad 3y_t \leq 600, \quad t = 1, 2, 3, 4 \quad (\text{labor constraint})$$

$$x_t \geq 0, \quad y_t \geq 0, \quad s_t \geq 0, \quad t = 1, 2, 3, 4 \quad (\text{non-negativity constraints})$$

Note that the constraints on the  $s_t$  ensure that we always satisfy the demand.

*Objective function*

$$\min 400(x_1 + x_2 + x_3 + x_4) + 300(y_1 + y_2 + y_3 + y_4) + 100(s_1 + s_2 + s_3 + s_4)$$

## Mixing Fertilizer

(a) SUM(E16:E17) – SUM(B19:C19)

(b) B11\*\$D16

(c) Target cell E20 (maximize). Changing cells B16:C17. Constraints: B16:C17  $\geq$  B23:C24 (this is the key blending constraint), B18:C18  $\leq$  B3:C3. Assume nonnegative, assume linear model.

## Production and Shipment Optimization

(a) =D6+D12/D7

(b) =SUM(C18:C20)

(c) =F10\*F21

(d) D24 = SUMPRODUCT(B3:B5,F18:F20),  
D25 = SUM(D12:F12),  
D26 = SUMPRODUCT(D3:F5,C18:E20)

(e) =D9\*F21, =F24-D27

(f) target cell F26; minimize; constraints: F18:F20 <= C3:C5, C21:E21 <= F22, D27 <= D10, C21:E21 <= D14:F14, assume linear model, assume nonnegative

## DigiNav

### Decision Variables

$x_1$  – number of standard models produced per week  
 $x_2$  – number of deluxe models produced per week

### Constraints

$$\begin{aligned}\frac{1}{250}x_1 + \frac{1}{200}x_2 &\leq 1 && \text{(line capacity – 1 week)} \\ 4x_1 + 7x_2 &\leq 1200 && \text{(quality control minutes)} \\ 0.95x_2 &\leq 0.25(0.96x_1 + 0.95x_2) && \text{(units shipped no more than 25\% deluxe)} \\ x_1, x_2 &\geq 0\end{aligned}$$

### Objective Function

$$\max (800)(0.96)x_1 + (1200)(0.95)x_2 - \frac{30000}{250}x_1 - \frac{38000}{200}x_2 - \frac{120}{60}(4x_1 + 7x_2)$$

## Goofy Park

(a) (Diagram omitted from this answer key.)

(b)

### Variables

$s_A$  – start of activity A  
 $s_B$  – start of activity B  
 $s_C$  – start of activity C  
 $s_D$  – start of activity D  
 $s_E$  – start of activity E  
 $s_F$  – start of activity F  
 $s_G$  – start of activity G  
 $s_H$  – start of activity H  
 $t_A$  – end of activity A  
 $t_B$  – end of activity B  
 $t_C$  – end of activity C  
 $t_D$  – end of activity D

$t_E$  – end of activity E  
 $t_F$  – end of activity F  
 $t_G$  – end of activity G  
 $t_H$  – end of activity H  
 $T$  – end of the whole project

*Constraints*

Relation of starting and end times:

$$\begin{aligned}t_A &= s_A + 4 \\t_B &= s_B + 2 \\t_C &= s_C + 2 \\t_D &= s_D + 8 \\t_E &= s_E + 5 \\t_F &= s_F + 2 \\t_G &= s_G + 2 \\t_H &= s_H + 8\end{aligned}$$

Predecessors:

$$\begin{aligned}s_A &\geq 0 \\s_B &\geq t_A \\s_C &\geq t_B \\s_D &\geq t_B \\s_E &\geq t_B \\s_F &\geq t_C \\s_G &\geq t_D \\s_G &\geq t_E \\s_G &\geq t_F \\s_H &\geq t_A\end{aligned}$$

Final time:

$$T \geq t_A, \quad T \geq t_B, \quad T \geq t_C, \quad T \geq t_D, \quad T \geq t_E, \quad T \geq t_F, \quad T \geq t_G, \quad T \geq t_H$$

*Objective Function*

$$\min T$$

**(c)**

*Additional Variables*

$r_A$  – reduction of time of activity A  
 $r_C$  – reduction of time of activity C  
 $r_D$  – reduction of time of activity D

Other variables the same.

*Modified Constraints*

Relation of starting and end times:

$$t_A = s_A + 4 - r_A$$

$$t_C = s_C + 2 - r_C$$

$$t_D = s_D + 8 - r_D$$

*Additional Constraint*

$$T \leq 12$$

The other constraints remain the same.

*Objective Function (in thousands of dollars)*

$$\min 10r_A + 5r_B + 12r_C.$$

(d) Add a new binary variable

$$z = \begin{cases} 1 & \text{buy the prefabricated court} \\ 0 & \text{do not buy} \end{cases}$$

and modify the start/end relation for E as follows:

$$t_E = s_E + 5 - 4z.$$

Also, add the constraint  $z \in \{0, 1\}$ .

*Objective Function*

$$\min 10r_A + 5r_B + 12r_C + 30z.$$

**Note:** Again, to get a formulation similar to that used in Professor Eckstein's sections, just eliminate the  $t_X$  variables using the equality constraints. This comment applies to all parts of this question. Either form is acceptable.

## Memory Manufacturing and Purchasing

*Decision Variables*

$x_1$  – number of Type 1 SIMM produced per week

$x_2$  – number of Type 2 SIMM produced per week

$x_3$  – number of DIMM produced per week

$x_4$  – number of X-DIMM produced per week

$y_1$  – number of chips purchased from Sleeman

$y_2$  – number of chips purchased from Chang

$y_3$  – number of chips purchased from Malaya

### Constraints

$$\begin{aligned}0.03x_1 + 0.02x_2 + 0.04x_3 + 0.06x_4 &\leq 120 && \text{(labor hours)} \\0.025x_1 + 0.025x_2 + 0.05x_3 + 0.05x_4 &\leq 125 && \text{(machine hours)} \\4x_1 + 4x_2 + 8x_3 + 8x_4 &= y_1 + y_2 + y_3 && \text{(memory chips)} \\y_1 &\leq 9000 \\y_2 &\leq 8500 \\y_3 &\leq 8000 \\y_1 &\leq (0.4)(y_1 + y_2 + y_3) \\y_2 &\leq (0.4)(y_1 + y_2 + y_3) \\y_3 &\leq (0.4)(y_1 + y_2 + y_3) && \text{(blending constraints)} \\x_1, x_2, x_3, x_4, y_1, y_2, y_3 &\geq 0\end{aligned}$$

### Objective Function

$$\begin{aligned}\max & (75x_1 + 70x_2 + 155x_3 + 165x_4) \\& - (14.50y_1 + 12.60y_2 + 12.25y_3) \\& - 20.50(0.03x_1 + 0.02x_2 + 0.04x_3 + 0.06x_4) \\& - 15.00(0.025x_1 + 0.025x_2 + 0.05x_3 + 0.05x_4)\end{aligned}$$

## Grading Language Tests

### Decision Variables

$x_{11}$  – number of written tests assigned to senior raters  
 $x_{12}$  – number of computer tests assigned to senior raters  
 $x_{13}$  – number of recorded tests assigned to senior raters  
 $x_{21}$  – number of written tests assigned to junior raters  
 $x_{22}$  – number of computer tests assigned to junior raters  
 $x_{23}$  – number of recorded tests assigned to junior raters  
 $x_{31}$  – number of written tests assigned to outside raters  
 $x_{32}$  – number of computer tests assigned to outside raters  
 $x_{33}$  – number of recorded tests assigned to outside raters

### Constraints

$$\begin{aligned}x_{11} + x_{21} + x_{31} &= 800 && \text{(written tests)} \\x_{12} + x_{22} + x_{32} &= 800 && \text{(computer tests)} \\x_{13} + x_{23} + x_{33} &= 800 && \text{(recorded tests)} \\20x_{11} + 5x_{12} + 15x_{13} &\leq 40 \times 5 \times 60 && \text{(time of senior raters)} \\30x_{21} + 6x_{22} + 18x_{23} &\leq 40 \times 9 \times 60 && \text{(time of junior raters)}\end{aligned}$$

$$\begin{aligned}
x_{11} &\geq 0.10(x_{11} + x_{21}) && \text{(10\% senior in house raters for written tests)} \\
x_{12} &\geq 0.10(x_{12} + x_{22}) && \text{(10\% senior in house raters for computer tests)} \\
x_{13} &\geq 0.10(x_{13} + x_{23}) && \text{(10\% senior in house raters for recorded tests)} \\
x_{11}, \dots, x_{33} &\geq 0 && \text{(all variables nonnegative)}
\end{aligned}$$

*Objective Function*

$$\min 25x_{31} + 15x_{32} + 20x_{33}$$

## Mall

(a)

*Variables*

$s_A$  – start of activity A  
 $s_B$  – start of activity B  
 $s_C$  – start of activity C  
 $s_D$  – start of activity D  
 $s_E$  – start of activity E  
 $s_F$  – start of activity F  
 $s_G$  – start of activity G  
 $s_H$  – start of activity H  
 $t_A$  – end of activity A  
 $t_B$  – end of activity B  
 $t_C$  – end of activity C  
 $t_D$  – end of activity D  
 $t_E$  – end of activity E  
 $t_F$  – end of activity F  
 $t_G$  – end of activity G  
 $t_H$  – end of activity H  
 $T$  – end of the whole project

*Constraints*

Relation of starting and end times:

$$\begin{aligned}
t_A &= s_A + 25 \\
t_B &= s_B + 10 \\
t_C &= s_C + 30 \\
t_D &= s_D + 22 \\
t_E &= s_E + 17 \\
t_F &= s_F + 10 \\
t_G &= s_G + 14 \\
t_H &= s_H + 5
\end{aligned}$$

Predecessors:

$$\begin{aligned}s_A &\geq 0 \\s_B &\geq 0 \\s_C &\geq t_A \\s_C &\geq t_B \\s_D &\geq t_C \\s_E &\geq t_C \\s_F &\geq t_D \\s_F &\geq t_E \\s_G &\geq t_E \\s_G &\geq t_E \\s_H &\geq t_F \\s_H &\geq t_G\end{aligned}$$

Final time:

$$T \geq t_A, \quad T \geq t_B, \quad T \geq t_C, \quad T \geq t_D, \quad T \geq t_E, \quad T \geq t_F, \quad T \geq t_G, \quad T \geq t_H$$

*Objective Function*

$$\min T$$

**(b)**

*Additional Binary*

$z_B$  – reduce time of activity B?

$z_C$  – reduce time of activity C?

$z_D$  – reduce time of activity D?

$z_E$  – reduce time of activity E?

$z_F$  – reduce time of activity F?

$z_G$  – reduce time of activity G?

*Additional Continuous Variables*

$r_B$  – reduction of time of activity B

$r_C$  – reduction of time of activity C

$r_D$  – reduction of time of activity D

$r_E$  – reduction of time of activity E

$r_F$  – reduction of time of activity F

$r_G$  – reduction of time of activity G

Other variables the same.

*Modified Constraints*

Relation of starting and end times:

$$t_B = s_A + 10 - r_A$$

$$t_C = s_C + 30 - r_C$$

$$t_D = s_D + 22 - r_D$$

$$t_E = s_E + 17 - r_D$$

$$t_F = s_F + 17 - r_F$$

$$t_G = s_G + 17 - r_G$$

### Logical Upper Bounds

$$0 \leq r_B \leq 5z_B$$

$$0 \leq r_C \leq 12z_C$$

$$0 \leq r_D \leq 6z_D$$

$$0 \leq r_E \leq 8z_E$$

$$0 \leq r_F \leq 8z_F$$

$$0 \leq r_G \leq 8z_G$$

### Additional Constraint

$$2000z_B + 3000z_C + 800z_D + 1200z_E + 600z_F + 1000z_G + 400r_B + 250r_C + 300r_D + 200r_E + 150z_F + 350r_G \leq 25,000$$

The other constraints and the objective remain the same.

## Germanium

### Decision Variables

$x$  - raw germanium processed

$y$  - regular refined germanium remelted

### Constraints

$$x - 0.25x - 0.1y \geq 50 \quad \text{refined germanium produced}$$

$$0.3x + 0.9y \geq 35 \quad \text{premium germanium produced}$$

$$y \leq 0.45x \quad \text{we cannot remelt more than we have}$$

Also, all variables are nonnegative.

### Objective:

$$\min(44 + 23)x + 18y + 12(0.25x + 0.1y)$$

## Sausages

### Decision Variables

$a_1$  - amount of meat A in sausages 1 (total)

$a_2$  - amount of meat A in sausages 2 (total)

$a_3$  - left over meat A

$b_1$  - amount of meat B in sausages 1 (total)

$b_2$  - amount of meat B in sausages 2 (total)

$b_3$  - left over meat B

$x_1$  - number of chickens I bought

$x_2$  - number of chickens II bought

All these variables are restricted to be nonnegative.

*Constraints*

$$a_1 \geq 75\%(a_1 + b_1) \quad (\text{meat A in Sausages 1})$$

$$a_2 \geq 65\%(a_2 + b_2) \quad (\text{meat A in Sausages 2})$$

$$a_1 + b_1 \leq 100 \quad (\text{sales of Sausages 1})$$

$$a_2 + b_2 \leq 80 \quad (\text{sales of Sausages 2})$$

$$2x_1 + x_2 = a_1 + a_2 + a_3 \quad (\text{balance of meat A})$$

$$x_1 + 2x_2 = b_1 + b_2 + b_3 \quad (\text{balance of meat B})$$

*Objective:*

$$\max 4(a_1 + b_1) + 3(a_2 + b_2) + 0.5a_3 + 0.3b_3 - 10x_1 - 6x_2$$

**Atlas Valve**

*Decision Variables*

$$x_i = \begin{cases} 1 & \text{make valve } i \\ 0 & \text{do not make valve } i \end{cases} \quad i = 1, 2, 3, 4, 5$$

$$y_i - \text{number of valves } i \text{ made; } i = 1, 2, 3, 4, 5$$

*Constraints*

Logical upper bounds

$$0 \leq y_i \leq 100x_i, \quad i = 1, \dots, 5.$$

Line time:

$$16x_1 + 10x_2 + 15x_3 + 8x_4 + 20x_5 + 2y_1 + 2.4y_2 + 1.5y_3 + 1.25y_4 + 3y_5 \leq 320.$$

Labor

$$4y_1 + 3y_2 + 3y_3 + 2.3y_4 + 3.5y_5 \leq 600.$$

Raw material:

$$15y_1 + 20y_2 + 30y_3 + 12y_4 + 40y_5 \leq 4000.$$

Logical restriction

$$x_2 + x_4 \leq 1.$$

$x_i \in \{0, 1\}$  (binary) for all  $i = 1, \dots, 5$ .

*Objective Function*

$$\max 741y_1 + 800y_2 + 620y_3 + 545y_4 + 1025y_5$$

## Investment Planning

Note: parts (f) and (g) were inadvertently omitted, and may be found in the “addendum” handout.

(a) =SUM(B18:B21)

(b) = - SUMPRODUCT(B3:D3,B17:D17)

(c) =SUMPRODUCT(B4:D4,B18:D18)

(d) =SUMPRODUCT(E3:F3,E\$13:F\$13)

(e) =G9 + E17 + F17 - G3

(f) The formula in G18 is = G17\*(1+D\$9) + E18 + F18 - G4

(g) Target cell G21; maximize; changing cells are B17:D21, E13:F13;  
constraints: B17:D21 binary, B22:D22 <= B17:D17 (sell assets that you bought),  
G17:G21 >=0 (keep nonnegative cash balance); assume linear model; assume nonnegative

## Dining Halls

*Decision Variables*

$$x_i = \begin{cases} 1 & \text{open hall } i \\ 0 & \text{do not open hall } i \end{cases} \quad i = 1, 2, 3, 4$$

$y_i$  - meals served in hall  $i = 1, 2, 3, 4$

$z$  - meals served by the caterer.

*Constraints*

Serve all guests:

$$y_1 + y_2 + y_3 + y_4 + z \geq 3000.$$

Logical upper bound on the number of meals served in halls:

$$0 \leq y_1 \leq 1200x_1$$

$$0 \leq y_2 \leq 900x_2$$

$$0 \leq y_3 \leq 600x_3$$

$$0 \leq y_4 \leq 500x_4$$

Caterer's capacity:

$$0 \leq z \leq 400$$

Binary variables:

$$x_1, x_2, x_3, x_4 \in \{0, 1\}.$$

*Objective Function*

$$\min 2000x_1 + 3y_1 + 1400x_2 + 5y_2 + 1000x_3 + 3y_3 + 800x_4 + 4y_4 + 7z.$$

## Inventory with Setup Costs

- (a) =E12 + C13 - B2
- (b) =B13\*E\$4
- (c) =B13\*E\$5
- (d) =D13\*F\$5
- (e) =C13\*D\$3 + B13\*B\$2 + E13\*F\$3 + D13\*F\$2
- (f) =SUM(I13:I18)
- (g) target cell I19; minimize; changing cells B13:D18;  
 constraints: B13:B18 binary, D13:D18 binary, C13:C18 <= G13:G18, C13:C18 >= F13:F18,  
 E13:E18 >= 0, E13:E18 <= H13:H18, D13:D18 <= B13:B18; assume linear model; you may,  
 but need not, assume nonnegative

## Housing Developments

*Decision variables:*

$$x_i = \begin{cases} 1 & \text{accept offer } i \\ 0 & \text{do not accept} \end{cases} \quad i = 1, \dots, 7.$$

*Constraints*

$$\begin{aligned} x_1 + x_2 + \dots + x_7 &= 3 \\ 40x_1 + 60x_2 + 55x_3 + 65x_4 + 35x_5 + 75x_6 + 50x_7 &\leq 150 \\ x_2 + x_3 + x_6 &\geq 1 \\ x_2 + x_3 + x_4 + x_7 &\geq 1 \\ x_1 + x_5 &= 1 \\ x_1, \dots, x_7 &\in \{0, 1\} \end{aligned}$$

*Objective function*

$$\begin{aligned} \max \quad & (40)(370)x_1 + (20 \cdot 180 + 40 \cdot 140)x_2 + (25 \cdot 200 + 30 \cdot 150)x_3 \\ & + (65)(160)x_4 + (35)(420)x_5 + x_6(75)(130) + x_7(50)(190) \end{aligned}$$

## Siting Restaurants

- (a) SUMPRODUCT(B6:E9,B12:E15)
- (b) SUMPRODUCT(\$A12:\$A15,B12:B15)
- (c) B5\*B18
- (d) Target cell B21 (maximize). Changing cells: B12:E15. Constraints: B12:E15 - binary,  
 B16:E16=1, F18:F19 <= B1:B2, B18 <= C18.

## Assigning People

We denote projects A, B, C, and D as 1, 2, 3, and 4, respectively.

*Decision variables:*

$$x_{ij} = \begin{cases} 1 & \text{assign exactly } j \text{ people to project } i \\ 0 & \text{do not assign exactly } j \text{ people} \end{cases} \quad i = 1, 2, 3, 4; j = 1, 2, 3.$$

*Constraints:*

$$\begin{aligned} x_{11} + x_{12} + x_{13} &= 1 \\ x_{21} + x_{22} + x_{23} &= 1 \\ x_{31} + x_{32} + x_{33} &= 1 \\ x_{41} + x_{42} + x_{43} &= 1 \end{aligned}$$

$$(x_{11} + x_{21} + x_{31} + x_{41}) + 2(x_{12} + x_{22} + x_{32} + x_{42}) + 3(x_{13} + x_{23} + x_{33} + x_{43}) \leq 6 \quad (\text{people available})$$
$$x_{11}, \dots, x_{43} \in \{0, 1\}$$

*Objective*

$$\min 5x_{11} + 3x_{12} + 2x_{13} + 7x_{21} + 5x_{22} + 3x_{23} + 9x_{31} + 8x_{32} + 7x_{33} + 11x_{41} + 8x_{42} + 8x_{43}.$$

## Locating Restaurants

(a)

*Decision variables:*

$$x_j = \begin{cases} 1 & \text{locate restaurant in town } j \\ 0 & \text{do not locate} \end{cases} \quad j = 1, \dots, 6.$$

*Constraints*

We convert the distances to 0–1 data (0 if distance more than 5 miles, 1 if no more than 5 miles) and we formulate covering constraints:

$$\begin{aligned} x_1 + x_3 &\geq 1 \\ x_2 + x_4 &\geq 1 \\ x_1 + x_3 + x_4 &\geq 1 \\ x_2 + x_3 + x_4 + x_5 &\geq 1 \\ x_4 + x_5 + x_6 &\geq 1 \\ x_5 + x_6 &\geq 1 \end{aligned}$$

In addition we have the logical constraint

$$x_2 = x_5$$

and, of course

$$x_1, \dots, x_6 \in \{0, 1\}.$$

*Objective function*

$$\min x_1 + x_2 + \dots + x_6$$

**(b)**

*Additional decision variables:*

$$y_i = \begin{cases} 1 & \text{cover town } i \\ 0 & \text{do not cover} \end{cases} \quad i = 1, \dots, 6.$$

*Constraints*

$$\begin{aligned} x_1 + x_3 &\geq y_1 \\ x_2 + x_4 &\geq y_2 \\ x_1 + x_3 + x_4 &\geq y_3 \\ x_2 + x_3 + x_4 + x_5 &\geq y_4 \\ x_4 + x_5 + x_6 &\geq y_5 \\ x_5 + x_6 &\geq y_6 \\ x_1 + x_2 + \dots + x_6 &= 2 \\ x_2 &= x_5 \\ x_1, \dots, x_6 &\in \{0, 1\} \\ y_1, \dots, y_6 &\in \{0, 1\} \end{aligned}$$

*Objective function*

$$\max 11y_1 + 20y_2 + \dots + 12y_6$$

### Data Network

**(a)** G26 = SUM(B26:F26)

**(b)** B31 = SUM(B26:B30)

**(c)** C37 = \$B\$4\*B16

**(d)** B22 = SUMPRODUCT(B8:F12,B16:F20)

**(e)** The target cell is B22 (minimize); changing cells are B16:F20 and B26:F30; constraints are B16:F20 = integer, B26:F30 ≤ B37:F41 (logical upper bounds), and B33:F33 = B2:F2 (flow balance); assume nonnegative, assume linear model.

### Modular Software

*Decision variables:*

$$x_j = \begin{cases} 1 & \text{market package } j \\ 0 & \text{do not market} \end{cases} \quad j = 1, \dots, 7.$$
$$y_l = \begin{cases} 1 & \text{write module } l \\ 0 & \text{do not write} \end{cases} \quad l = A, \dots, F.$$

*Constraints*

$$x_1 \leq y_A, \quad x_1 \leq y_B$$

$$x_2 \leq y_A, \quad x_2 \leq y_E$$

and similarly for all packages, up to

$$x_7 \leq y_E, \quad x_7 \leq y_F.$$

*Objective*

$$\max 7x_1 + 10x_2 + \dots + 8x_7 - (4y_A + 5y_B + \dots + 5y_F)$$

## Pension Fund

(a) =sum(b18:e18);

(b) =\$g\$6 \* b17;

(c) =\$g\$2 \* b17;

(d) =sumproduct(b\$18:e\$18,b3:e3);

(e) =b12\*\$g\$18;

(f) =sumproduct(b8:e8,b18:e18);

(g) target cell b18 - maximize; changing cells b17:e18; constraints: b17:e17 binary; b18:e18 >= b19:e19, b18:e18 <= b20:e20; b24:b26 >= c24;c26, b24:b26 <= d24:d26; g18 <= g2; assume linear model, assume nonnegative.

## Schedules

*Decision variables:*

$x_j$  – number of people working on schedule  $j = 1, \dots, 8$

They are restricted to be integer and nonnegative.

*Constraints*

$$x_1 + x_3 + x_5 \geq 5 \quad (\text{Mon AM people})$$

$$x_2 + x_4 + x_6 \geq 5 \quad (\text{Mon PM people})$$

$$x_1 + x_4 + x_5 \geq 5 \quad (\text{Tue AM people})$$

and so on, for each shift.

*Objective*

$$\min 500(x_1 + x_2 + \dots + x_6) + 150(x_7 + x_8)$$

### **Pollution**

- (a) =A8\*A9
- (b) =sumproduct(b5:f5,b\$10:f\$10)
- (c) B16=b9+c9, B17=d9+e9+f9
- (d) changing cells b9:f10, target b19=sumproduct(b2:f3,b9:f10)
- (e) b9:f9 - binary, b10:f10 <= b12:f12, g5:g6 >= h5:h6, b16;b17 >= c16:c17, b16:b17 <= d16:d17.

### **Warehouses**

- (a) = sum(b21:e21)
- (b) = sum(b21:b24)
- (c) = 4\*b16
- (d) B28=sumproduct(b3:e3,b16:e16), B29=sumproduct(b8:e11,b21:e24), B30=B28+B29
- (e) target B30 minimize, changing b16:e16, b21:e24, constrains b16:e16 binary, b21:e24 binary, f21:f24=1, b25:e25 <= b26:e26.

### **Semiconductors**

- (a) B8=RISKSIMTABLE(E8:E12), B9 = B1 - B8
- (b) C8=RISKBINOMIAL(B8,B4), copy to C9
- (c) =MIN(INT(C8/2),C9))
- (d) D14, D16
- (e) 12 slots for SCA and 8 for SCB, because the mean number of SMM's is the largest for Simulation 3.
- (f) 0.6441 (the mean value of the output cell D16) for Simulation 1. This can also be estimated roughly by 65% (which is 100% - 35%) by using the percentile output table in the detail statistics report

### **Bouquets**

- (a) RISKBINOMIAL(B9,B1) (because each of B9 bouquets fails with probability B1).
- (b) RISKPOISSON(B2)
- (c) MIN(B11,B12) (because we can't sell more than we have and more than people want)
- (d) B21
- (e) RISKSIMTABLE(D4:D8)

## Managing Capital

- (a)  $B6 = \text{RISKPOISSON}(B2)$
- (b)  $B9 = \text{RISKNORMAL}(B7, B8)$
- (c)  $B11 = \text{RISKSIMTABLE}(E5: E9)$
- (d)  $B12 = \text{MIN}(B9, B11)$
- (e)  $B13 = B9 - B12$ ;  $B14 = B13 * E3$
- (f)  $E11 = B1 - B11$
- (g)  $E12 = \text{RISKUNIFORM}(E1, E2)$
- (h)  $E13 = E11 * E12$
- (i)  $E15 = B1 - B9 - B14 + E13$
- (j)  $E15, B9$
- (k) Simulation 3: 4,900,000 Reserve, rest in bonds.
- (l) 5,218,756 (the 90% Percentile of Cell B9)

## Wild West

- (a)  $=\text{RISKPOISSON}(E\$1)$
- (b)  $=\text{RISKPOISSON}(E\$2)$
- (c)  $=\text{MIN}(C19, D19)$
- (d)  $=\text{MIN}(C19 - E19, C\$6)$
- (e)  $=E19 * E\$3 - F19 * E\$4 - C\$7$
- (f) Rent the small corral, because the mean profit per day is then the highest: 480.3672

## Mixing Funds

- (a)  $=\text{RISKSIMTABLE}(G11: G14)$
- (b)  $=\text{SUMPRODUCT}(C4: E4, C\$11: E\$11)$
- (c)  $=\text{RISKNORMAL}(H4, I4), =\text{RISKNORMAL}(H5, I5), =\text{RISKUNIFORM}(H7, I7)$
- (d)  $=\text{SUMPRODUCT}(C15: C17, E15: E17)$
- (e)  $=\text{IF}(E19 < H7, 1, 0)$
- (f)  $E19, E20$

(g) The fourth case; prob = 0.112

(h) The second case

### Gambling

(a) =IF(C12>0,RISKDISCRETE(B\$1:D\$1,B\$2:D\$2),0)

(b) =C12+ B13

(c) 7.187 (or roughly \$7.19)

(d) 0.4465 from the summary statistics for B8 (or one may estimate 40%-45% from the percentile detail statistics for B6).

(e) 0.305 from the summary statistics for B9 (or one may estimate slightly more than 30% since the 70th percentile of B6 is 10.25).

### Shuttle Buses

(a) B10 = RISKSIMTABLE(B6:E6)

(b) B17 = RISKPOISSON(E\$1)

(c) C17 = G16+B17

(d) D17 = MIN(C17,B\$10)

(e) E17 = C17 - D17

(f) We gave full credit for F17 = RISKBINOMIAL(E17,E\$3), which ought to work; however, due to a bug in @Risk, this formula will cause errors if E17 contains 0 (it should just return 0 in this case). To avoid these errors, you may use IF(E17=0,0,RISKBINOMIAL(E17,E\$3)) instead.

(g) G17 = E17 - F17

(h) F10 = E2\*SUM(D17:D61); F11 = F10 - B11

(i) Bus for 20 people (Simulation 2), average number left is 0.2458.