Advanced Operations Management: Solutions to Homework 5a

Fall, 2000

Problem 1

From problem 2 on page 920 (problem 5 of the Homework 4): $q = 107.5$, $E[X] = 40$ and $\sigma_X = 10$. Since X is normally distributed, we will use a Normal Loss Function in the expression for $E[B_r]$. Thus,

$$NL\left(\frac{r - E[X]}{\sigma_X}\right) = \frac{q(1 - SLM_1)}{\sigma_X}.$$

$SLM_1 = 80\%$

$$NL\left(\frac{r - 40}{10}\right) = \frac{107.5(1 - 0.8)}{10} = 2.15$$

Since $NL(-2.14) = NL(2.14) + 2.14 = 2.145788$,

$$r = 40 - 2.14 \times 10 = 18.6.$$

$SLM_1 = 90\%$

$$NL\left(\frac{r - 40}{10}\right) = \frac{107.5(1 - 0.9)}{10} = 1.075$$

Since $NL(-0.99) = NL(0.99) + 0.99 = 1.07491$,

$$r = 40 - 0.99 \times 10 = 30.1.$$

$SLM_1 = 95\%$

$$NL\left(\frac{r - 40}{10}\right) = \frac{107.5(1 - 0.95)}{10} = 0.538$$

Since $NL(-0.25) = NL(0.25) + 0.25 = 0.5363$,

$$r = 40 - 0.25 \times 10 = 37.5.$$

$SLM_1 = 99\%$

$$NL\left(\frac{r - 40}{10}\right) = \frac{107.5(1 - 0.99)}{10} = 0.108$$

Since $NL(0.86) = 0.1080$,

$$r = 40 + 0.86 \times 10 = 48.6.$$

For $SLM_2 = s_0 = 0.5$:

$$P(X > r) = \frac{s_0 q}{E[D]} = \frac{2 \times 107.5}{1040} = 0.207$$

After standardizing X :

$$P\left(Z \leq z^* = \frac{r - 40}{10}\right) = P(X \leq r) = 1 - P(X > r) = 0.793$$

we get $z^* = 0.82$ and $r = 40 + 0.82 \times 10 = 48.2$.

Problem 2

a. For $r = 25$:

$$E[B_r] = \frac{1}{4} \times 5 = 1.25.$$

and

$$SLM_1 = 1 - \frac{E[B_r]}{q} = 1 - \frac{1.25}{100} = 98.75\%.$$

b. We have to find such r that

$$E[B_r] = q(1 - SLM_1) = 100 \times (1 - 0.95) = 5$$

For $15 \leq r \leq 20$:

$$E[B_r] = \frac{1}{4}(20 - r) + \frac{1}{12}(25 - r) + \frac{1}{4}(30 - r) = \frac{175}{12} - \frac{7}{12}r.$$

Thus

$$r = \frac{175 - 12 \times 5}{7} = 16.4.$$

c. Pick the smallest r such that

$$P(X > r) \leq \frac{s_0 q}{E[D]} = \frac{2 \times 100}{1000} = 0.2$$

For $r \geq 30$ we have $P(X > r) = 0$, for $30 > r \geq 25$: $P(X > r) = 0.25 > 0.2$. Thus, pick $r = 30$.