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MODELING OPPORTUNITIES IN AUCTIONS

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Abstract. This paper argues that the answers to interesting questions about real auctions depend, often critically, on the particular mathematical assumptions that go into a model of an auction situation. It then suggests some understudied areas for fruitful mathematical research on competitive bidding. These include asymmetry, financially constrained bidders, complicated information structures, bidder decisions about auction participation, the effect of repeated auctions involving the same participants, auctioning items with interrelated values, and transaction costs. The paper also discusses two major areas where new, complicated auctions are being designed—combinatorial spectrum auctions and electricity and transmission rights auctions.

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1 Introduction

Bidders have been trying to figure out how to bid and auctioneers have been designing auctions for centuries. In the last half-century, academics of various kinds have started to study these matters. The first Ph.D. in operations research was for a dissertation on competitive bidding.[1] Economists of all kinds (experimentalists, econometricians, and especially game theorists), including at least two Nobel Prize winners,[2] have been involved, and so have a few sociologists.[3]

The first purpose of this paper is to argue that modeling and context matter. In other words, the answers to interesting questions about auctions depend, often critically, on the particular mathematical assumptions to go into a model of an auction situation. This is not a new topic for me,[4] but it is important, and it bears review here to motivate the second purpose of this paper. That purpose is to suggest some areas for fruitful mathematical research on competitive bidding. The bad news about this second purpose is that mathematical convenience is often at cross-purposes with useful results about real auctions. This is also the good news, at least for mathematically sophisticated researchers, since it implies that there is a lot of worthwhile work still to be done.

2 Modeling and Context Matter

Models, by definition, abstract from reality, including some aspects and ignoring others. For example, physicists build models that neglect friction. Such models are often useful for real phenomena because the neglected aspects are relatively unimportant. However, the history of auction theory is replete with examples of neglected factors turning out to be extremely important. The examples below illustrate this. For a fuller treatment, see Rothkopf and Harstad (1994a).

2.1 Should a Bidder with More Competition Bid More Aggressively?

Let's start with a basic simple question. Should a bidder in a standard sealed bid auction bid more aggressively if she is faced with additional competitors? Auction theory's initial answer to this question was an unambiguous "yes." The answer came from a model in which bidders had independent private values for the object being auctioned. The optimal bid for a bidder in such a model is below her value for the object. To maximize her expected winnings, she must trade off her profit if she wins (which is her value for the object less her bid) against her chance of winning. When her competition increases, her chance of winning with any bid decreases, so she has incentive to increase her bid part of the way towards her value.

For fifteen years, all bidding models were private value models. However, people concerned with construction bidding[5] and, especially, offshore oil lease auctions[6] started

[1] L. Friedman, 1956 and 1957.
[2] See M. Friedman 1960 and W. Vickrey 1961.
[3] C. W. Smith 1990.
[4] See Rothkopf, Teisberg and Kahn 1990, pp.107, Rothkopf 1991, Rothkopf and Harstad 1994a, and Harstad and Rothkopf 2000.
[5] Rothkopf 1969, See also Wilson 1969.

considering common value models. In a common value model, the value of what is being auctioned off is uncertain, but what ever it is, it has the same value to each bidder. For example, the value of an offshore oil tract is highly uncertain, but for who ever wins it, the amount of oil in it, the cost of extracting that oil, and the price of oil will be essentially the same. A bidder in a common value situation faced with an increase in competition beyond two competitors should bid *less* aggressively. The reason is that the bidder must compensate for selection bias. In a common value situation, the bidders' estimating errors are the independent random variables. The bidder who is likely to win is the bidder who has made the greatest overestimate of the value.[7] Bidders have to compensate for this effect by bidding less aggressively. The larger the number of bidders, the larger the compensation that is needed. In all of the bidding models I have seen, this need to compensate for selection bias (or the "winner's curse," as is it commonly called) dominates any other effects when a bidder has two or more competitors.

2.2 Why Are Vickrey Auctions Rare?

In 1961, William Vickrey proposed and analyzed what he apparently though was a novel auction form.[8] It was a sealed bid auction in which the bidder making the highest bid would win the object being sold, but the price would not be the amount of that bid, but rather the amount of the best losing bid. Such auctions are called "second-price" or Vickrey auctions. It is easy to show that in a single, isolated Vickrey auction it is a dominant (not merely equilibrium) strategy for a bidder to bid exactly what the object being sold is worth to her.[9] If all bidders follow this dominant strategy, the auction result will be perfectly efficient in that the bidder with the highest value will always win. In addition, Vickrey proved a "revenue neutrality theorem." This theorem showed that (in the independent private values context of the day), the average revenue received by the seller would be the same as it would in standard sealed bid auction or a progressive oral auction.

Given these results, Vickrey auctions appear to be a superior auction form, but to this day they remain unusual. Why? The answer is that two modeling assumptions are too strong. One of these assumptions is that it is sufficient to analyze an isolated auction. The other assumption is that bidders will believe that the rules of the auction will always be followed. Rothkopf, Teisberg and Kahn 1990 modeled a Vickrey auction in a broader context. They assumed that the winning bidder would have to negotiate with third parties and that these third parties would be able to capture some fraction of the "economic rent" revealed by the auction. For example, if in an auction with publicly opened bids, the winning bidder offered \$5 million but only had to pay \$3 million, everyone she subsequently had to negotiate with would know that she had \$2 million that she didn't

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[6] Capen Clapp, and Cambell 1971.

[7] If bidders with higher estimates bid more, a very reasonable assumption about actual, individually optimal, and Nash equilibrium strategies, then the bidder with the highest estimate will win. Even if all of the estimates are unbiased, the estimate of the winning bidder, the only one that matters economically, will be highly biased.

[8] Vickrey 1961. In fact, the auction form was unusual rather than new. See Lucking-Reiley 2000.

[9] No matter what other bidders do, if she loses because she bids less than her value or wins because she bids more than her value, she is worse off.

"need." Because of this, they would be able to get from her some extra fraction of that $2 million. It turns out that if the logic of Myerson's revenue equivalence theorem[10] can be applied to situations like this with third party rent capture. It shows that what is equivalent is not the bid taker's expected revenue, but the expected amount paid by the bidders. Hence, on average all of the extra money captured by third parties comes out of the pocket of the bid taker.[11]

In addition to this problem from the bid taker's point of view, bidders are wary of Vickrey auctions because they worry about whether the bid taker will follow the rules. If the bid taker in the example above is not completely trustworthy, he may concoct an imaginary or insincere bid of, say, $4 million, thus increasing his revenue by $1 million. Rothkopf and Harstad 1995 developed two models of bid taker cheating in Vickrey auctions. In both, even a small probability of cheating by bid takers makes sustained, honest Vickrey auctions impossible. Recently, Lucking-Reiley 2000 documented cheating in a Vickrey auction.

2.3 Are Sealed Bid Auctions Efficient?

Generally, economists do not believe that standard sealed bid auctions are perfectly efficient. They are aware that occasionally the bidder with the highest value will get too greedy and lose the auction to a bidder with a lower value. Ironically, however, *a priori* symmetric sealed bid auctions are perfectly efficient in the economists' standard models of them.[12] The reason is that in these models the equilibrium bidding strategies are monotonic in the bidder's private estimate of the value to her of what is being sold. Recently, Harstad, Rothkopf and Waehrer[13] showed that this unrealistic result is an artifact of the assumption that a bidder's private information is a scalar. When bidders have a vector of private information, say an estimate of the value to themselves and estimates of the value to each other bidder, the monotonicity of bids in own value estimates disappears.

2.4 Are There n Bidders or N Potential Bidders?

Most game theoretic models of auctions, and all of the early ones, assumed that there were a known number, n, of bidders. These models were used to examine the relative revenue to be obtained from different auction forms. See, for example, Milgrom and Weber 1982 who explored this issue using the concept of affiliated values.[14] They found that on average revenue from progressive auctions was greater than or equal to the expected revenue of a Vickrey auction, which in turn was greater than or equal to the

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[10] Myerson 1981 presents a more general version of Vickrey's original revenue equivalence theorem.

[11] In reaction to this analysis, Nurmi and Salomaa 1993 published a cryptographic protocol for Vickrey auctions that limits who knows what about bids. This may deal with this concern about Vickrey auctions, but it does not deal with bidders' concern about cheating by bid takers.

[12] See, for example, the most cited model of Milgrom and Weber 1982 or any of the other models discussed in McAfee and McMillan's 1987 survey paper.

[13] Harstad, Rothkopf and Waehrer, 1996

[14] Bidders with (strictly) affiliated values will raise their estimate of the value of what is being sold if they find out that another bidder's value is higher. Both independent private values and common values are limiting cases of affiliated values.

expected revenue from standard sealed bidding or a Dutch auction. In addition, in their model having more bidders was better for the bid taker.

However, these conclusions are not necessarily true about real auctions. If instead of assuming that there are n bidders, we assume that there is a pool of potential bidders who must decide whether to incur the cost of participating in the auction, two things may happen. First, the auction form producing the greatest expected revenue can change. Second, the bid taker generally prefers a limited number of bidders to a large number. Implicitly, he is paying the cost of auction participation by the bidders.[15]

2.5 If a Bidder Offers Her Maximum Bid, Does She Care if it Wins?

Until recently, auction theory assumed that bidders were indifferent between winning and losing an item at the maximum bid they were willing to make. With perfect capital markets, that might indeed be the case. However, a few years ago, I consulted for a bidder who valued a license he was bidding on at about $85 million dollars. However, he was only able to arrange financing for the license that would allow him to bid $65 million. He cared a lot about whether his maximum bid of $65 million won. In this auction, which had significant bid increments, he wanted to be sure to be the first to bid $65 million.[16] Recently, the auction literature has been augmented by models that consider bidders facing financing constraints.[17]

2.6 Does a Bidder's Bid Affect the Future Bids of Her Competitors?

The vast bulk of the literature on auction theory deals with models of single, isolated auctions. However, auctions that are an important part of commerce are, almost by definition, not single and isolated. An Auction theory that deals only with single, isolated auctions makes no more sense than would chemistry if dealt only with single atoms and not with molecules.

This was brought home to me in the 1960s when my then employer, Shell, asked me to evaluate an analysis of bidding strategy. The situation was that Shell sold a solvent it manufactured, methyl ethyl ketone (MEK), by standard sealed bid to government agencies several times each year. Shell's analyst had done some careful statistical work to back out freight costs and quantity effects. After correcting for these effects, he obtained a narrow probability distribution for the best competitive bid Shell had faced. His analysis, which used this distribution in a decision theory model of a single isolated auction,[18] suggested that Shell could make much more profit from these auctions if it would bid a little more aggressively. The reason that it seemed so much more profitable to bid a bit more aggressively is that, with the narrow distribution of the best competitive bid, doing so would apparently raise Shell's probability of winning from about 50% to

[15] See Harstad 1990, Harstad 1993.

[16] Even though most progressive auctions have strictly controlled bid increments and no progressive auction allows infinitesimal bid increments, the bulk of auction theory assumes that bids can vary continuously. For a discussion of the effect of finite bid increments, see Rothkopf and Harstad 1994b.

[17] See particularly Che and Gale 1998.

[18] He was using Friedman's (1956) approach and following suggestions in a number of academic and trade papers of the day that discussed using historical data on auctions to estimate the probability distribution of the best competitive bid for use in it.

about 99.5% at little cost in profit if it won. The problem with this analysis is that there was only one other manufacturer of MEK, Esso (now ExxonMobile). It was unlikely that Esso would let Shell win 99.5% of the government business. Rather, Esso would probably continue to win about half of the auctions at any reasonable price. Hence, the only long run effect of bidding a bit more aggressively would be to lower the price.

A few years later, Shmuel Oren and I wrote a paper in which we considered the effect of a bidder's bid in one auction on the bids of its competitors in future auctions.[19] Rather than solve a sequential game, we used control theory reaction functions, a construct from nineteenth century economics. In some ways, it would have been nice if we had solved a full sequential game model in which every bidder's dynamic strategy is optimal with respect to every other bidder's dynamic strategy. However, not only would that have been hard mathematically, it is not clear that it would have been behaviorally realistic. Such a model would depend upon what bidders believe their competitors would do in situations that have never occurred. When the same bidders interact regularly, sociology may have as much to say as game theory about how they behave.[20]

3 Some Modeling Opportunities

I hope that these examples have brought home the idea that modeling and context matter. Next, I want to discuss some of the mathematically difficult issues that arise in attempting to build useful models of real auctions. In the examples above, I have already alluded to several issues such as non-scalar information, endogenous bidder entry determination, capital limitations of bidders, and the strategic effect of sequential auctions. These are all good issues and, while there has been some recognition of them, there is still much to do to integrate them fully them into auction theory. Among them, I think that non-scalar information and sequential auctions will prove the hardest to deal with. Both appear to add complications that are not easily simplified while maintaining some generality.

There are other issues I will discuss as well, and then I want to address two major auction design problems: spectrum auctions and electricity and transmission rights auctions.

3.1 Asymmetry

In most game theoretic auction models, a bidder's strategy is a general function of her private information. The Nash equilibrium that is considered the solution to such game models requires that each bidder's strategy function be optimal with respect to the functions chosen by all other bidders. Finding optimal functions is difficult enough. Finding an equilibrium set of functions is even harder. Often, the only solutions that game theorists have been able to find have been for the special case in which all of the bidders are identical *a priori* and use the same strategy function.[21] This is useful theory,

[19] Oren and Rothkopf 1975.

[20] See, for example, Smith 1990.

[21] There are a few exceptions. See for example McAfee and McMillan 1989, Maskin and Reiley forthcoming. In addition, Rothkopf 1969 deals with asymmetry in a limited strategy game model and Klemperer 1998 has dealt with small asymmetries.

but there are many situations in which there are commonly known asymmetries. Evaluating how different auction rules favor bidders with higher values or the effect of subsidizing high-cost bidders requires asymmetric models.[22] In my view, *a priori* asymmetry is sufficiently difficult to deal with in generality that exploring various approximate methods and limited special cases is useful.

3.2 Auctioning Items with Interrelated Values

When multiple items are auctioned, things can get complicated. In general, we have to think about situations in which a bidder's value for an item can depend upon which other items she wins.[23] In some cases, items may be truly independent. In some, the dependency may take on a particularly simple form. For example, a bidder may have a value for only one item. Similarly, a bidder may have a budget constraint that limits its ability to buy many items. In some cases, however, we may be tempted to apply auctions to the sale of highly complicated assets. For example, we may wish to auction off the right to use various segments of a region's railroad tracks during particular time segments. Someone who wants to buy the right to send a train from city A to city B during a particular time period needs to collect a complicated pattern of rights. There may be alternative paths and time periods that meet her need, but she has use for only one workable collection. There are also intermediate cases in which different items have synergistic values, but there are no absolute requirements for certain combinations.[24]

In any event, a seller offering multiple items with interrelated values must decide whether to sell them sequentially or simultaneously. If they are to be sold sequentially, in what order will they be sold? If they are to be sold simultaneously, will it be an iterative process or a one-time sealed bid? Will bidders be able to submit bids on combinations of items, e.g., a bid on A and B? Will bidders be able to submit contingent bids, e.g., a bid on A if I don't win B; budget constrained bids, e.g., all other bids are withdrawn once I have spent $1,000; or OR-bids, e.g., $X for A or $Y for B, but not both? Whatever the form of the sale, bidders need to be able to figure out their chances of acquiring synergistic combinations or of staying within their budgets. Will the allocation of items to bidders tend to be efficient? Will the auction allow bidders wanting multiple items to collude (tacitly or explicitly) in order to avoid competing and driving up prices? Where bidders have private information that is of use to other bidders, will the auction form

[22] Rothkopf, Harstad and Fu have used Rothkopf's 1969 limited-strategy model to evaluate the surprising effects of subsidizing high-cost bidders.

[23] Also of potential significance and of great difficulty, but not discussed further here, is the situation in which a bidder's value for an item depends upon who wins some other item. For example, if there are two broadcasting licenses for a region being sold, my value for a license might depend upon whether a particularly tough competitor wins the other license.

[24] No discussion of bidding on combinations of items would be complete without pointing out that the definitions of items are not preordained. Rather, the seller defines them. That definition may be critical. Should a seller sell each one-bedroom condo in a building separately, or should he pool them and sell the right to choose one of the unsold condos from the pool? Should the Federal government sell coal leases or a one-year option to sign such a lease? The answer to this question, discussed in Rothkopf and Englebrecht-Wiggans 1992, could be worth many millions of dollars to the US treasury. Should the FCC sell three 10 MHz licenses in a region, two 15 MHz licenses, or one 30 MHz License?

encourage revelation or will it produce incentives for bidders to keep the information private?[25]

Both the FCC spectrum license auctions and electricity auctions discussed below involve multiple items with interrelated values.

3.3 Transaction Costs

Most auction theory has been developed on the assumption that transaction costs are negligible. Even when the items being sold are highly valuable, that assumption is suspect, as bidders will often spend a few percent of the value at stake to evaluate the items and participate in the auction. When the items are inexpensive, the assumption can be quite misleading. Just the time of the auction participants can be significant. The daily produce auctions held in Vineland, New Jersey are simple progressive auctions that, on average, take over 20 seconds per transaction. Many transactions involve less than $100 worth of produce. Typically, there are 40 to 50 buyers in the auction room, several staffers supporting the auctioneer, and up to 100 farmers queued up for the chance to sell their produce. In order to reduce the time required to complete a transaction, the auctioneer delivers the sale document to the winning bidder by throwing it tucked into a slit cut in an old tennis ball. By contrast, the Dutch auctions used in the Dutch flower markets average four seconds per transaction.[26]

The role of transaction costs cries out for analytic attention. Arguably, the Internet has lowered the cost of holding an auction by an order of magnitude, making auctions relatively more attractive compared to posted prices and negotiations. This needs to be studied and modeled as well as being checked empirically.[27]

Transaction costs can affect the relative attractiveness of different auction forms. Recently, Lucking-Reiley 1999 found to his surprise that in a controlled experiment with real Internet auctions, Dutch auctions produced significantly more revenue than did standard sealed bidding. The strategic and revenue equivalence of Dutch auctions and standard sealed bidding has been unchallenged in the theoretical literature. In each, selecting a bid involves exactly the same trade off between the probability of winning and the profit from winning, and in each the choice must be made before observing any competitive bid. I suspect that Lucking-Reiley's result subtlety involves transaction costs. Lucking-Reiley's Internet Dutch auctions are "slow Dutch auctions," more like Filene's basement where unsold goods are periodically reduced in price than the four-second Dutch flower auctions. If I see something in Filene's basement I think will be attractively priced in two weeks, I can buy it now or come back in two weeks when the price is lower. However, if I wait, I not only risk that the item will be sold, but I also have to pay the cost of returning. This cost can make me decide to pay more now when, if returning were free, I would wait. Lucking-Reiley's Dutch auction bidders had to log back in at a later time in order to bid, and the magnitude of the revenue difference is about that of the cost of doing so. I doubt that this is an isolated example. Online auctions provide a

[25] See Hausch 1986.

[26] Kambil and van Heck 1998 discuss the Dutch flower auctions. The Vineland information if from personal observation.

[27] Kambil and van Heck 1998 provide an empirically based framework for analyzing regular auction exchanges but no mathematical models.

whole new context for auctions. Old assumptions about auction form will have to be reexamined in it. This will surely require good observation and modeling. I am unsure whether the mathematics will be particularly difficult.

3.4 Combinatorial Spectrum Rights Auctions

The FCC's spectrum rights auctions were a wake up call for auction theorists. Several companies filed briefs with the FCC (FCC Docket 93-253) accompanied by papers by economists suggesting alternative rules for the spectrum auctions. Five different papers by five sets of leading economists all argued that existing theory implied that there was only one way to hold the auctions, but they proposed five different ways. Much discussion made it clear that existing bidding theory had little to say about situations as complicated as the one faced by the FCC. What evolved was a series of simultaneous progressive auctions that had their roots in a proposal by Milgrom and Wilson. The history of these auctions is discussed extensively elsewhere.[28]

One aspect of the auction design, however, in the view of Congress and the FCC, needed additional attention. The simultaneous progressive auction design did not allow bidders to place bids on combinations of licenses.[29] However, the FCC has now announced an auction design for a simultaneous progressive auction with combinatorial bids for a March 2001 sale. Although the FCC hosted a conference of theorists to discuss the design of combinatorial auctions, the actual design is quite different from what was discussed. Their auction design and alternatives to it need study by theorists.[30] Among the questions I view as open are

1. Should an auction with combinatorial bids allowed by a one-time sealed bid or a simultaneous progressive auction? What will be the effect on efficiency and revenue?
2. Should bidders be allowed to offer unrestricted OR-bids, budget based OR-bids, or no OR-bids at all? Does this answer depend upon whether the auction is progressive or a one-time sealed bid?

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[28] See Cramton 1995, 1997; Cramton and Schwartz 2000; McAfee and McMillan 1996; McMillan 1994 and Bykowsky, Cull and Ledyard 2000.

[29] When the original FCC auctions were designed, the most important argument against allowing combinatorial bids was mathematical. The problem of picking the revenue maximizing set of bids when bids on any combination are allowed is NP complete. Since the FCC sold as many as 1500 licenses in a single sale, this was taken as a reason to disallow bids on any combinations. However, Rothkopf, Pekec and Harstad 1998 point out that winner determination is not NP complete for many economically interesting kinds of combinations. Furthermore, algorithms, including standard integer programming codes like CPLEX, normally perform much better than the worst-case assumption involved in the definition of NP completeness. (This is both my own experience and consistent with Anderson, Tenhuen and Ygge 2000). Thus it may make sense to allow arbitrary combinatorial bids provided that there is a reasonable contingency plan for the unlikely event that winner determination is difficult. One possibility is that bidders prioritize their combinations and that the winner determination calculation start with each bidder's highest priority combination and add additional combinations in priority order until it runs out of calculation time. See Park and Rothkopf 2000. Another proposal discussed in Rothkopf, Pekec and Harstad 1998 is to allow bidders a limited time to challenge a potentially suboptimal winner determination by proposing one that produces more revenue.

[30] Much material is posted on the FCC website http://combin.fcc.gov/papers.html#comments.

3. What, in general, should be done if winner determination in not provably optimal?[31]

3.5 Electrical Power and Transmission Rights

The realization that there are no longer economies of scale in electricity generation has led, in many places, to the deregulation of generation and to auction for the purchase of electricity. Auctions are, or soon will be, involved as ell with the allocation of electricity transmission capacity as well as the provision of "ancillary services" to electricity generation.[32]

It is harder to imagine a more demanding challenge for auction design. The demand for electricity varies substantially by time of day, is uncertain, and in the short run, is extremely inelastic. With unimportant exceptions, electricity cannot be stored, and demand must be met the instant it occurs.[33] However, much of the lowest cost generating equipment takes many hours and incurs substantial costs to start or stop. Furthermore, electricity transmission adds greatly to the challenge. One cannot transmit electricity in a modern grid from A to B without causing flows in essentially every link of the grid. Ignoring line losses, the fact that electricity uses alternating rather than direct current, and the nonlinear constraints associated with reliability concerns, we can construct a linear model to describe the flows. However, all three of these factors are nonlinear and sometimes quite significant. Finally, our electrical system is not a single closed system under a single management, but rather a set of interconnected separate systems. What goes on in the New England system and the Pennsylvania-New Jersey-Maryland (PJM) system affects the New York system but is not under its control.

The problems and disagreements start at the most basic level. What are the commodities to be auctioned? An answer being tried in California is that the basic commodity is electric power during a given hour delivered in a given zone. Twenty-four hours of supply are being procured simultaneously, but separately. This is simple, but it presents several problems. First, generators must deal with the fact that their cost in a given hour may depend heavily upon which other hours it is generating.[34] Second, the appropriate definitions of zones depend upon where the transmission grid is congested, but this depends upon which power is purchased. Attempts to define uncongested zones *a priori* have a spotty record.

Another approach is for bidders to submit bids for both power and start up conditions and costs. The bid taker then solves, exactly or approximately, an integer

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[31] The March 2001 auction has only 12 licenses, so this is not an issue in this auction. It might be for subsequent ones. The issue is more important than the size of the inefficiency that might result. Auctions are used to guarantee fairness. If different suboptimal alternatives favor different bidders, the fairness issue must be addressed.

[32] These services include automatic frequency control, spinning and stand by reserves, and cold start capabilities. They are supplied typically by equipment that might otherwise be supplying electricity itself.

[33] One implication of inelastic demand for a nonstorable product is that when demand gets near generating capacity a generator with even a modest amount of capacity can drive the price extremely high by withholding part of it. Indeed, price spikes orders of magnitude higher than normal prices have been observed even when capacity is apparently sufficient to meet demand.

[34] An attractive alternative suggested by Elmaghraby and Oren 1999 is to make the commodity a given amount of power starting at time A and running until time B. This can viewed as purchasing horizontal rather than vertical slices of the load-duration curve. It would allow generators to know their start up costs.

programming problem to select the bids that meet the projected demand at the lowest cost while respecting the various transmission and operating constraints. Payment is based upon market clearing prices adjusted for locational marginal prices. Variants of this approach are being tried in New York and PJM.

Neither approach is ideal. Evaluations of the approaches have gotten entangled with ideological concerns about the role of the central system operator. Furthermore, each approach is subject to variants in how it is implemented. England is in the process of switching from market clearing prices used in California, NY and PJM to pay-your-bid. In the context of the daily repetition of the auctions, this may not be a bad idea.[35]

There is currently an active debate on the difficult issue of best way to sell transmission rights. Some focus on the many constraints in the network and some want a liquid market in a few popular or representative transmission routes. The problem is complicated by the fact that there are a lot of potential constraints. Even without considering AC issues and reliability contingency constraints, the capacity of every link in the grid is constrained.[36] There are further issues in defining what is to be sold. Should it be an option or a right and duty? The distinction matters because failure to use a right can limit counter flows. An option suggested by O'Neill *et al.* 2000 is to hold an auction that includes option and rights-and-duties on each constraint and on each popular transmission route, including them all in one constrained optimization problem. Such an auction could be held annually, leaving the redistribution of transmission rights to an aftermarket, or daily or even hourly.

4 Conclusions

There are two major approaches to the creation of useful mathematical models of auctions. One is to start with a simple mathematical model and see how much can be solved. It is a common approach, congenial to mathematicians, and it has produced some interesting, beautiful, and occasionally useful mathematics. This has some value. However, it has repeatedly missed important factors that affect how real auctions work, at time misleading practitioners who tried to use the results of the mathematics for decision making.[37] The alternative approach, which I advocate, is to pay attention to auctions—study their context, purposes, particulars, and peculiarities—and then try to develop mathematical models and theories that, at least approximately, deal with these realities. This is hard and will often lead to less sweeping results. However, the results are much more likely to be useful for answering important questions about real auctions.

As deregulation and lowered transaction costs of the Internet push auctions into new applications and contexts, challenges arise for those trying to develop theories useful for decision making. I have cataloged some factors that need to be incorporated into the analysis of auctions and the mathematical models used in them. These include asymmetry, financially constrained bidders, complicated information structures, bidder decisions about participation, the effects of repeated auctions, auctioning items with interrelated values, and transaction costs. I have also discussed two major areas where

[35] See Rothkopf 1999.

[36] See Chow and Peck 1996.

[37] In addition to the economists' briefs to the FCC and Shell's experience with MEK mentioned above, see Capen, Clapp and Campbell 1971.

new, complicated auctions are being designed—combinatorial spectrum auctions and electricity and transmission rights auctions. In both of these areas, theory to guide decision makers is woefully inadequate. I hope that these challenges will lead the mathematically talented to undertake some work of great potential value.

References

Andersson, Arne, Mattias Tehhunen and Fredrik Ygge, "Integer Programming for Combinatorial Auctions Winner Determination," International Conference on Multi-Agent Systems, 2000.

Bykowsky, Mark M., Robert J. Cull and John O. Ledyard, "Mutually Destructive Bidding: The FCC Auction Design Problem," *Journal of regulatory Economics* **17**(3), pp. 205-228, 2000.

Capen, Edward R., Robert Clapp, and William Campbell, "Biding in High Risk Situations," *J. Petroleum Technology* **23**, pp. 641-53, 1971.

Chao, Hung-po and Stephen Peck, "A Market Mechanism for Electric Power Transmission," *Journal of Regulatory Economics* **10**(1), 1996.

Che, Yeon-Koo and Ian Gale, "Standard Auctions with Financially Constrained Bidders," *Review of Economic Studies* **65**, pp. 1-21, 1998.

Cramton, Peter, "Money Out of Thin Air: The Nationwide Narrowband PCS Auction," *Journal of Economics and Management Strategy* **4**, pp. 431-495, 1995.

Cramton, Peter, "The FCC Spectrum Auctions: An Early Assessment," *Journal of Economics and Management Strategy* **6**, pp. 267-343, 1997.

Cramton, Peter and Jesse A. Schwartz, "Collusive Bidding: Lessons from the FCC Spectrum Auctions," *Journal of Regulatory Economics* **17**(3), pp. 229-252, 2000.

Elmaghraby, Wedad and Shmuel Oren, "The Efficiency of Multi-Unit Electricity Auctions," The Energy Journal 20(3) pp. 89-116, 1999.

Friedman, Lawrence, "A Competitive Bidding Strategy," Operations *Research* **4**, pp. 104-12, 1956.

Friedman, Lawrence, "Competitive Bidding Strategies, Ph.D. Dissertation, Case Institute of Technology, 1957.

Friedman, Milton, *A Program for Monetary Stability*, Fordham University Press, New York, 1960.

Harstad, Ronald M., "Alternative Common-Value Auction Procedures: Revenue Comparisons with Free Entry," *Journal of Political Economy* **98**, pp. 421-29, 1990.

Ronald M. Harstad and Michael H. Rothkopf, "An 'Alternating Recognition' Model of English Auctions," *Management Science* **46**, pp. 1-12, 2000.

Harstad, Ronald M., Michael H. Rothkopf, and Kieth Waehrer, "Efficiency in Auctions When Bidders Have Private Information About Competitors," *Advances in Applied Micro-Economics, Volume 6*, Michael Baye, Ed., JAI Press, pp. 1-13, 1996.

Hausch, Donald B., "Multi-Object Auctions: Sequential vs. Simultaneous Sales," *Management Science* **32**, pp. 1599-1610, 1986.

Kambil, Ajit, and Eric van Heck, "Reengineering the Dutch Flower Auctions: A Framework for Analyzing Exchange Operations," *Information Systems Research* **9**, pp. 1-19, 1998.

Klemperer, Paul, "Auctions with Almost Common Values: The 'Wallet Game' and its Applications," European Economic Review 42, pp. 757-769, 1998

Levin, Dan and James L. Smith, "Equilibrium in Auctions with Entry," *American Economic Review* 84, pp. 585-599.

Lucking-Reiley, David, "Using Field Experiments to Rest Equivalence Between Auction Formats: Magic on the Internet," *American Economic Review* **89**, pp. 1063-1080, 199.

Lucking-Reiley, David, "Vickrey Auctions in Practice: From Nineteenth-Century Philately to Twenty-First Century E-Commerce," *Journal of Economic Perspectives* **14**(3), pp. 183-192, 2000.

Maskin, Eric and John Riley, "Asymmetric Auctions," *Review of Economic Studies*, forthcoming.

McAfee, R. Preston and John McMillan, "Auctions and Bidding," *J. Econ. Literature* **25**, pp. 699-738, 1987.

McAfee, R. Preston and John McMillan, "Government Procurement and International Trade," *Journal of International Economics* **26**, pp291-308, 1989.

McMillan, John, "Selling Spectrum Rights," *Journal of Economic Perspectives* **8**, pp. 145-162, 1994.

Milgrom, Paul R. and Robert J. Weber, "A Theory of Auctions and Competitive Bidding," *Econometrica* **50**, pp. 1089-1122, 1982.

Myerson, Roger B., "Optimal Auction Design," *Mathematics of Operations Research* **6**, pp. 58-73, 1981.

Nurmi, Hannu and Arto Salomaa, "Cryptographic Protocols for Vickrey Auctions, *Group Decision and Negotiation* **4**, pp. 363-373, 1993.

O'Neill, Richard P., Benjamin Hobbs, William Stewart, Michael H. Rothkopf and Udi Helman, "A Bidder-Specified Transmission Rights Contracts Auction," paper at MEET/EPRI Conference, Stanford, University, August 17, 2000.

Shmuel S. Oren and Michael H. Rothkopf, "Optimal Bidding in Sequential Auctions," *Operations Research* **23**, pp. 1080-1090, 1975.

Michael H. Rothkopf, "A Model of Rational Competitive Bidding," *Management Science* **15**, pp. 362-373, 1969.

Michael H. Rothkopf, "On Auctions with Withdrawable Winning Bids," *Marketing Science* **10**, pp. 40-57, 1991.

Michael H. Rothkopf, "Daily Repetition: A Neglected Factor in the Analysis of Electricity Auctions," *The Electricity Journal* **11**, pp. 60-70, April 1999.

Michael H. Rothkopf and Richard Engelbrecht-Wiggans, "Innovative Approaches to Competitive Mineral Leasing," *Resources and Energy* **14**, pp. 233-248, 1992.

Michael H. Rothkopf and Ronald Harstad, "Modeling Competitive Bidding: A Critical Essay," *Management Science* **40**, pp. 364-384, 1994a.

Michael H. Rothkopf and Ronald M. Harstad, "On the Role of Discrete Bid Levels in Oral Auctions," *European Journal of Operational Research* **74**, pp. 572-581, 1994b.

Michael H. Rothkopf and Ronald M. Harstad, "Two Models of Bid-Taker Cheating in Vickrey Auctions," *Journal of Business* **68**, pp. 257-267, 1995.

Michael H. Rothkopf, Ronald M. Harstad and Yuhong Fu, "Is Subsidizing Inefficient Bidders Actually Costly?" RUTCOR Research Report #38-96, Rutgers University, New Brunswick, N.J., 1996, revised 2000.

Michael H. Rothkopf, Aleksandar Pekec, and Ronald M. Harstad, "Computationally Manageable Combinational Auctions," *Management Science*.**44**, pp. 1131-1147, 1998.

Michael H. Rothkopf, Thomas J. Teisberg and Edward P. Kahn, "Why Are Vickrey Auctions Rare?" *Journal of Political Economy* **98**, pp. 94-109, 1990.

Smith, Charles W. *Auctions: The Social Construction of Value,* University of California Press, Berkeley, California, 1990.

Vickrey, William, "Counterspeculation, Auctions and Competitive Sealed Tenders," *J. of Finance* **41**, pp. 8-37, 1961.

Wilson, Robert B., "Competitive Bidding with Disparate Information," *Management Science* **15**, pp. 446-8, 1969.