

The Moore of the Moore–Penrose Inverse

1. Introduction

This Appendix is based on [76].

E. H. Moore (1862–1932) introduced and studied the *general reciprocal* during the decade 1910–1920. He stated the objective as follows:

“The effectiveness of the reciprocal of a non–singular finite matrix in the study of properties of such matrices makes it desirable to define if possible an analogous matrix to be associated with each finite matrix κ^{12} even if κ^{12} is not square or, if square, is not necessarily non–singular.” [572, p. 197],

Moore constructed the general reciprocal, established its uniqueness and main properties, and justified its application to linear equations. This work appears in [571], [572, Part 1, pp. 197–209].

The general reciprocal was rediscovered by R. Penrose [637] in 1955, and is nowadays called the *Moore–Penrose inverse*. It had to be rediscovered because Moore’s work was sinking into oblivion even during his lifetime: it was much too idiosyncratic, and used unnecessarily complicated notation, making it illegible for all but very dedicated readers.

Much of Moore’s work is today of interest only for historians. One of the exceptions is his work on the general reciprocal, that may still interest, and benefit, mathematical researchers. It is summarized below, and – where necessary – restated in plain English and modern notation.

To illustrate the difficulty of reading the original Moore, and the need for translation, here is a theorem from [572, Part 1, p. 202]

(29.3) **Theorem.**

$$\mathfrak{U}^C \mathfrak{B}^1 \Pi \mathfrak{B}^2 \Pi \kappa^{12}.) . \\ \exists | \lambda^{21} \text{ type } \mathfrak{M}_{\kappa^*}^2 \overline{\mathfrak{M}}_{\kappa}^1 \ni . S^2 \kappa^{12} \lambda^{21} = \delta_{\mathfrak{M}_{\kappa}^1}^{11} . S^1 \lambda^{21} \kappa^{12} = \delta_{\mathfrak{M}_{\kappa^*}^2}^{22}$$

One symbol needs explanation: \mathfrak{U} stands for the *number system* used throughout, and \mathfrak{U}^C denotes a number system of *type C*, that is a *quasi–field* with a *conjugate* and an *order relation*, see [572, Part 1, p. 174] for details. All results below are for type *C* number systems, so this assumption will not be repeated. The rest of the theorem, in plain English, is:

(29.3) **Theorem.**

For every matrix A there exists a unique matrix $X : R(A) \rightarrow R(A^H)$ such that

$$AX = P_{R(A)} , \quad XA = P_{R(A^H)} .$$

The plan of this appendix:

- Section 2 summarizes the results of Moore’s lecture to the American Mathematical Society in 1920 [571].
- Section 3 is a translation of the main results in [572, Part 1, pp. 197–209].

2. The 1920 lecture to the AMS

This is an abstract of a lecture given by E. H. Moore at the Fourteenth Western Meeting of the American Mathematical Society, held at the University of Chicago in April 9–10, 1920. There were 19 lectures in

two afternoons; only the abstracts, written by Arnold Dresden (Secretary of the Chicago Section) appear in the *Bulletin*. Dresden writes

“In this paper Professor Moore calls attention to a useful extension of the classical notion of the reciprocal of a nonsingular square matrix.” [571, p. 394].

The details: Let A be any $m \times n$ complex matrix. Then there exists a unique $n \times m$ matrix A^\dagger , the *reciprocal* of A , such that:

- (1) the columns of A^\dagger are linear combinations of the conjugate of the rows of A ,
- (2) the rows of A^\dagger are linear combinations of the conjugate of the columns of A ,
- (3) $AA^\dagger A = A$.

If A is of rank r , then A^\dagger is given explicitly as follows:

($r \geq 2$):

$$A^\dagger[j_1, i_1] = \frac{\sum_{\substack{i_2 < \dots < i_r \\ j_2 < \dots < j_r}} A \begin{pmatrix} i_2 \cdots i_r \\ j_2 \cdots j_r \end{pmatrix} \overline{A \begin{pmatrix} i_1 & i_2 \cdots i_r \\ j_1 & j_2 \cdots j_r \end{pmatrix}}}{\sum_{\substack{k_1 < \dots < k_r \\ \ell_1 < \dots < \ell_r}} A \begin{pmatrix} k_1 \cdots k_r \\ \ell_1 \cdots \ell_r \end{pmatrix} \overline{A \begin{pmatrix} k_1 \cdots k_r \\ \ell_1 \cdots \ell_r \end{pmatrix}}},$$

($r = 1$):

$$A^\dagger[j, i] = \frac{\overline{A[i, j]}}{\sum_{k\ell} A[k, \ell] \overline{A[k, \ell]}},$$

($r = 0$):

$$A^\dagger[j, i] = 0,$$

where $A \begin{pmatrix} g_1 \cdots g_k \\ h_1 \cdots h_k \end{pmatrix}$ denotes the determinant of the k^2 numbers $A[g_i, h_j]$ and \bar{x} denotes the conjugate of x .

The relation between A and A^\dagger is mutual: A is the reciprocal of A^\dagger , viz.,

(4),(5): the columns (rows) of A are linear combinations of the conjugates of rows (columns) of A^\dagger ,

(6) $A^\dagger AA^\dagger = A^\dagger$.

The linear combinations of the columns of A (A^\dagger) are the linear combinations of the rows of A^\dagger (A) and constitute the m -dimensional vectors \mathbf{y} (n -dimensional vectors \mathbf{x}) of an r -dimensional subspace M (N) of \mathbb{C}^m (\mathbb{C}^n). Let \overline{M} (\overline{N}) denote the conjugate space of the conjugate vectors $\overline{\mathbf{y}}$ ($\overline{\mathbf{x}}$). Then the matrices A, A^\dagger establish 1-1 linear vector correspondences between the spaces M, \overline{M} and the respective subspaces N, \overline{N} ; $\mathbf{y} = A\mathbf{x}$ is equivalent to $\mathbf{x} = A^\dagger\mathbf{y}$ and $\overline{\mathbf{x}} = \overline{\mathbf{y}}A$ is equivalent to $\overline{\mathbf{u}} = \overline{\mathbf{v}}A^\dagger$.

3. The general reciprocal in *General Analysis*

The centerpiece of Moore's work on the general reciprocal is Section 29 of [572], his treatise on *General Analysis*, edited by R. W. Barnard and published posthumously. These results were since rediscovered, some more than once.

For a matrix A denote:

A^H the conjugate transpose of A ,

$R(A)$ the range of A .

For index sets I, J :

A_{I*} or $A[I, *]$ the submatrix of rows indexed by I ,

A_{*J} or $A[* , J]$ the submatrix of columns indexed by J ,

A_{IJ} the submatrix of A with rows in I and columns in J .

If A is non-singular, its inverse A^{-1} satisfies,

$$AX = I, \quad XA = I .$$

Moore begins by constructing *generalized identity matrices* to replace the identity matrices above. This is done in Lemma (29.1) and Theorem (29.2). The *general reciprocal* is then constructed in Theorems (29.3) and (29.4), and its properties are studied in the sequel.

(29.1) **Lemma.**

Let A be a non-zero $m \times n$ matrix, and let A_{IJ} be a maximal non-singular submatrix of A .

- (1) $A_{*J}^H A_{*J}$ is Hermitian, positive-definite¹.
- (2) $(A_{*J}^H A_{*J})^{-1}$ is Hermitian, positive-definite.
- (3) $A_{I*} A_{I*}^H$ is Hermitian, positive-definite.
- (4) $(A_{I*} A_{I*}^H)^{-1}$ is Hermitian, positive-definite.
- (5) $P_{R(A)} := A_{*J} (A_{*J}^H A_{*J})^{-1} A_{*J}^H$ (the *generalized identity* on $R(A)$).
- (6) $P_{R(A^H)} := A_{I*}^H (A_{I*} A_{I*}^H)^{-1} A_{I*}$ (the *generalized identity* on $R(A^H)$).
- (7) $P_{R(A)} \mathbf{x} = \mathbf{x}$ for all $\mathbf{x} \in R(A)$.
- (8) $\mathbf{x}^H P_{R(A)} = \mathbf{x}^H$ for all $\mathbf{x} \in R(A)$.
- (9) $P_{R(A^H)} \mathbf{x} = \mathbf{x}$ for all $\mathbf{x} \in R(A^H)$.
- (10) $\mathbf{x}^H P_{R(A^H)} = \mathbf{x}^H$ for all $\mathbf{x} \in R(A^H)$.
- (11) Let

$$\begin{aligned} X &:= A_{I*}^H (A_{I*} A_{I*}^H)^{-1} A_{IJ} (A_{*J}^H A_{*J})^{-1} A_{*J}^H \\ &= A_{I*}^H (A_{I*}^H A_{I*})^{-1} P_{R(A)} [I, *] \\ &= P_{R(A^*)} [* , J] (A_{*J}^H A_{*J})^{-1} A_{*J}^H, \quad (\text{the } \textit{general reciprocal} \textit{ of } A) . \end{aligned}$$

- (12) X maps $R(A^H)$ onto $R(A)$.
- (13) $AX = P_{R(A)}$.
- (14) $XA = P_{R(A^H)}$.

(29.2) **Theorem.**

Let M be a finite dimensional subspace.

- (1) There exists a unique linear operator² P_M such that

$$P_M \mathbf{x} = \mathbf{x}, \quad \mathbf{x}^H P_M = \mathbf{x}^H, \quad \text{for all } \mathbf{x} \in M .$$

- (2) P_M is positive semidefinite, Hermitian and idempotent.
- (3) $M = R(P_M)$.
- (4) For all \mathbf{x} : $P_M \mathbf{x} \in M$, $(\mathbf{x} - P_M \mathbf{x}) \in M^\perp$.
- (5) $\mathbf{x} \perp M \iff P_M \mathbf{x} = \mathbf{0}$.
- (6) For any matrix A

$$\begin{aligned} A = P_M &\iff \begin{cases} A\mathbf{x} = \mathbf{x}, \text{ for all } \mathbf{x} \in M \\ R(A^H) \subset M \end{cases} \\ &\iff \begin{cases} A\mathbf{x} = \mathbf{x}, \text{ for all } \mathbf{x} \in M \\ A\mathbf{x} = \mathbf{0}, \text{ for all } \mathbf{x} \in M^\perp \end{cases} . \end{aligned}$$

¹Moore calls it *proper* (i.e., the determinants of all principal minors are non-zero), *positive* (i.e., the corresponding quadratic form is non-negative) and Hermitian.

²Called the *generalized identity matrix* for the space M , and denoted by δ_M , [572, p. 199].

(29.3) Theorem.

For every matrix A there exists a unique matrix $X : R(A) \rightarrow R(A^H)$ such that

$$AX = P_{R(A)}, \quad XA = P_{R(A^H)}.$$

We call X the *general reciprocal* and denote it by A^\dagger .

(29.4) Theorem.

For every matrix A the general reciprocal A^\dagger satisfies:

- (1) $A^\dagger AA^\dagger = A^\dagger, AA^\dagger A = A$.
- (2) $\text{rank } A = \text{rank } A^\dagger$.
- (3) $R(A) = R(A^{\dagger H}), R(A^H) = R(A^\dagger)$.
- (4) $A^{\dagger H} = (A^H)^\dagger, A = (A^\dagger)^\dagger$.

(29.45) Corollary.

If $A[I, J]$ is a maximal nonsingular submatrix of A then:

- (1) $A^\dagger = P_{R(A^H)}[* , J]A_{IJ}^{-1}P_{R(A)}[I , *]$.
- (2) $\mathbf{x}^H A^\dagger \mathbf{y} = \mathbf{x}_I^H A_{IJ}^{-1} \mathbf{y}_J$.

(29.5) Theorem.

For any matrix A , the following statements on a matrix X are equivalent:

- (a) $X = A^\dagger$
- (b) $R(X) \subset R(A^H), AX = P_{R(A)}$
- (c) $R(X) \subset R(A^H), R(X^H) \subset R(A), AXA = A$.

(29.55) Corollary.

If $A = \begin{bmatrix} B & O \\ O & C \end{bmatrix}$ then $A^\dagger = \begin{bmatrix} B^\dagger & O \\ O & C^\dagger \end{bmatrix}$.

(29.6) Theorem.

Let the matrix A be Hermitian. Then

- (1) A^\dagger is Hermitian.
- (2) If A is positive semi-definite then so is A^\dagger . □

Consider a square matrix A . Then for any principal submatrix A_{II} ,

$$A_{II} = A_{II}A_{II}^\dagger A_{II}$$

More can be said if A is Hermitian positive semi-definite:

(29.7) Theorem.

Let A be Hermitian positive semi-definite. Then for any principal submatrix A_{II}

- (1) $A_{II}A_{II}^\dagger A_{I*} = A_{I*}$.
- (2) $A_{*I}A_{II}^\dagger A_{II} = A_{*I}$.

(29.8) Theorem.

Let A be Hermitian positive semi-definite. Then the following statements, about a vector \mathbf{x} , are equivalent.

- (a) $\mathbf{x}^H A \mathbf{x} = 0$,
- (b) $\mathbf{x} \perp R(A)$,
- (c) $\mathbf{x} \perp R(A^\dagger)$,
- (d) $\mathbf{x}^H A^\dagger \mathbf{x} = 0$.

The general reciprocal can be used to solve linear equations

$$A\mathbf{x} = \mathbf{b},$$

that are assumed consistent, i.e. $\mathbf{b} \in R(A)$, or the way Moore expresses consistency: $\text{rank } A = \text{rank } [A \ \mathbf{b}]$.

(29.9) **Theorem.**

Let A be a matrix, \mathbf{b} a vector in $R(A)$. Then the general solution of $A\mathbf{x} = \mathbf{b}$ is

$$A^\dagger \mathbf{b} + \{\mathbf{y} : \mathbf{y} \perp R(A^H)\}.$$

Note: Moore avoids the concept of null-space, and the equivalent form of the general solution, $A^\dagger \mathbf{b} + N(A)$. Also, Moore does not consider the case where $A\mathbf{x} = \mathbf{b}$ is inconsistent. A. Bjerhammar [99], R. Penrose [638] and Yuan–Yung Tseng³ [828] would later use A^\dagger to obtain least-squares solutions. This has become the major application of the Moore–Penrose inverse.

Suggested further reading

Bliss ([111], [112]), Parshall [628], Parshall and Rowe [629], Siegmund-Schultze ([768], [769]).

³Tseng, a student of Barnard at Chicago (1933), extended the Moore–Penrose inverse to linear operators.

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