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OPTIMAL BUSINESS POLICIES FOR A SUPPLIER-TRANSPORTER-BUYER CHANNEL WITH A PRICE-SENSITIVE DEMAND

Lei Lei Qiang Wang Chunxing Fan

Abstract. As the third party logistics partners (carriers) taking a more and more significant role in supply chain practices and customer service performance improvement, there is an emerging need for the studies on optimal channel coordination policies for business processes involving not only supplier and buyer, but also transportation partners. In this paper, we explicitly add a transportation partner with concave cost functions into the analysis for supplier-buyer channel coordination policies, and analyze the impact of coordination and pricing policies on supply chain profitability. The market demand ($D(x)$) is assumed to be a decreasing convex function of buyer's selling price (x), or $D(x)=d/x^e$, where $e > 1$ stands for the index of price elasticity. Under this assumption, we quantify the improvement on total supply chain profitability when moving from a non-cooperative environment to a cooperative environment. Our result demonstrates the importance of the transporter's role in this collaboration. Using $D(x)=d/x^2$ as an example, we show that the joint annual profit of three partners in a fully cooperative environment can be at least twice of what may be achieved by three independently operated companies in a leader-follower business game. While in a real world business environment, a perfect collaboration is hard to achieve, this result can be used to provide a quick estimation on the upper bound on the budget for profit sharing among the supply chain partners or discount offers.

***Key words**: Supplier, buyer and transporter coordination policies, pricing, ordering quantity, concave transportation cost, price-sensitive demand.*

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1 Introduction

Buyer-supplier channel coordination to achieve better joint profit by optimizing the pricing and inventory policies has received a significant attention of researchers during the past two decades. Available results in this regard can be found in the work by Goyal [1,2], Monahan [3], Lee and Rosenblatt [4], Banerjee [5], Abad [6], Goyal and Gupta [7], Benjamin[8], Anupindi and Akella [9], Kohli and Park [10], Lau and Lau [11], Weng [12,13,14,15], Li and Huang [16], Corbett and Groote [17], Chen, Federgruen and Zheng [18], Cheung and Lee [19], Abad and Jaggi [20], Abad [21], Shinn and Hwang [22], and Viswanathan and Wang [23], etc. Among these studies, many assumed the market demand is price- sensitive and a decreasing function of the buyer (or retailer)'s selling price, which in turn depends on the supplier's selling price, trade credit offering, quantity or volume discount policies, or other incentive programs to the buyer. With this assumption, Li and Huang [16] proved the improvement on annual profit by the buyer-seller cooperation when the buyer is in a monopolistic market, and investigated the mechanisms to achieve this cooperation. Weng [12,13,14] developed optimal quantity discount policies, analyzed their benefit to both the supplier and the buyer, and shown that with price-sensitive demand there are two incentives in offering quantity discount: increasing demand and ensuring Pareto-efficient transactions. Abad and Jaggi [20] developed the optimal supplier's selling price and the period length of trade credit to the buyer for a non-cooperative environment, and proposed a procedure for characterizing the Pareto-efficient solutions for a cooperative environment. Viswanathan and Wang [23] analyzed the effectiveness of quantity discount and volume discount as channel coordination mechanisms with respect to the level of sensitivity of demand to the price, and proved that a perfect coordination can be achieved when volume and quantity discounts are offered simultaneously.

To our knowledge, most of the existing literature on pricing and coordination analysis does not explicitly include logistics or transportation partners which, however, have been taking an increasingly important role toward a stronger supply chain collaboration in practice. Carter and Ferrin [24] pointed out that the supplier-buyer coordination cannot possibly optimize supply chain profit without the involvement of transportation carrier in the process. In today's competition, businesses cannot afford to leave any order unfulfilled or to deliver imperfect orders to their customers. A critical factor to ensure the customer satisfaction is a strong supply chain connectivity and collaboration, supplier and carrier compliance, and improved communications across all the partners (Lynch [25]). This means that it is more important than ever to achieve a stronger partnership with logistics partners to ensure a smooth flow between the supplier and the buyer. A recent example of this integration is the strategic alliance of a group of third party carriers with Sheetz Corporation – a food company that operates more than 285 convenience stores in Maryland, Ohio, Pennsylvania, Virginia and West Virginia. Another example of this is the newly formed partnership between Transplace and AutoZone Inc. – a national auto parts retail chain that has a sales network with more than 3,000 stores in United States and Canada served by its logistics partner Transplace. One of the key factors that enable Transplace and AutoZone to work together as if one is an extension of the other is the profit sharing (Harps [26]). More recent examples of this strategic alliance among carriers, suppliers, and buyers are listed in Table 1 below (Annual Survey, 2003, *Inbound Logistics*).

Carriers	**Strategic supply chain partners** (as a supplier or a buyer in the channel)
UPS	Birkenstock, Honeywell, Lucent, National semiconductor, Nikon, etc
Schneider	Ford, GM, Miller Brewing, Ocean Spray, Kimberly-Clark, etc
Menlo	Cisco, Dow Chemical, Nike Golf, Imation, Sears, etc
Ryder	D. Chrysler, Delphi, Applied Materials, John Deere, Snapple, HP, etc
Fedex	DirectTV, Fairchild Semiconductor, Mitsubishi, Philips, etc
CH Robinson	Wal-Mart, AOL, International Paper, Best Buy, Clorox, Dana Corp., etc
Americold	H. J. Heinz, Tyson Foods, Kraft Foods, Sara Lee, General Mills, etc
Penske	Whirlpool, International Truck and Engine, Panasonic, etc
TNT	Home Depot, Honda, BMW, Michelin, etc
Exel	BP, Coors, Goodyear, Sun Microsystems, Unilever, etc
Transplace	Autozone, Grainger, J.C. Penney, Pfizer, Office Depot, Weyerhaeuser, etc
Landstar	Southwestern Bell, Hitachi, Verizon, Glazers Wholesale, etc
TLC	Guinness, Yoplait, Campbell's, Welch's, Dean Foods, etc

Table 1. Strategic partnerships (sampled from *Inbound Logistics*, Vol. 23, No. 7, 2003)

Motivated by the needs for optimal policies that coordinate the operations of multiple partners, we present in this study an analysis for a single-product supply process involving a supplier, a buyer (a retailer), and a third party logistics partner (a *transporter*) with concave transportation cost functions. The buyer purchases the product from the supplier, sells in the market, and bears a fixed ordering cost (i.e., overhead for placing order, quality assurance, contract management, etc) and holding cost for the resulting inventories. The supplier controls the selling price to the buyer, pays for the shipping cost (i.e., the free-on-board deal with buyer), and bears the processing and shipping cost for each order placed by the buyer, and the holding cost for the inventory required for a continuous supply to the buyer. The transporter charges supplier a shipping rate, be responsible for transporting the product from the supplier to the buyer, and bears his/her own transportation cost, $a+b\cdot Q$, per shipment, where parameter a stands for the fixed cost per shipment (e.g., insurance per trip, truck driver's cost, mileage cost, truck usage, etc) and parameter b stands for the unit shipping variable cost. The market demand, $D(x)$, is assumed to be a commonly used decreasing convex function of buyer's selling price (x) or $D(x)=d/x^e$ (see Li and Huang [16], Shinn and Hwang [22], Abad and Jaggi [20], and Abad [21]), where $e>1$ stands for the index of price elasticity and $d>0$ stands for the scaling factor. Under this assumption, we develop simple and close-form formulas to quantify the improvement on supply chain profitability that may be achieved by better cooperation and having the logistics partner to join the supply chain alliance. One potential application of our results is to offer a quick estimation for budgeting the profit sharing or quantity discount offers to partners in business processes where the market demand can be approximated by a decreasing convex function of the buyer's selling price, x, to the ultimate customers.

We assume that the operation costs, including fixed/setup cost per order and the holding cost (per unit per year) incurred to all the partners are known, and let

x: The buyer's (or the retailer's) unit selling price to the market (as the buyer's decision variable);

p: The buyer's unit purchasing price ($p<x$) from the supplier(as the supplier's decision variable);

c: The supplier's variable cost for manufacturing($c<p$);

S_b: The buyer's fixed cost per order;

S_s: The supplier's fixed setup and processing cost per order;

a: The transporter's fixed cost per shipment (or per order);

b: The transporter's unit transportation cost;

H_b: The buyer's unit holding cost per year;

H_s: The supplier's unit holding cost per year;

Q: The buyer's order size (or the shipping quantity) per order (as the buyer's decision variable);

g: Average shipping rate charged by the transporter (as the transporter's decision variable), and $g > b + a/Q$, where transporter's operation cost per order is assumed to be $a + b \cdot Q$.

$D(x)$: The annual market demand, as a decreasing convex function of x, $D(x) = d / x^e$, where d is the scaling factor(>0) and e is the index of price elasticity (>1).

We assume that the supply process is a free-on-board (FOB) destination process where the supplier's selling price p to the buyer must cover his/her variable production cost (c), average fixed cost per unit, and the shipping cost (g). This means $c + g < p < x$. The underlying supply process is depicted in Figure 1 below.

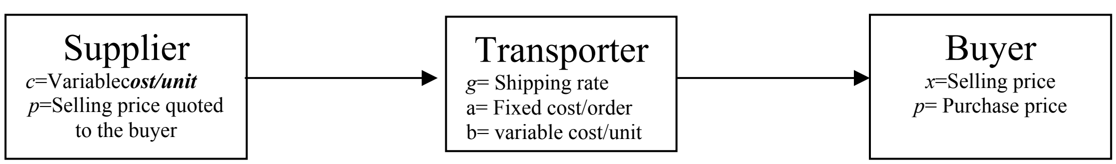

Figure 1. A supply channel involving a buyer, a supplier and a transporter.

With these assumptions, the yearly profit of the supplier (Π_s), the transporter (Π_t), and the buyer (Π_b) can be represented as

$$\Pi_s(p) = (p - c - g)D(x) - S_s\, D(x)/Q - H_s\, Q/2 \quad (1)$$

$$\Pi_t(g) = gD(x) - (a + bQ)\, D(x)/Q = (g - b)D(x) - aD(x)/Q \quad (2)$$

$$\Pi_b(x, Q) = (x - p)D(x) - S_b\, D(x)/Q - H_b\, Q/2 \quad (3)$$

Given the definition of yearly profit functions and assuming that the market demand can be approximated by a decreasing convex function of selling price x, we proceed to the following analysis on optimal pricing policies, starting with $D(x) = d / x^2$. Later in Section 4, we extend the analysis to a more general demand function of type $D(x) = d / x^e$, with $e > 1$.

2 Optimal Business Policies in a Non-Cooperative Business Environment

When each partner acts for his/her own interests without joint decisions, we have a non-cooperative business environment. The specific non-cooperative environment underlying our analysis in this section is based on the following leader-follower process:

Definition 1. A *non-cooperative business environment involving a buyer, a supplier and a transporter* is a process where the transporter decides on the shipping rate g that is acceptable to the supplier and will be paid by the supplier. The supplier chooses his/her own selling price p to the buyer to maximize supplier's yearly profit with the knowledge that the buyer will act to maximize his/her own profit. For any given supplier's selling price p, the buyer makes decisions on ordering quantity, or Q_b, that minimizes buyer's yearly fixed ordering and inventory holding cost and determines market selling price x that maximizes buyer's yearly profit.

Assuming that the buyer's inventory policy follows the Economic Ordering Quantity (EOQ), which, in a non-cooperative business environment, becomes $Q_b = (2S_b\, D(x)/H_b)^{1/2}$. Replacing Q by Q_b and $D(x)$ by d/x^2, the buyer's yearly profit (3) becomes

$$\Pi_b(x \mid Q_b) = (x-p)D(x) - S_b\, D(x)/Q - H_b\, Q/2 = \frac{d}{x} - \frac{dp}{x^2} - \frac{[2dS_bH_b]^{1/2}}{x} \tag{3a}$$

which, for any given p, is maximized when the market selling price is set at

$$x_b = \frac{2dp}{d - [2dS_bH_b]^{1/2}} \tag{4}$$

Given $Q_b = (2S_b\, D(x)/H_b)^{1/2}$ and x_b defined in (4), the supplier's yearly profit becomes

$$\begin{aligned}\Pi_s(p) &= (p-c-g)\frac{d}{x_b^2} - (S_s/S_b + H_s/H_b)(S_bH_b\, d/2x_b^2)^{1/2} \\ &= \frac{[d^{1/2} - (2S_bH_b)^{1/2}]^2}{4p} - (c+g)\frac{[d^{1/2} - (2S_bH_b)^{1/2}]^2}{4p^2} \\ &\quad - (S_s/S_b + H_s/H_b)(S_bH_b/2)^{1/2}\frac{d^{1/2} - (2S_bH_b)^{1/2}}{2p}\end{aligned} \tag{1a}$$

Let $d\Pi_s/dp = 0$ and solve for p, we have

$$p = \frac{2(c+g)(d^{1/2} - (2S_bH_b)^{1/2})}{d^{1/2} - \theta \cdot (2S_bH_b)^{1/2}} \tag{5}$$

where $\theta = 1 + S_s/S_b + H_s/H_b$. Similarly, given Q_b, x_b, and p, where x_b, and p, are defined in (4), and (5), respectively, the transporter's yearly profit becomes

$$\Pi_t(g) = (g-b)D(x) - aD(x)/Q = (g-b)\frac{d}{x_b^2} - \frac{a}{S_b}[S_b H_b/2]^{1/2}\frac{d^{1/2}}{x_b}$$

$$= \frac{[d^{1/2} - \theta\cdot(2S_b H_b)^{1/2}]^2}{16(c+g)} - (c+b)\frac{[d^{1/2} - \theta\cdot(2S_b H_b)^{1/2}]^2}{16(c+g)^2} \qquad (2a)$$

$$-(a/S_b)[S_b H_b/2]^{1/2}\frac{[d^{1/2} - \theta\cdot(2S_b H_b)^{1/2}]}{4(c+g)}$$

Let $d\Pi_t/dg = 0$, we obtain

$$g^*_{non-cp} = \frac{2(c+b)[d^{1/2} - \theta\cdot(2S_b H_b)^{1/2}]}{d^{1/2} - (\theta + 2a/S_b)(2S_b H_b)^{1/2}} - c \qquad (6)$$

Replacing g in (5) by g^*_{non-cp}, we obtain p^*_{non-cp}, and replacing p in (4) by p^*_{non-cp}, we obtain x^*_{non-cp}, which then leads to the following results (proof skipped).

Lemma 1: In a non-cooperative business environment with market demand $D(x) = d/x^2$, the supplier's optimal selling price to the buyer, the transporter's optimal shipping rate quoted to the supplier, and the buyer's optimal market selling price are

$$x^*_{non-cp} = 8(c+b)\frac{d^{1/2}}{d^{1/2} - (\theta + 2a/S_b)(2S_b H_b)^{1/2}} \qquad (4a)$$

$$p^*_{non-cp} = 4(c+b)\frac{d^{1/2} - (2S_b H_b)^{1/2}}{d^{1/2} - (\theta + 2a/S_b)(2S_b H_b)^{1/2}} \qquad (5a)$$

$$g^*_{non-cp} = \frac{2(c+b)[d^{1/2} - (1 + S_s/S_b + H_s/H_b)(2S_b H_b)^{1/2}]}{d^{1/2} - (\theta + 2a/S_b)(2S_b H_b)^{1/2}} - c \qquad (6a)$$

The results in (4a)-(6a) show that, when the magnitude of market demand (measured by factor d) to the product is sufficiently large, we have $p^*_{non-cp} \approx 4(c+b)$, $x^*_{non-cp} \approx 2p^*_{non-cp}$, and $g^*_{non-cp} \approx 2b + c$. Furthermore, if we replace p^*_{non-cp}, x^*_{non-cp} *and* g^*_{non-cp} in (1a), (2a) and (3a), we obtain the following maximum annual profitability that the supplier, the buyer, and the transporter alone may achieve in this non-cooperative business environment:

$$\Pi^*_s = \frac{1}{32(c+b)}[d^{1/2} - \theta(2S_b H_b)^{1/2}][d^{1/2} - (\theta + 2a/S_b)(2S_b H_b)^{1/2}] \qquad (1b)$$

$$\Pi^*_t = \frac{1}{64(c+b)}[d^{1/2} - (\theta + 2a/S_b)(2S_b H_b)^{1/2}]^2 \qquad (2b)$$

$$\Pi_b^* = \frac{1}{16(c+b)}[d^{1/2} - (2S_bH_b)^{1/2}][d^{1/2} - (\theta + 2a/S_b)(2S_bH_b)^{1/2}] \qquad (3b)$$

Since $\theta > 1$, we have $\Pi_b^* > 2\Pi_s^*$, and $\Pi_s^* > 2\Pi_t^*$. That is, in the assumed independent business environment, for any purchasing contract signed by the supplier, the buyer and the transporter, the transporter has the least incentive to take the responsibility for the overall channel performance improvement, which in turn reveals the potential weakness of a non-cooperative business process in competing for logistics services.

Example 1: Consider a non-cooperative business environment with S_b=\$500, S_s=\$15,000, a=\$500, H_b=\$2, H_s=\$2, $d = 1\times 10^8$, c=\$5, b=\$1, $Q_b = 3{,}950$, and a demand function $D(x) = d/x^2$. According to 4a – 6a, we have $p^*_{non-cp} = \$28.18$, $x^*_{non-cp} = \$56.61$, $g^*_{non-cp} = \$7.13$. By using the results in (1b) – (3b), we have yearly maximum profit for the supplier, the buyer, and the transporter, respectively, as following:

$$\Pi_t^* = \frac{1}{64(c+b)}[d^{1/2} - (\theta + 2a/S_b)(2S_bH_b)^{1/2}]^2 = \$0.187\times 10^6,$$

$$\Pi_b^* = \frac{1}{16(c+b)}[d^{1/2} - (2S_bH_b)^{1/2}][d^{1/2} - (\theta + 2a/S_b)(2S_bH_b)^{1/2}] = \$0.879\times 10^6,$$

$$\Pi_s^* = \frac{1}{32(c+b)}[d^{1/2} - \theta(2S_bH_b)^{1/2}][d^{1/2} - (\theta + 2a/S_b)(2S_bH_b)^{1/2}] = \$0.378\times 10^6.$$

$$\Pi_s^* + \Pi_b^* + \Pi_t^* = \$1.444\times 10^6.$$

3 Optimal Business Policies for a Cooperative Business Environment

In this section, we analyze the impact of business policies and cooperation strategies on total annual profit with respect to two different levels of collaboration among partners. First, we consider a *partially cooperative environment* where the buyer and the supplier cooperate while leaving the transporter to act independently. Then, we consider a *totally cooperative environment* where the three partners act as if they belong to the same company and agree to share the profit and not to charge each other, except the buyer (the retailer) charges the ultimate customers at the market place. For each environment, we develop the optimal business policies for the partners and quantify the improvement on supply chain profitability, assuming the optimal policies can be fully executed.

Definition 2. A *partially cooperative environment* is a business process where the buyer and the supplier agree to share the profit and make joint decisions on *ordering quantity* Q_{sb} and *market selling price* x_{sb}, that together maximize the buyer and the supplier's joint yearly profit. The transporter acts independently and controls the shipping rate g that will be paid by the supplier under a FOB destination agreement.

Such a partially cooperative environment is common for business processes where the buyer and the supplier are the sales and the manufacturing divisions of the same company while the transporter is a contracted third party carrier. According to a recent survey by Langley, Allen and Tyndall [28], about 71% firms being interviewed for the survey study outsourced the transportation needs to third party logistics partners, which makes this assumed partially cooperative environment a common business process in supply chain practices.

Consider such a partially cooperative environment. Let $S_{sb} = S_s + S_b$, $H_{sb} = H_s + H_b$, and define the supplier-buyer joint economic ordering quantity as $Q_{sb} = (2S_{sb}\, D(x)/H_{sb})^{1/2}$, which leads to their joint yearly profit function

$$\Pi_{sb}(x \mid Q_{sb}) = (x - c - g)D(x) - S_{sb}\, D(x)/Q - H_{sb}\, Q/2 = \frac{d}{x} - \frac{d(c+g)}{x^2} - \frac{[2dS_{sb}H_{sb}]^{1/2}}{x} \tag{7}$$

which is maximized by setting the market selling price x at

$$x_{sb} = \frac{2d(c+g)}{d - [2dS_{sb}H_{sb}]^{1/2}} \tag{8}$$

Given Q_{sb} and x_{sb}, the transporter's yearly profit becomes

$$\Pi_t(g) = (g-b)D(x) - aD(x)/Q = (g-b)\frac{d}{x_{sb}^2} - \frac{a}{S_{sb}}[\omega/4]^{1/2}\frac{d^{1/2}}{x_{sb}} = \frac{[d^{1/2} - \omega^{1/2}]^2}{4(c+g)} - (c+b)\frac{[d^{1/2} - \omega^{1/2}]^2}{4(c+g)^2} - (a/S_{sb})[\omega/4]^{1/2}\frac{[d^{1/2} - (2S_{sb}H_{sb})^{1/2}]}{2(c+g)} \tag{9}$$

where $\omega = 2S_{sb}H_{sb}$. Let $d\Pi_t / dg = 0,$ we obtain

$$g^*_{Partial} = \frac{2(c+b)[d^{1/2} - \omega^{1/2}]}{d^{1/2} - (1 + a/S_{sb})\omega^{1/2}} - c \tag{10}$$

Replacing g in (8) by $g^*_{Partial}$, we obtain $x^*_{Partial}$ and the following results (proof skipped).

Lemma 2: In a partially cooperative environment, if the market demand can be approximated by $D(x) = d / x^2$, then the optimal market selling price of the supplier-buyer channel, and the transporter's optimal shipping rate are as follows:

$$x^*_{Partial} = 4(c+b)\frac{d^{1/2}}{d^{1/2} - (1 + a/S_{sb})\omega^{1/2}} \tag{8a}$$

$$g^*_{Partial} = \frac{2(c+b)[d^{1/2} - \omega^{1/2}]}{d^{1/2} - (1 + a/S_{sb})\omega^{1/2}} - c \tag{10a}$$

The result of (8a) shows that in this partially cooperative environment, the buyer (retailer)'s optimal market selling price is lower than that in the non-cooperative business environment (see (4a)). In particular, we can show (see Lei, Wang and Fan [27]) that the following relationship holds.

Observation 1. If the market demand can be approximated by $D(x) = d / x^2$, and $S_s / S_b \geq H_s / H_b$, then the optimal market selling price, $x^*_{Partial}$, as defined in (8a), that maximizes the joint yearly profit for the supplier-buyer channel should be much lower than x^*_{non-cp} which maximizes the buyer's yearly profit in a non-cooperative business environment, and

$$x^*_{Partial} < \frac{1}{2} x^*_{non-cp} \qquad (11)$$

Note that condition $S_s / S_b \geq H_s / H_b$ holds for many practical situations, especially when the supplier is a manufacturer who has to pay for a production line setup cost for each order the buyer places. In this case, the inventory holding costs for the same product should be similar at the manufacturer's and at the buyer's warehouses. However, the manufacturer's setup cost (S_s) could be easily hundreds or even thousands times higher than buy's overhead cost for placing an order (S_b).

The decrease in optimal market selling price, when changing from a non-cooperative business environment to a partially cooperative environment, will in turn increases the market demand to the product, and thus improves revenues of the partners. To see this, let's replace $x^*_{Partial}$ *and* $g^*_{Partial}$ for x and g in (7) and (9), respectively, we have

$$\Pi^*_{sb} = \frac{1}{8(c+b)}[d^{1/2} - \omega^{1/2}][d^{1/2} - (1 + a/S_{sb})\omega^{1/2}] \qquad (7a)$$

and $$\Pi^*_{t(sb)} = \frac{1}{16(c+b)}[d^{1/2} - (1 + a/S_{sb})\omega^{1/2}]^2 \qquad (9a)$$

Comparing (9a) and (2b), we can easily verify that $\Pi^*_{t(sb)} > 4\Pi^*_t$ when $S_s / S_b \geq H_s / H_b$. This is due to the use of supplier-buyer joint economic ordering quantity Q_{sb} and the increase in market demand. In practice, this supplier-buyer joint EOQ, if can be implemented, could offer a stronger incentive to the transporter which in turn may lead to a better coordination and better logistics services. However, since $\omega > 0$, $\Pi^*_{sb} > 2\Pi^*_{t(sb)}$. That is, in this partially cooperative environment, the transporter again receives the least share in the total profit by three partners.

In addition, we have:

Observation 2. If the market demand can be approximated by $D(x) = d / x^2$ and the optimal market selling price, $x^*_{Partial}$, as defined in (8a), is applied, then the sum of yearly profit of the supplier, the buyer and the transporter is much higher than the sum of these individual partners' profit in a non-cooperative business environment, and

$$\Pi_{sb}^* + \Pi_{t(sb)}^* > \frac{12}{7}(\Pi_s^* + \Pi_b^* + \Pi_t^*) \quad (12)$$

Observation 2 indicates that at least 70% increases in the sum of yearly profit achieved by the three partners can be expected when moving from a non-cooperative business environment to a partially cooperative environment where the supplier and the buyer agree to jointly optimize the ordering quantity (Q_{sb}), apply the joint optimal market selling price, and share the profit.

Example 2. Consider a partially cooperative environment with S_b=\$500, S_s=\$15,000, a=\$500, H_b=\$2, H_s=\$2, $d = 1\times 10^8$, c=\$5, b=\$1, $Q_{sb} = 35{,}348$. Suppose $D(x) = d/x^2$. Use the results in (8a) and (10a), we have $x_{partial}^* = \$24.91$, $g_{partial}^* = \$7.01$. Now, applying (7a) and (9a), we have yearly maximum profit for the supplier-buyer channel and the transporter, respectively, as follows:

$$\Pi_{sb}^* = \frac{1}{8(c+b)}[d^{1/2} - \omega^{1/2}][d^{1/2} - (1 + a/S_{sb})\omega^{1/2}] = \$1.937\times 10^6,$$

$$\Pi_{t(sb)}^* = \frac{1}{16(c+b)}[d^{1/2} - (1 + a/S_{sb})\omega^{1/2}]^2 = \$0.967\times 10^6.$$

Comparing the results in Example 1 and Example 2, we have:

$$\Pi_{sb}^* + \Pi_{t(sb)}^* = \$2.904\times 10^6 > \frac{12}{7}(\Pi_s^* + \Pi_b^* + \Pi_t^*) = \$2.475\times 10^6.$$

The improvement on supply yearly profit can be further increased when the transporter joins the alliance or say when the three partners agree to share the profit and to act in a totally cooperative mode. To do so, let $S_J = S_s + a + S_b$, $H_J = H_s + H_b$, and define the joint economic ordering quantity of three partners as $Q_J = (2S_J\, D(x)/H_J)^{1/2}$, which leads to the joint yearly profit function

$$\Pi_J(x \mid Q_J) = (x - c - b)D(x) - S_J\, D(x)/Q - H_J\, Q/2$$
$$= (x - c - b)\frac{d}{x^2} - \sqrt{2S_J H_J \frac{d}{x^2}}$$

which is maximized at

$$\Pi_J^* = \frac{1}{4(c+b)}[d^{1/2} - (2S_J H_J)^{1/2}]^2 \quad (13)$$

by

$$x_J^* = \frac{2d(c+b)}{d - [2dS_J H_J]^{1/2}} \quad (14)$$

Given (14), the following result holds (see Lei, Wang and Fan [27] for details).

Observation 3. If the market demand can be approximated by $D(x) = d/x^2$, then the optimal market selling price, x_J^*, as defined in (14), that maximizes the joint yearly profit of the supplier, the buyer (the retailer) and the transporter, should be much lower than $x^*_{Partial}$ which maximizes the yearly profit for supplier and buyer in a partially cooperative environment, and

$$x_J^* < \frac{1}{2} x^*_{Partial} \qquad (15)$$

Furthermore, we can quantify the improvement on total annual profit of three partners when moving from a non-cooperative environment to a partially cooperative environment, and then to a totally cooperative environment (see Appendix for details).

Observation 4. If the market demand can be approximated as $D(x) = d/x^2$ and the optimal market selling price, x_J^*, as defined in (14), is applied, then the following relationship holds

$$\Pi_J^* > 1\frac{1}{3}(\Pi_{sb}^* + \Pi_{t(sb)}^*) > 2\frac{2}{7}(\Pi_s^* + \Pi_b^* + \Pi_t^*) \qquad (16)$$

Example 3. Consider a totally cooperative environment with S_b=\$500, S_s=\$15,000, a=\$500, H_b=\$2, H_s=\$2, $d = 1\times 10^8$, c=\$5, b=\$1, $Q_J = 71{,}869$. The results in (13) and (14) lead to $x_J^* = \$12.46$ and

$$\Pi_J^* = \frac{1}{4(c+b)}[d^{1/2} - (2S_J H_J)^{1/2}]^2 = \$3.874\times 10^6$$

Comparing this result with that in Examples 1 and 2, we have:

$$\Pi_J^* = \$3.874\times 10^6 > 1\frac{1}{3}(\Pi_{sb}^* + \Pi_{t(sb)}^*) = \$3.872\times 10^6$$

$$> 2\frac{2}{7}(\Pi_s^* + \Pi_b^* + \Pi_t^*) = \$3.301\times 10^6$$

$$> \Pi_s^* + \Pi_b^* + \Pi_t^* = \$1.444\times 10^6.$$

This result shows that, as the level of partners' cooperation increases by one level, the total supply profitability has a potential to increase by at least one-third (1/3). In practice, this estimate on the amount of improvement in annual profit can be used to serve as a budget for profit sharing among partners or as a guideline for decisions on discount offers.

4 Optimal Policies for General $D(x) = d/x^e$

When the market demand can be approximated by $D(x) = d/x^e$, where $e > 1$, the maximum yearly profits of the supplier, the transporter, and the buyer in a non-cooperative environment become

$$\Pi_s(p) = (p-c-g)\frac{d}{x^e} - (S_s/2S_b + H_s/2H_b)\frac{(2dS_bH_b)^{1/2}}{x^{e/2}} \qquad (17)$$

$$\Pi_t(g) = (g-b)\frac{d}{x^e} - \frac{a}{2S_b}\cdot\frac{(2dS_bH_b)^{1/2}}{x^{e/2}} \qquad (18)$$

$$\text{and} \qquad \Pi_b(x\,|\,Q_b) = (x-p)\frac{d}{x^e} - \frac{[2dS_bH_b]^{1/2}}{x^{e/2}} \tag{19}$$

respectively, which added together yields

$$\Pi_s^* + \Pi_t^* + \Pi_b^* = (x_{non-cp}^* - c - b)\frac{d}{x_{non-cp}^{*e}} - [1 + (S_s/S_b + H_s/H_b)/2 + a/2S_b]\frac{(2dS_bH_b)^{1/2}}{x_{non-cp}^{*e/2}} \tag{20}$$

In a partially cooperative environment, with $Q_{sb} = (2S_{sb}\,D(x)/H_{sb})^{1/2}$, the yearly profits of the supplier-buyer channel and the transporter increase to

$$\Pi_{sb}(x\,|\,Q_{sb}) = (x-c-g)\frac{d}{x^e} - \frac{(2dS_{sb}H_{sb})^{1/2}}{x^{e/2}} \quad (21) \text{ and}$$

$$\Pi_{t(sb)}(g) = (g-b)\frac{d}{x^e} - \frac{a}{2S_{sb}}\cdot\frac{(2dS_{sb}H_{sb})^{1/2}}{x^{e/2}} \tag{22}$$

respectively, and

$$\Pi_{sb}^* + \Pi_{t(sb)}^* = (x_{Partial}^* - c - b)\frac{d}{x_{Partial}^{*e}} - [1 + a/2S_{sb}]\frac{(2dS_{sb}H_{sb})^{1/2}}{x_{Partial}^{*e/2}} \tag{23}$$

In a totally cooperative environment, with $Q_J = (2S_J\,D(x)/H_J)^{1/2}$, the joint yearly profit of three partners becomes

$$\Pi_J(x\,|\,Q_J) = (x-c-b)\frac{d}{x^e} - \frac{(2dS_JH_J)^{1/2}}{x^{e/2}} \tag{24}$$

which is maximized at

$$\Pi_J^* = (x_J^* - c - b)\frac{d}{x_J^{*e}} - \frac{(2dS_JH_J)^{1/2}}{x_J^{*e/2}} \tag{25}$$

Given the maximum total channel profit achieved under three different levels of cooperation (see (20), (23) and (25)), the following results can be shown to hold (proof skipped).

Observation 5. If the market demand can be approximated as $D(x) = d/x^e$, then the following relationship holds

$$\Pi_J^* > \Pi_{sb}^* + \Pi_{t(sb)}^* > \Pi_s^* + \Pi_b^* + \Pi_t^* \tag{26}$$

This general result shows that, as long as the market demand can be modeled by $D(x) = d/x^e$, for any $e > 1$, the total supply profitability increases as the level of cooperation between the supplier, the buyer and the transporter increases. However, in this general case, the improvement on profitability cannot be exactly quantified unless the value of parameter *e* is fixed.

5 Empirical Results

Our empirical observations obtained in this study show the impact of business collaboration policies on channel performance (i.e., resulting operation cost and annual profit). Base values of cost parameters used in this empirical study are summarized in Table 2 below. As we can see, the supplier's fixed/processing cost per order is significantly higher than that of the buyer. This is because 1) we assumed a FOB contract between the supplier and the buyer so that the supplier has to bear for the transportation cost for each order the buyer placed; and 2) for most JIT manufacturers (suppliers), this fixed cost also include production line setup cost which can be easily several hundred times higher than the fixed ordering cost of a buyer.

Name of parameter	Value
Buyer's fixed cost per order S_b	$1250
Supplier's fixed cost per order S_s	$25000
Buyer's unit holding cost per H_b	$25
Supplier's unit holding cost H_s	$25
Market scaling factor d	4E+10
Supplier's variable cost per unit c	$40
Transporter's fixed cost per shipment a	$2,000
Transporter's variable cost b	$5
Transporter's profit margin k	$2

Table 2. Base values of parameters used in empirical study.

Figure 2a shows the impact of changes in manufacturer's variable production cost c (affected by material procurement cost, labor cost, utility cost, etc) on the resulting optimal product market selling price x^*. In practice, increases in manufacturing cost, especially in northeast region of US, has been a main driving force for outsourcing the production to low cost countries. However, outsourcing to foreign country is not always the best solution due to potential risk in slow responsiveness, increases in shipping cost, and higher variability in order lead times. As manufacturing cost increases, the supplier's selling price p to the buyer also increases in a leader-follower type independent business environment. This in turn drives the buyer's market selling price, x, to increase and makes the optimal value of x become extremely vulnerable. For channels with low market shares, such a vulnerable market selling price may seriously affect the channel's position in market competition. Nevertheless, the impact of manufacturing cost on the optimal market selling price is no longer that significant under a more cooperative environment, especially when the three partners collaborate in a totally cooperative environment. This means that, in a cooperative environment, the channel's market share (measured by the demand) becomes more stable even when the manufacturing cost increases.

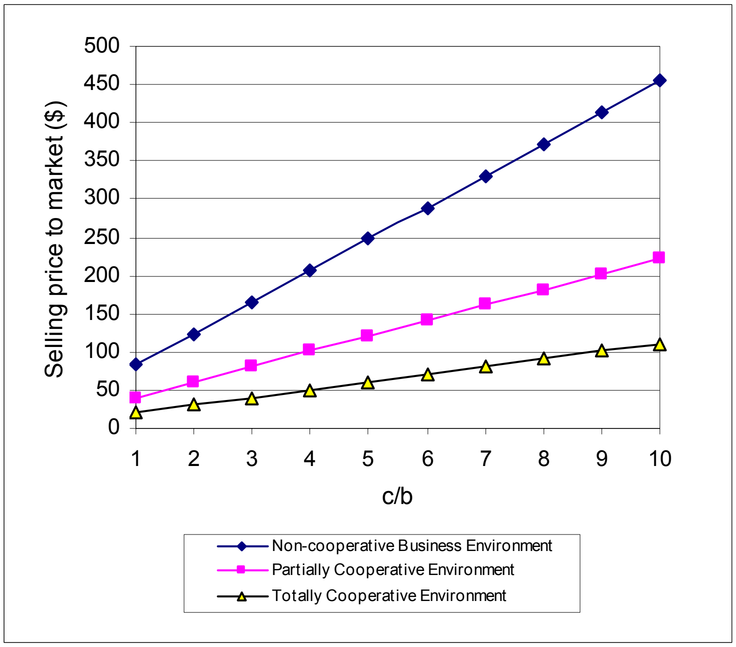

Figure 2a. The impact of manufacturing variable cost (c) on the optimal market selling price, where parameter b is held as constant.

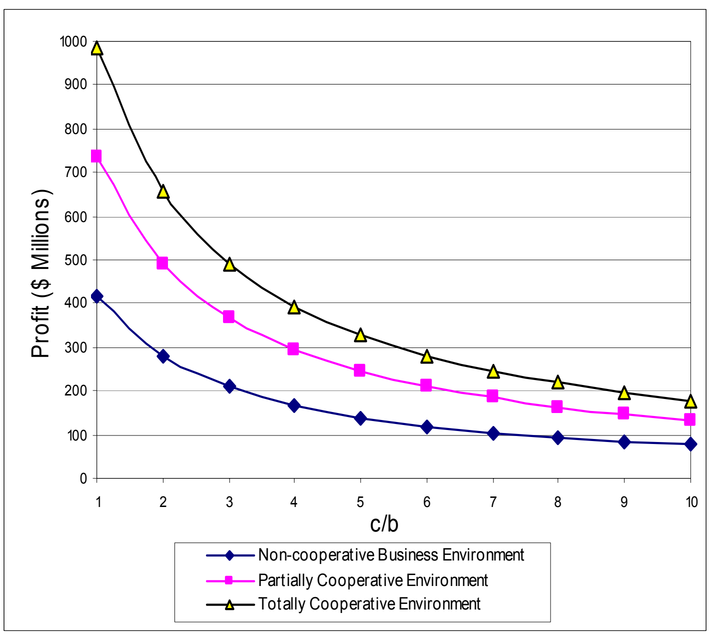

Figure 2b. The impact of manufacturing variable cost (c) on the channel annual profitability, where parameter b is held as constant.

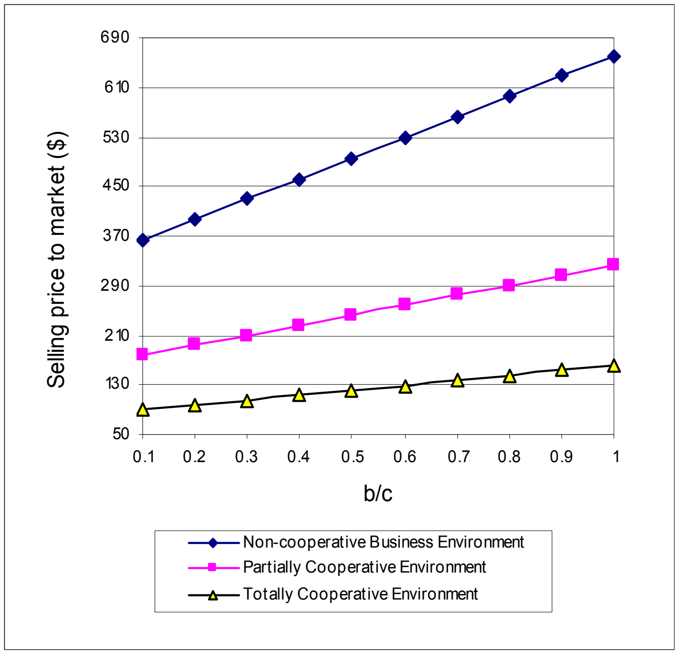

Figure 3a. The impact of transportation variable cost (b) on market selling price, where parameter c is held as constant.

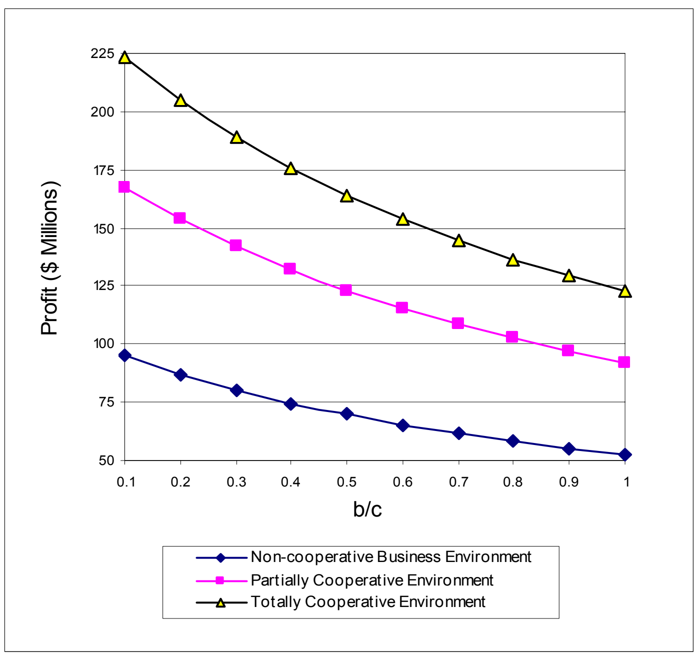

Figure 3b. The impact of transportation variable cost (b) on channel annual profitability, where parameter c is held as constant.

Figure 2b shows the impact of manufacturing cost on total channel profit achieved by supplier, buyer and transporter. As *c* value increases, the channel profit decreases. However, the total profit achieved under a totally cooperative environment is consistently and significantly higher than that achieved in a less cooperative environment.

Figures 3a and 3b show the impact of transportation cost, affected by gasoline prices, delays at boarder for security check, and increases in trucking or international ocean vessel/container insurances, etc, on the optimal product market selling price and channel annual profitability with respect to different channel cooperation levels. As we can see, similar observations are obtained in this case as well. Figures 4a and 4b show the impact of collaboration with transportation partner on product market selling price (and consequently the market share of the channel), where parameter *k=g-[b+a/Q]* stands for the average profit margin that transporter charges for each unit shipped from supplier's inventory to buyer's warehouse. As we can see, if the transporter joins the strategic alliance of the supplier-buyer channel's and if the three partners can operate in a fully cooperative by adapting an effective incentive program, then the optimal product market selling price (*x*) has a potential to drop be nearly 30% within the sampled data range, which in turn increases the channel's market chare (demand) and the channel annual profit. As we can also see, the sum of partners' annual profit is very vulnerable and decrease quickly as the transporter's profit margin increases, and could potentially decrease by 22% within the sampled data range.

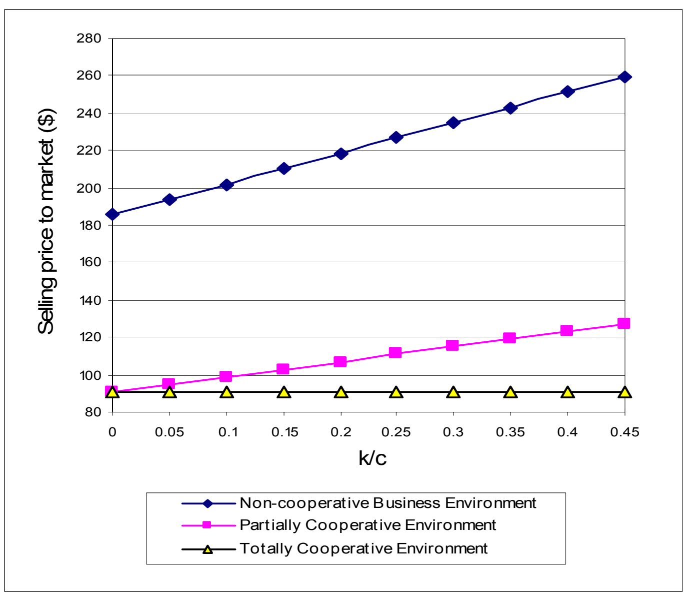

Figure 4a. The impact of transporter's marginal profit (k) on market selling price, where k=g-[b+a/Q] and c is held constant.

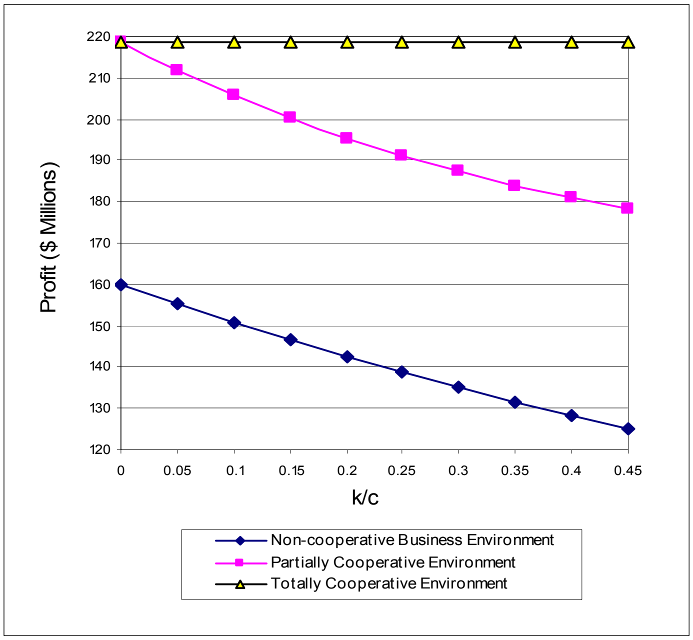

Figure 4b. The impact of transporter's marginal profit (k) on channel profitability, where k=g-[b+a/Q] and c is held constant.

6 Conclusion and extension

Assuming the market demand follows $D(x) = d / x^2$, where x stands for the unit market selling price of the product, we quantified the improvement on total channel annual profitability and product market selling price that may be jointly achieved by the supplier, the buyer and the transporter with a concave transportation cost function. We also presented the observations from an empirical study to show the impact of manufacturing cost, shipping cost and transporter's marginal profit on both market shares and channel profit in business environment with different levels of channel cooperation. An extension to the general decreasing convex demand function $D(x) = d / x^e$, $e > 1$, is also discussed.

When multiple independent partners are involved in a supply chain process, a "solution" that integrates their business operation policies for an overall performance optimality requires all involved channel partners to be willing to collaborate to implement the business process prescribed by the optimal solution. This totally collaborative environment, however, is not easy to achieve without a strong incentive program and effective implementation mechanisms in place. Nevertheless, the results developed in this study can always be used to develop the budget and guidelines when planning for such incentive programs. Meanwhile, it is also an interesting extension of this study to analyze the discount and credit policies for channels involving not only supplier and buyer, but also third logistics partners.

References:

[1] Goyal, SK. An integrated inventory model for a single supplier–single customer problem. International Journal of Production Research 1976;15:107-11.

[2] Goyal SK. A joint economic lot size model for purchaser and vendor: a comment. Decision Science 1988;19:236-41.

[3] Monahan JP. A quality discount pricing model to increase vendor's profits. Management Science 1984;30:720-26.

[4] Lee HL, Rosenblatt MJ. A generalized quantity discount model to increase supplier profit. Management Science 1986;32(9):1177-85.

[5] Banerjee A. A joint economic lot size model for purchaser and vendor. Decision Science 1986;17:292-311.

[6] Abad PL. Determining optimal selling price and the lot Size when the supplier offers all-unit quantity discounts. Decision Science 1988;19(3):622-34.

[7] Goyal SK, Gupta YP. Integrated inventory models: the buyer-vendor coordination. European Journal of Operational Research 1989;41:261-69.

[8] Benjamin J. An analysis of mode choice for shippers in a constrained network with applications to just-in-time inventory. Transportation Research (series B) 1990;24B(3):229-45.

[9] Anupindi R, Akella R. Diversification under supply uncertainty. Management Science 1993;39(8):944-63.

[10] Kohli, R, Park H. Coordinating buyer-seller transactions across multiple products. Management Science 1994;40(9):45-50.

[11] Lau HS, Lau AHL. Coordination of two suppliers with offsetting lead time and price performance. Journal of Operations Management 1994;11:327-37.

[12] Weng ZK. Channel coordination and quantity discount. Management Science1995;41(9):1509-22.

[13] Weng ZK. Modeling quantity discount under general price-sensitive demand functions: optimal policies and relationships. European Journal of Operational Research 1995;86:300-14.

[14] Weng ZK. Pricing and ordering strategies in manufacturing and distribution alliances. IIE Transaction 1997;29:681-92.

[15] Weng ZK. Coordinating order quantities between the manufacturer and the buyer: a generalized newsvendor model. European Journal of Operational Research 2003;(to appear).

[16] Li SX, Huang Z. Managing buyer-seller system cooperation with quantity discount considerations. Computer and Operations Research 1995;22(9):947-58.

[17] Corbett CJ, Groote Xde. A supplier's optimal quantity discount policy under asymmetric information. Management Science 2000;46(3):444-50.

[18] Chen F, Federgruen A, Zheng YS. Coordination mechanisms for a distribution system with one supplier and multiple retailers. Management Science 2001;47:693-708.

[19] Cheung KL, Lee H. The inventory benefit of shipment coordination and stock rebalancing in a supply chain. Management Science 2002;48(2):300-6).

[20] Abad PL, Jaggi CK. A joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive. International Journal of Production Economics 2003;83:115-22.

[21] Abad PL. Optimal price and lot size when the supplier offers a temporary price reduction over an interval. Computers & Operations Research 2003;30:63-74.
[22] Shinn SW, Hwang H. Optimal pricing and ordering policies for retailers under order-size-dependent delay in payments. Computers & Operations Research 2003;30(1):35-50.
[23] Viswanathan S, Wang Q. Discount pricing decisions in distribution channels with price-sensitive demand. European Journal of Operational Research 2003;149:571-87.
[24] Carter JR, Ferrin BG. The impact of transportation costs on supply chain management. Journal of Business Logistics 1995;16(1):189-212.
[25] Lynch K. The missing link in the prefect order cycle. Inbound Logistics 2003;23(1):44.
[26] Harps LH. The nature of change. Inbound Logistics2003;23(1):76-132. Editor's Report. Inbound Logistics 2003;23(7):49-51.
[27] Lei L, Wang Q, Fan C. Three-Partner Coordination Models with Price-Sensitive Demand. Working paper 2002; Rutgers Business School, Rutgers University.
[28] Langley, C. J., G. R. Allen and G. R. Tyndall. Third-party-logistics services: views from the customers. Georgia Institute of Technology, Cap Gemini Ernst & Young and Ryder System, Inc. 2001.

Appendix. Proofs for Observation 2 and Observation 4

Observation 2. If the market demand can be approximated by $D(x) = d / x^2$ and the optimal market selling price, $x^*_{Partial}$, as defined in (8a), is applied, then the sum of yearly profit of the supplier, the buyer and the transporter is much higher than the sum of these individual partners' profit in a non-cooperative business environment, and

$$\Pi^*_{sb} + \Pi^*_{t(sb)} > \frac{12}{7}(\Pi^*_s + \Pi^*_b + \Pi^*_t) \tag{12}$$

Proof. From (7a) and (9a), we have the yearly joint profit in a partially cooperative supply process

$$\Pi^*_{sb} + \Pi^*_{t(sb)} = \frac{3}{16(c+b)}[d^{1/2} - (1 + a/S_{sb})(2S_{sb}H_{sb})^{1/2}][d^{1/2} - (1 + a/3S_{sb})(2S_{sb}H_{sb})^{1/2}]$$

$$> \frac{3}{16(c+b)}[d^{1/2} - (1 + a/S_{sb})(2S_{sb}H_{sb})^{1/2}]^2$$

and add (1b), (2b) and (3b) together, we have the maximum total yearly profit of the three individual partners in a non-cooperative business environment

$$\Pi^*_s + \Pi^*_b + \Pi^*_t = \frac{7}{64(c+b)}[d^{1/2} - (1 + (3S_s/S_b + 3H_s/H_b + 2a/S_b)/7)(2S_bH_b)^{1/2}]$$

$$[d^{1/2} - (1 + S_s/S_b + H_s/H_b + 2a/S_b)(2S_bH_b)^{1/2}]$$

$$< \frac{7}{64(c+b)}[d^{1/2} - (2S_bH_b)^{1/2}][d^{1/2} - (1 + S_s/S_b + H_s/H_b + 2a/S_b)(2S_bH_b)^{1/2}]$$

The claim is verified if we can show that

$$[d^{1/2}-(1+a/S_{sb})(2S_{sb}H_{sb})^{1/2}]^2 > [d^{1/2}-(2S_bH_b)^{1/2}][d^{1/2}-(1+S_s/S_b+H_s/H_b+2a/S_b)(2S_bH_b)^{1/2}]$$

Since

$$[d^{1/2}-(1+a/S_{sb})(2S_{sb}H_{sb})^{1/2}]^2$$

$$= d-2d^{1/2}(1+a/S_{sb})(2S_{sb}H_{sb})^{1/2}+(1+a/S_{sb})^2(2S_{sb}H_{sb})$$

$$[d^{1/2}-(2S_bH_b)^{1/2}][d^{1/2}-(1+S_s/S_b+H_s/H_b+2a/S_b)(2S_bH_b)^{1/2}]$$

$$= d-2d^{1/2}[1+(S_s/S_b+H_s/H_b)/2+a/S_b)(2S_bH_b)^{1/2}+(1+S_s/S_b+H_s/H_b+2a/S_b)(2S_bH_b)$$

$$(1+a/S_{sb})(2S_{sb}H_{sb})-(1+S_s/S_b+H_s/H_b+2a/S_b)(2S_bH_b)=2S_sH_s+4aH_s+\frac{2aH_{sb}}{S_{sb}}>0$$

Now we need to show that

$$[1+(\frac{S_s}{S_b}+\frac{H_s}{H_b})/2+\frac{a}{S_b}]\sqrt{2S_bH_b} > [1+\frac{a}{S_{sb}}]\sqrt{2S_{sb}H_{sb}}$$

or

$$[1+(\frac{S_s}{S_b}+\frac{H_s}{H_b})/2]\sqrt{2S_bH_b}+\frac{a}{S_b}\sqrt{2S_bH_b} > \sqrt{2S_{sb}H_{sb}}+\frac{a}{S_{sb}}\sqrt{2S_{sb}H_{sb}}$$

Since $[1+(\frac{S_s}{S_b}+\frac{H_s}{H_b})/2]^2\cdot 2S_bH_b-2S_{sb}H_{sb}=\frac{1}{2}(\frac{S_s}{S_b}-\frac{H_s}{H_b})^2S_bH_b\geq 0$, we have

$$[1+(\frac{S_s}{S_b}+\frac{H_s}{H_b})/2]\sqrt{2S_bH_b} > \sqrt{2S_{sb}H_{sb}} \qquad \text{(i)}$$

and since $\frac{S_s}{H_s}\geq\frac{S_b}{H_b}$, we have

$$\frac{a^2}{S_b^2}\cdot 2S_bH_b-\frac{a^2}{S_{sb}^2}\cdot 2S_{sb}H_{sb}=2a^2(\frac{H_b}{S_b}-\frac{H_{sb}}{S_{sb}})=2a^2\frac{S_sH_b-H_sS_b}{S_{sb}S_b}\geq 0$$

or

$$\frac{a}{S_b}\sqrt{2S_bH_b} > \frac{a}{S_{sb}}\sqrt{2S_{sb}H_{sb}} \qquad \text{(ii)}$$

Observations in (i) and (ii) together show the following inequality holds

$$[1+(\frac{S_s}{S_b}+\frac{H_s}{H_b})/2+\frac{a}{S_b}]\sqrt{2S_bH_b} > [1+\frac{a}{S_{sb}}]\sqrt{2S_{sb}H_{sb}}$$

From

$$[d^{1/2}-(1+a/S_{sb})(2S_{sb}H_{sb})^{1/2}]^2 > [d^{1/2}-(2S_bH_b)^{1/2}][d^{1/2}-(1+S_s/S_b+H_s/H_b+2a/S_b)(2S_bH_b)^{1/2}]$$

we have

$$\Pi_s^* + \Pi_t^* + \Pi_b^* < \frac{7}{64(c+b)}[d^{1/2} - (1 + a/S_{sb})(2S_{sb}H_{sb})^{1/2}]^2$$

$$= \frac{7}{12}(\Pi_{sb}^* + \Pi_{t(sb)}^*)$$

That is

$$\Pi_{sb}^* + \Pi_{t(sb)}^* > \frac{12}{7}(\Pi_s^* + \Pi_b^* + \Pi_t^*) \qquad \blacksquare$$

Observation 4. If the market demand can be approximated as $D(x) = d/x^2$ and the optimal market selling price, x_J^*, as defined in (14), is applied, then the following relationship holds

$$\Pi_J^* > 1\frac{1}{3}(\Pi_{sb}^* + \Pi_{t(sb)}^*) > 2\frac{2}{7}(\Pi_s^* + \Pi_b^* + \Pi_t^*) \qquad (16)$$

Proof. From (14), we have the yearly joint profit in a totally cooperative supply process

$$\Pi_J^* = \frac{1}{4(c+b)}[d^{1/2} - (2S_J H_J)^{1/2}]^2$$

and add (7a) and (9a) together, we have the maximum total yearly profit of the three individual partners in a partially cooperative environment

$$\Pi_{sb}^* + \Pi_{t(sb)}^* = \frac{3}{16(c+b)}[d^{1/2} - (1 + a/S_{sb})(2S_{sb}H_{sb})^{1/2}][d^{1/2} - (1 + a/3S_{sb})(2S_{sb}H_{sb})^{1/2}]$$

$$< \frac{3}{16(c+b)}[d^{1/2} - (2S_{sb}H_{sb})^{1/2}][d^{1/2} - (1 + a/S_{sb})(2S_{sb}H_{sb})^{1/2}]$$

Claim

$$\Pi_J^* > 1\frac{1}{3}(\Pi_{sb}^* + \Pi_{t(sb)}^*)$$

is verified if we can show that

$$[d^{1/2} - (2S_J H_J)^{1/2}]^2 > [d^{1/2} - (2S_{sb}H_{sb})^{1/2}][d^{1/2} - (1 + a/S_{sb})(2S_{sb}H_{sb})^{1/2}]$$

Since

$$[d^{1/2} - (2S_J H_J)^{1/2}]^2 = d - 2d^{1/2}(2S_J H_J)^{1/2} + 2S_J H_J$$

$$[d^{1/2} - (2S_{sb}H_{sb})^{1/2}][d^{1/2} - (1 + a/S_{sb})(2S_{sb}H_{sb})^{1/2}]$$

$$= d - 2d^{1/2}(1 + a/2S_{sb})(2S_{sb}H_{sb})^{1/2} + (1 + a/S_{sb})(2S_{sb}H_{sb})$$

and

$$(1 + a/S_{sb})(2S_{sb}H_{sb}) = \frac{S_J}{S_{sb}} \cdot 2S_{sb}H_{sb} = 2S_J H_J$$

$$(1 + \frac{a}{2S_{sb}})^2(2S_{sb}H_{sb}) - 2S_J H_J = \frac{a^2}{4S_{sb}^2}(2S_{sb}H_{sb}) > 0$$

We have

$$[d^{1/2} - (2S_J H_J)^{1/2}]^2 > [d^{1/2} - (2S_{sb}H_{sb})^{1/2}][d^{1/2} - (1 + a/S_{sb})(2S_{sb}H_{sb})^{1/2}]$$

So

$$\Pi^*_{sb} + \Pi^*_{t(sb)} < \frac{3}{16(c+b)}[d^{1/2} - (2S_J H_J)^{1/2}]^2 = \frac{3}{4}\Pi^*_J$$

That is

$$\Pi^*_J > \frac{4}{3}(\Pi^*_{sb} + \Pi^*_{t(sb)})$$

From Observation 2, we obtain

$$\Pi^*_{sb} + \Pi^*_{t(sb)} > \frac{12}{7}(\Pi^*_s + \Pi^*_b + \Pi^*_t)$$

So

$$\Pi^*_J > 1\frac{1}{3}(\Pi^*_{sb} + \Pi^*_{t(sb)}) > 2\frac{2}{7}(\Pi^*_s + \Pi^*_b + \Pi^*_t) \blacksquare$$