Advanced Operations Management: Solutions to Homework 4

Fall, 2000

Problem 1

$$\begin{array}{rl} d \le q : & c(d,q) = 0.15q - 0.3d \;\Rightarrow\; c_o = 0.15 \\ d \ge q+1 : & c(d,q) = 0.15q - 0.3q = -0.15q \;\Rightarrow\; c_u = 0.15 \end{array}$$

Pick the smallest q^* such that

$$F(q^*) \ge \frac{c_u}{c_o + c_u} = \frac{0.15}{0.3} = 0.5 \;\Rightarrow\; q^* = 90$$

Problem 2

It is easier to think of q as the number of books printed above the quantity already in stock. Consider a case when we print a non-zero number of books first.This incurres a fixed cost of \$50000 plus a variable cost of \$15 per book printed. For $d \le q + 2000$:

$$\begin{aligned} c(d,q) &= 50000 + 15q - 35d - 5(q + 2000 - d) \\ &= 10q - 30d + 40000 \;\Rightarrow\; c_o = 10 \end{aligned}$$

For $d \ge q + 2001$:

$$\begin{aligned} c(d,q) &= 50000 + 15q - 35(q + 2000) \\ &= -20q - 20000 \;\Rightarrow\; c_u = 20 \end{aligned}$$

Pick the smallest q^* such that

$$F(q^* + 2000) \ge \frac{c_u}{c_o + c_u} = \frac{2}{3},$$

then $q^* + 2000 = 7000$ and $q^* = 5000$. Plug in this value back into the cost function:

$$\begin{array}{rl} d \le 7000 : & c(d,q^*) = 90000 - 30d \\ d \ge 7001 : & c(d,q^*) = -120000 \end{array}$$

Remember, that we think of the cost as negative profit. Thus

$$\begin{aligned} E[profit] &= E[-c(D,q^*)] = \\ &= 30(0.3 \times 5000 + 0.2 \times 6000 + 0.4 \times 7000) \\ &\quad -0.9 \times 90000 + 0.1 \times 120000 = 96000 \end{aligned}$$

If we do not print any extra books then our expected profit is

$$E[profit] = 35 \times 2000 = 70000.$$

Thus, our decision is to print 5000 books.

For a case of 4000 books in stock we have

$$\begin{aligned} d \le q+4000: &\qquad c(d,q) = 10q - 30d + 30000 \\ d \ge q+4001: &\qquad c(d,q) = -20q - 90000 \end{aligned}$$

Pick the smallest q^* such that

$$F(q^*+4000) \ge \frac{c_u}{c_o+c_u} = \frac{2}{3} \quad \Rightarrow \quad q^* = 7000 - 4000 = 3000.$$

Calculation of the expected profit similar to the one above yields

$$E[profit] = 126000.$$

Without printing any extra books:

$$profit = 35 \times 4000 = 140000,$$

so our solution is not to print any extra books.

Problem 3

$$\begin{aligned} d \le q: &\qquad c(d,q) = 10q - 25d \quad \Rightarrow \quad c_o = 10 \\ d \ge q+1: &\qquad c(d,q) = -15q \quad \Rightarrow \quad c_u = 15 \end{aligned}$$

Pick q^* that solves an equation

$$P(D \le q^*) = \frac{c_u}{c_o+c_u} = \frac{15}{10+15} = 0.6$$

After standardizing normal random variable D :

$$P\left(Z \le z^* = \frac{q^*-100}{30}\right) = 0.6$$

$z^* = 0.25$ gives

$$q^* = 100 + 30z^* = 100 + 7.5 = 107.5$$

Problem 4

Assume that the cost of producing 1000 Tv's is \$100000.

$$\begin{aligned} d \le q: &\qquad c(d,q) = 100q - 250d \quad \Rightarrow \quad c_o = 100 \\ d \ge q+1: &\qquad c(d,q) = -150q \quad \Rightarrow \quad c_u = 150 \end{aligned}$$

Pick q^* that solves an equation

$$P(D \le q^*) = \frac{c_u}{c_o+c_u} = \frac{15}{10+15} = 0.6$$

After standardizing normal random variable D :

$$P\left(Z \le z^* = \frac{q^*-6000}{2000}\right) = 0.6$$

$z^* = 0.25$ gives

$$q^* = 6000 + 2000 \times 0.25 = 6500$$

Problem 5

$$
\begin{aligned}
h &= h_p p = 0.3 \times 60 = 18, \\
q^* &= EOQ = \sqrt{\frac{2KE[D]}{h}} = \sqrt{\frac{2 \times 1000 \times 1040}{18}} = 107.5, \\
E[X] &= E[D]L = 1040 \times \frac{1}{26} = 40, \\
\sigma_X &= \sigma_D \sqrt{L} = 50.99 \times \sqrt{\frac{1}{26}} = 10.
\end{aligned}
$$

a. When all demand is backlogged,

$$P(X \geq r^*) = \frac{hq^*}{c_B E[D]} = \frac{18 \times 107.5}{50 \times 1040} = 0.0372,$$

thus,

$$P(X \leq r^*) = P\left(Z \leq z^* = \frac{r^* - 40}{10}\right) = 1 - 0.0372 = 0.9628.$$

$z^* = 1.78$ gives a reorder point

$$r^* = 40 + 10 \times 1.78 = 57.8$$

and a safety stock level of

$$r^* - E[X] = 17.8.$$

b. In a case of lost sales,

$$c_{LS} = c_B + \text{lost profit from one item} = 50 + 100 - 60 = 90.$$

Reoreder point r^* is a solution of

$$P(X \geq r^*) = \frac{hq^*}{hq^* + c_{LS} E[D]} = \frac{18 \times 107.5}{18 \times 107.5 + 90 \times 1040} = 0.0203,$$

thus,

$$P(X \leq r^*) = P\left(Z \leq z^* = \frac{r^* - 40}{10}\right) = 1 - 0.0203 = 0.9797.$$

$z^* = 2.05$ gives a reorder point

$$r^* = 40 + 10 \times 2.05 = 60.5$$

and a safety stock level of

$$r^* - E[X] = 20.5.$$