RUTCOR RESEARCH REPORT

ON THE INTEGRATED PRODUCTION, INVENTORY, AND DISTRIBUTION ROUTING PROBLEM

Lei Lei[a], Shuguang Liu[b], Andrzej Ruszczynski[a],

Sunju Park[a]

RRR-41-2003 NOVEMBER 2003

RUTCOR
Rutgers Center for
Operations Research
Rutgers University
640 Bartholomew Road
Piscataway, New Jersey
08854-8003
Telephone: 732-445-3804
Telefax: 732-445-5472
Email: rrr@rutcor.rutgers.edu
http://rutcor.rutgers.edu/~rrr

[a] Rutgers Business School & ROTCOR, Rutgers University, 180 University Avenue, Newark, NJ 07102
[b]School of Business, State University of New York at New Paltz

RUTCOR RESEARCH REPORT

RRR-41-2003 NOVEMBER 2003

ON THE INTEGRATED PRODUCTION, INVENTORY, AND DISTRIBUTION ROUTING PROBLEM

Lei Lei, Shuguang Liu, Andrzej Ruszczynski, and Sunju Park

Abstract. The integrated production, inventory and distribution routing problem (PDRP) is concerned with coordinating the production, inventory and delivery operations to meet customer demand with an objective to minimize the cost. The particular PDRP that we consider in this study also involves heterogeneous transporters with non-instantaneous traveling times and many customer demand centers each with its own inventory capacities. Optimally solving such an integrated problem is in general not easy due to its combinatorial nature, especially when transporter routing is involved.

In this paper, we propose a two-phase solution approach to this problem. Phase I solves a mixed integer programming model which includes all the constraints in the original model except the transporter routings are restricted to direct shipment between facilities and customer demand centers. The resulting optimal solution to the Phase I problem is always feasible to the original model. Phase II solves an associated consolidation problem to handle the potential inefficiency of direct shipment. The delivery consolidation problem is formulated as a capacitated transportation problem with additional constraints and is solved by an efficient heuristic routing algorithm. The main advantage of this proposed approach, over the classical decoupled approach, is its ability to *simultaneously* optimize the production, inventory and transportation operations (subject to restricted routing/direct shipments) without the needs for aggregating the demand and relaxing the constraints on transportation capacities. We evaluate the performance of this proposed two-phase approach and report its application to a real-life supply network which motivated this study.

(Integrated production, inventory and distribution routing problem; Mixed integer programming; Direct shipment; Heterogeneous transporters; Heuristic routing algorithm)

Acknowledgements: The jointed sponsored by Rutgers Business School and ROTCOR is gratefully acknowledged.

1 Introduction

As more and more companies become aware of their supply chain performance and the importance of their performance improvement, coordination and integration of the production (supply), inventory, and distribution (demand) operations have been known as the next source of competitive advantage (see Thomas and Griffin (1996), Fumero and Vercellis (1999), Brown et al (2001), Lee and Whang (2001), Gupta et al (2002), and Bloomquist et al (2002)). This issue becomes especially critical after companies merge as it inevitably introduces redundancies (in terms of low capacity utilization) and inefficiencies (in terms of high distribution cost) into the combined supply/distribution network. A representative example of this is the recent merge of Nabisco and Kraft Food Inc., a tasty combination blending Kraft's cheese, dressings, and beverages with Nabisco's cookies and crackers. Merging the two giant food chains has been a huge undertaking for the new supply network in order to ensure a high level efficiency of consolidated distribution (Harps (2003)).

◎ In this study, we are interested in the following integrated production, inventory and distribution routing problem (PDRP). We are given a single product and set of plants, each has its own production capacity, inventory capacity, raw material supply contract, inventory holding cost, and production cost. Associated with each plant, there is a heterogeneous fleet of transporters, each has its own operation cost, required loading/unloading time, loading capacity, available time, and traveling speed. We are also given a set of customer demand centers (DCs) located over a wide geographical region, each has its own demand per time period in the planning horizon, its own inventory capacity, holding cost, and safety stock requirement. The problem is *to determine the operation schedules to coordinate the production, inventory, and transportation routing operations so that the customer demand, transporter travel time and capacity constraints, plant production and inventory constraints are all satisfied, while the resulting operation cost (i.e., the sum of production, inventory and transportation cost) over a given planning horizon is minimized*.

Our research on this problem was motivated by the after-merge demand-supply coordination problem encountered in the practice of a leading chemical company in North America. After acquiring a large manufacturing facility from one of the competitors, the company has experienced inefficiencies in the after-merge operation. First, there was no guideline as how much should be produced at which facility since the production costs, material cost, supply contract, logistics partner contract, production cost and inventory capacities are all different at different facilities. Second, there were a large number of customer DCs to be supplied. Different DCs have different inventory capacities, different geographical locations, different demand patterns and consequently different safety stock requirements over a year. It is unclear which DC should be supplied by which facility or which group of facilities, and how much ending-inventory for a particular month at a particular DC makes sense. Third, most distributions are based on water transportation via heterogeneous vessels owned by individual facilities. These vessels are different in terms of their capacity, operation cost, travel speed, loading/unloading time, and ability to access particular DCs in different seasons (as the chemical may freeze if the vessel is not equipped with adequate heaters). Since the vessel costs are usually very high (at \$350-\$500/hour), any inefficiency in solving this integrated planning/scheduling problem can easily cost the company millions of additional dollars.

If we deal with only a single product, then the associated coordination problem can be formulated as the following mixed integer program, which shall be denoted as problem **P** in remaining part of this paper.

Notations:

Model parameters

I Set of plants;

J Set of customer DCs;

K I union J;

T Set of time periods in the planning horizon;

$V(i)$ Set of transporters owned by plant i;

$N(v)$ Maximum number of trips by transporter v during each period, where each trip is defined as a sequence of DCs visited by the transporter between its consecutive returns to the home/base plant for loading;

$a_i(t)$ Production cost at plant i during time t;

$h_i(t)$ Inventory holding cost at plant i during time t;

$\hbar_j(t)$ Inventory holding cost at DC_j during time t;

c_v Variable shipping cost for transporter v (per hour);

$c^c_{i,j}$ Variable shipping cost per unit load for chartered transportation services from location i to location j;

C_v Maximum loading capacity for transporter v;

$T_v(t)$ Available time for transporter v to perform transportation operations during period t;

$t^v_{j,k}$ Traveling time from location j to location k by transporter v (include loading time if location j is a plant, unloading time if location k is a customer site);

$D_j(t)$ Demand of DC_j in time period t that must be satisfied by either the inventory at DC_j, or by the shipment arriving during period t, or by both;

$p^{\max}_i$ Maximum production capacity of plant i;

$s^{\max}_i, s^{\min}_i$ Maximum ending inventory and safety stock (i.e., the minimum inventory) requirement, respectively, at plant i

$z^{\max}_i, z^{\min}_i$ Maximum ending inventory and safety stock (i.e., the minimum inventory) requirement, respectively, at DC i

Variables

$p_i(t)$ Production quantity by plant i during time t;

$s_i(t)$ Ending inventory at plant i during time t;

$z_j(t)$ Ending inventory at DC_j during time t;

$x_{i,j,k}^{v,n}(t)$ Equal to 1, if transporter v of plant i visits DC_k immediately after visiting DC_j during its *nth* trip in time period t, $\forall i \in I, v \in V(i)$, $n \in N(v), j \neq k \in \{i\} \cup J, t \in T$, $if j \in I,\ k \notin I$;

$g_{i,j,k}^{v,n}(t)$ Quantity carried by transporter v of plant i traveling from DC_j to DC_k during its *nth* trip in time period t, $\forall i \in I, v \in V(i), n \in N(v)$, $j \neq k \in \{i\} \cup J, t \in T$, $if j \in I,\ k \notin I$;

$q_{i,j}^{v,n}(t)$ Quantity delivered by transporter v, $v \in V(i)$, to DC_j from plant i during its *nth* trip in period t;

$Q_{i,j}(t)$ Quantity delivered from plant i to DC_j by chartered transporters in period t;

$$\textbf{P}: \quad \text{Min} \quad \begin{array}{l} \sum_{t\in T}\sum_{i\in I}\sum_{v\in V(i)}\sum_{n\in N}\sum_{j,k\in K,\, j\neq k} c_v t_{j,k}^{v} x_{i,j,k}^{v,n}(t) + \sum_{t\in T}\sum_{i\in I}\sum_{j\in J} c_{i,j}^{c} Q_{i,j}(t) + \\ \sum_{t\in T}\sum_{i\in I} a_i(t) p_i(t) + \sum_{t\in T}\sum_{i\in I} h_i(t)(p_i(t)/2 + s_i(t)) + \sum_{t\in T}\sum_{j\in J} \hbar_j(t) z_j(t) \end{array} \qquad (1)$$

s.t.

Plant inventory balance constraints

$$s_i(t) = s_i(t-1) + p_i(t) - \sum_{j\in J}\sum_{v\in V(i)}\sum_{n\in N(v)} q_{i,j}^{v,n}(t) - \sum_{j\in J} Q_{i,j}(t) \quad \forall i \in I, t \in T \qquad (2)$$

Customer inventory balance constraints

$$z_j(t) = z_j(t-1) + \sum_{i\in I}\sum_{v\in V(i)}\sum_{n\in N(v)} q_{i,j}^{v,n}(t) + \sum_{i\in I} Q_{i,j}(t) - D_j(t) \quad \forall j \in J, t \in T \qquad (3)$$

Storage capacity and safety stock requirement constraints

$$s_i^{\min} \le s_i(t) \le s_i^{\max}, \quad \forall i \in I, t \in T \qquad (4)$$

$$z_j^{\min} \le z_j(t) \le z_j^{\max}, \quad \forall j \in J, t \in T \qquad (5)$$

Production capacity constraints

$$0 \le p_i(t) \le p_i^{\max}, \quad \forall i \in I, t \in T \qquad (6)$$

Trip integrity constraints

$$\sum_{\substack{j\in\{i\}\cup J \\ j\neq k}} x_{i,j,k}^{v,n}(t) = \sum_{\substack{j\in\{i\}\cup J \\ j\neq k}} x_{i,k,j}^{v,n}(t), \forall i \in I, v \in V(i), n \in N(v), k \in \{i\} \cup J, t \in T \quad (7)$$

$$\sum_{j\in J} x_{i,i,j}^{v,n}(t) \le 1 \quad \forall i \in I, v \in V(i), n \in N(v), t \in T \qquad (8)$$

Transporter capacity constraints

$$g_{i,j,k}^{v,n}(t) \le C_v x_{i,j,k}^{v,n}(t) \quad \forall i \in I, j \ne k \in \{i\} \cup J, v \in V(i), n \in N(v), t \in T \qquad (9)$$

Commodity flow conservation constraints

$$\sum_{\substack{j \in \{i\} \cup J \\ j \ne k}} g_{i,j,k}^{v,n}(t) - \sum_{\substack{l \in \{i\} \cup J \\ l \ne k}} g_{i,k,l}^{v,n}(t) = q_{i,k}^{v,n}(t) \quad \forall i \in I, v \in V(i), k \in J, n \in N(v), t \in T \qquad (10a)$$

$$\sum_{j \in J} g_{i,j,i}^{v,n}(t) - \sum_{l \in J} g_{i,i,l}^{v,n}(t) = -\sum_{j \in J} q_{i,j}^{v,n}(t) \quad \forall i \in I, v \in V(i), n \in N(v), t \in T \qquad (10b)$$

Transportation duration constraints

$$\sum_{n \in N(v)} \sum_{j \in \{i\} \cup J} \sum_{k \in \{i\} \cup J} t_{j,k}^{v} x_{i,j,k}^{v.n}(t) \le T_v(t) \quad \forall i \in I, v \in V(i), t \in T \qquad (11)$$

Non-negative and integer requirement

$$q_{j,k}^{v,n}(t) \ge 0,\ g_{i,j,k}^{v,n}(t) \ge 0, \ \forall i \in I, j \ne k \in \{i\} \cup J, v \in V(i), n \in N(v), t \in T \qquad (12a)$$

$$Q_{i,j}(t) \ge 0 \quad \forall i \in I, j \in J, t \in T \qquad (12b)$$

$$x_{i,j,k}^{v,n}(t) = binary \ \ \forall i \in I, j \ne k \in \{i\} \cup J, v \in V(i), n \in N(v), t \in T \qquad (12c)$$

In a similar way, we can extend **P** to a formulation with multiple products.

Directly solving **P** is not easy. The required computation time to verify the optimal solution could often become unbearable because of the enormous amount of integer variables involved. For example, with a problem of 2 plants, 8 owned transporters, at most 12 trips per transporter per period, 20 DCs, and 12-period/month planning horizon, one can easily verify that the number of integer variables gets close to 0.5 million (483,840 to be exact). The main reason behind such a large number of integer variables is that the model tempts to construct the optimal transporter routing, production and inventory schedules all at the same time.

In the literature, there are many known results available for either optimizing the inventory and distribution policies, or optimizing the production and inventory plans (e.g., see Williams (1981), Dror and Ball (1987), Goyal and Gupta (1989), Cohen and Lee (1988, 1989), Blumenfeld, Burns and Dagunzo (1991), Anily and Federgruen (1993), Chandra (1993), Pyke and Cohen (1993, 1994), Slats et al. (1995), Federgruen and Simchi-Levi (forthcoming)). Reviews on these related works can be found in Bhatnangar et al. (1993), Thomas and Griffin (1996), Vidal and Goetschalckx (1997), Baita and Ukovich (1998), and Erengüç et al. (1999). However, results that may be directly applied to solve model **P** are limited. Available approaches are either based on problem decomposition or heuristics. A seminal work in this regard involving multiple plants, identical transporters, and multiple products was contributed by Chandra and Fisher (1994). The authors analyzed and evaluated two approaches. The *decoupled approach* determines the production lot size to minimize the setup and inventory cost subject to the total demand per time period, and then schedules the transporter deliveries to meet customer demand subject to inventory availability determined by the given production lot size. The delivery schedules are based on well-known vehicle routing heuristics, and improved by combining the delivery to a customer at a later period with delivery to the same customer at an earlier period. This approach, however, does not allow modifying the given production lot sizes. The *coordinated approach* follows essentially the same process as the decoupled approach except that transportation decisions may entail changes in the production plans. Their computational results show a consistent improvement on the total cost by the coordinated approach.

Fumero and Vercellis (1999) proposed an integrated optimization model solved via Lagrangean relaxation for the PDRP involving multi-product, multi-period, single-plant, and identical-transporters. They used Lagrangean relaxation to produce four separate sub-problems, while at the same time preserving a global optimization perspective through the dual master problems, and compared the performance of the proposed approach with a decoupled approach extended from the one in Chandra and Fisher (1994). A consistent and significant improvement in cost savings by the integrated optimization approach over that by the decoupled approach was observed. Van Buer *et al*. (1999) studied a special version of PDRP with no inventories. Several heuristic algorithms, such as tabu search and simulated annealing, were used to solve the problem. They found no significant performance difference among these heuristics, and noted that re-using the trucks that had completed earlier routes could be an important way to achieve low-cost solutions. Özdamar and Yazgac (1999) considered a PDRP involving a central factory, a set of warehouses, and identical vehicles. They adopted a hierarchical approach to make use of medium-range aggregate information and to satisfy weekly fluctuating demand. Their study focused on optimizing the fleet size instead of transporter routing. Applications of optimization techniques for the integrated PDRP without the involvement of transporter routing were reported by Martin, Dent and Eckhart (1993), Gupta, Peters and Miller (2002), and Brown, *et al.* (2001).

In this paper, we propose a two-phase approach to solve **P**. In phase I, we solve a restricted coordination problem which keeps all the constraints in **P** except that the transporter routings are limited to direct shipment. We prove that the resulting optimal solution to the Phase I problem is always feasible to **P**, and thus gives an upper bound solution to **P**. Successfully solving this restricted problem determines the optimal quantity to be produced, inventoried, and transported by each transporter (per time period) from each plant to each DC. It also determines the optimal number of trips (per time period) performed by each individual transporter, in terms of its very own capacity, cost and speed. Note that for all such DCs with a demand higher than transporter capacity, direct shipment is needed anyway. The potential inefficiency with this phase I solution, however, comes from the fact that sometimes a consolidated delivery (routing) for less-than-transporter-load (LTL) assignments may further reduce the transportation cost. Therefore, in phase II, we propose a heuristic transporter routing algorithm, the *Load Consolidation (LC)* algorithm, that removes all the LTL assignments from the phase I solution and consolidates such assignments into transporter routing schedules subject to the transporter capacity and available time (after performing the full load assignment determined by the Phase I solution) constraints. In addition to evaluating the performance of this two-phase approach using a set of randomly generated test cases, we also apply this approach to a real-life supply network coordination problem.

The main advantage of this proposed approach, over the classical decoupled approach, is its ability to *simultaneously* optimize the production, inventory and transportation operations (while subject to restricted routing/direct shipments) without the needs for aggregating the demand of DCs and relaxing the constraints on transportation operations/capacities. The phase I search always ends up with a solution within the feasible region and the phase II then improves this feasible solution by further exploring its neighborhood areas.

Section 2 of this paper presents the proposed two-phase approach to PDRP. We formulate the phase I problem to explicitly include the period demand from each DC and the heterogeneity of transporters into the production and inventory optimization assuming direct shipment between plants and DCs. We then, for the Phase II problem, propose a heuristic search algorithm, the LC algorithm, to consolidate the LTL assignments caused by the potential inefficiency of direct shipment. Section 3 presents our performance evaluation for the two-phase approach. Section 4 reports the application of this approach to a real-life supply network with additional practical constraints. Finally, we conclude the study and discuss future research directions in Section 5.

2 The Two-Phase Approach to PDRP

Let the transporter routing be restricted to direct shipment between plants and DCs. That is, a transporter may only visit a single DC per trip (i.e., leaving a plant, visiting a DC, and then returning to the plant). Let $y_{i,j}^{v}(\mathrm{t})$ denote the number of direct shipment trips that transporter v, $v \in V(i)$, makes between plant i and $\mathrm{DC_j}$ in time period t. Let $q_{i,j}^{v}(\mathrm{t})$ be the total quantity delivered from plant i to $\mathrm{DC_j}$ by transporter v in period t through $y_{i,j}^{v}(\mathrm{t})$ trips, and replace $\sum_{n\in N(v)} q_{i,j}^{v,n}(t)$ in (2) and (3) by $q_{i,j}^{v}(t)$. Then, we can reformulate **P** into a more restricted problem, $\mathbf{P_1}$, as follows.

$$\mathbf{P_1}\text{: Min } \begin{array}{l} \sum_{t\in T}\sum_{i\in I}\sum_{v\in V(i)}\sum_{j\in J} c_v t_{i,j}^{v} y_{i,j}^{v}(t) + \sum_{t\in T}\sum_{i\in I}\sum_{j\in J} c_{i,j}^{c} Q_{i,j}(t) + \\ \sum_{t\in T}\sum_{i\in I} a_i(t)p_i(t) + \sum_{t\in T}\sum_{i\in I} h_i(t)(p_i(t)/2 + s_i(t)) + \sum_{t\in T}\sum_{j\in J} \hbar_j(t) z_j(t) \end{array} \quad (13)$$

Subject to

(2), (3), (4), (5), (6) and

Transporter capacity constraints

$$q_{i,j}^{v}(t) \le C_v y_{i,j}^{v}(\mathrm{t}) \quad \forall i \in \mathrm{I}, v \in V(i), j \in J, t \in T \quad (14)$$

Transportation duration constraints

$$\sum_{j\in J} t_{i,j}^{v} y_{i,j}^{v}(t) \le T_v(t) \quad \forall i \in I, v \in V(i), t \in T \quad (15)$$

$$y_{i,j}^{v}(t) \text{ integers} \quad (16)$$

Optimally solving model $\mathbf{P_1}$ determines, for each time period, the production quantity by each plant, the quantity to be delivered from each plant by each transporter to each DC, the amount of quantity to be carried at each inventory, and the optimal number of direct shipment trips that an individual transporter should make. Note that model $\mathbf{P_1}$ contains substantially less number of integer variables than model **P**, and is more computationally tractable because of the elimination of transporter routing constraints. For the same sized problem with 2 plants, 8 owned vessels, 20 DCs, and 12 time periods, the number of integer variables is now reduced from 483,840 to 3,840.

Let $G^*(P_1)$, and $G^*(P)$, be the optimal objective function value of $\mathbf{P_1}$, and **P**, respectively. The following relationship holds.

Proposition 1. A feasible solution to $\mathbf{P_1}$ is always a feasible solution to **P**, and $G^*(P) \le G^*(P_1)$.

Proof. Any feasible solution to $\mathbf{P_1}$ satisfies all the constraints in **P**. Since the optimal solution to $\mathbf{P_1}$ is a feasible solution to **P**, $G^*(P) \le G^*(P_1)$. ◊

The gap between $G^*(P)$ and $G^*(P_1)$ depends on the magnitude of cost savings that may be achieved by using tours to consolidate less-than-transporter-load (LTL) assignments in the transporter routing schedules. For this reason, we solve a follow-up Phase II problem to consolidate the LTL assignments. To do so, let H(t), $||H(t)|| \le ||J||$, denote the subset of DCs that receives LTL shipments in period t from the optimal solution to $\mathbf{P_1}$. Each DC_j, $DC_j \in H(t)$, has a LTL delivery in quantity $d_j(t) < C_v$, where C_v stands for the capacity of the transporter (v) assigned to perform the respective LTL assignment by the Phase I solution. Let $\tau_v(t)$, $\tau_v(t) \le T_v(t)$, be the available time for transporter v to perform LTL assignments in period t, where $\tau_v(t)$ equals to $T_v(t)$ minus the time needed by transporter v to perform full load transportation assignments determined by the optimal solution to $\mathbf{P_1}$.

Let a transporter *trip* be a *route* leaving a plant, visiting a sequence of DCs, and then returning to that plant. The problem in Phase II is then to *determine the set of trips for each transporter, in each period t, to minimize the total transportation cost subject to the satisfaction of LTL demand, the transporter capacity (C_v), and the transporter available time ($\tau_v(t)$) constraints*. We shall call this problem $\mathbf{P_2}$.

Let $G^*(\lfloor P_1 \rfloor)$ equal to $G^*(P_1)$ minus the traveling and holding cost associated to the LTL assignments. Then we have the following result (proof skipped).

Proposition 2. For any given feasible solution to $\mathbf{P_1}$, a feasible solution to the associated $\mathbf{P_2}$ plus the full-load assignments defined by that feasible solution to $\mathbf{P_1}$ gives a feasible solution to **P**, and

$$G^*(P) \le G(P_2) + G(\lfloor P_1 \rfloor) \le G(P_1).$$

To develop a heuristic solution procedure for solving $\mathbf{P_2}$, we start with the following subproblem $\mathbf{S_2}$.

S$_2$: We are given a single plant, a single transporter with capacity C, and a set of H demand centers, each with a demand d_i, $d_i < C, 1 \le i \le H$, $1 \le \text{H} \le \| \text{J} \|$. The cost (or the time) for the transporter to travel from DC$_i$ to DC$_j$ is known as $t_{i,j}$. Let π be a permutation of DCs and, without loss of generality, let

$$\pi = < DC_1,\ DC_2, ..., DC_H > .$$

The problem is to partition π into a sequence of transporter trips (see Figure 1 below) to minimize the total cost subject to C and $\{d_j \mid 1 \le j \le H\}$.

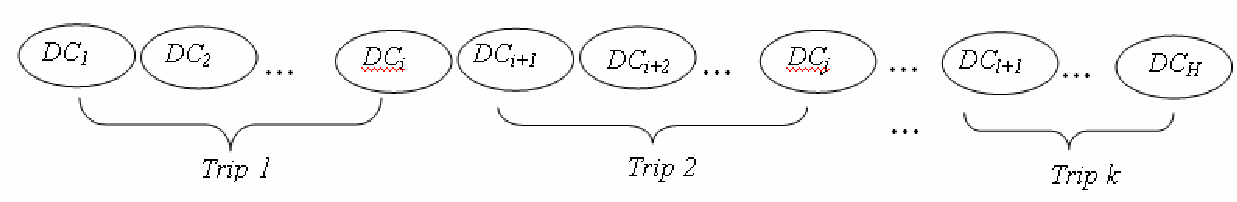

Figure 1. A sequence of trips.

Let $\pi^+ = < DC_0 \| \pi >$, where DC$_0$ denotes the plant. If we assume that each DC may be visited only once, then **S$_2$** can be solved in strongly polynomial time by a straight forward extension from the known Optimal Partitioning Procedure (Beasley, 1983; Altinkemer and Gavish, 1987; Li and Simchi-Levi, 1990). For the completeness of the paper, we present this *Extended Optimal Partitioning* (EOP) procedure as follows:

Procedure EOP: Construct a directed acyclic graph G with V(G)=$\{i \mid 0 \le i \le H\}$, where node i denotes DC$_i$, $0 \le i \le H$. Let E(G) be the set of directed arcs on G, where $(i, j) \in E(G)$ iff

$$d_{i+1} + d_{i+2} + ... + d_{j-1} + d_j \le C\,,$$

that is, the transporter has enough loading capacity to serve all the demand from node i+1 to node j. Each arc represents a feasible trip for the transporter. Add all such feasible arcs, (i, j), $0 \le i < j, 1 \le j \le H$, to G(V,E), and let the arc length be the resulting trip cost, $\tau_{i,j} = t_{i,0} + t_{0,i+1} + \sum_{h=i+1...j-1} t_{h,h+1}$.

Figure 2 illustrates a network with H=6 and π=<DC$_1$, DC$_2$,…,DC$_6$>, where the number on arc (i, j) denotes $t_{i,j}$, and the number on node j denotes d_j. Assuming C = 1200 units, then the resulting G(V,E) is given in Figure 3. As we can see, each path on graph G(V,E) from node 0 to node H defines a feasible sequence of transporter trips that satisfies all the DC demands and the transporter capacity constraints. For example, an arc from node 1 to node 4 represents the trip from node 1 to node 0 (to get loaded), node 0 to node 2, node 2 to node 3, and then node 3 to node 4, with an arc cost of 240+280+168+85=773. The shortest path on G(V,E) is 0-2-3-6 with a total cost of 1181 and defines three trips: 0-1-2-0, 0-3-0, and 0-4-5-6-0.

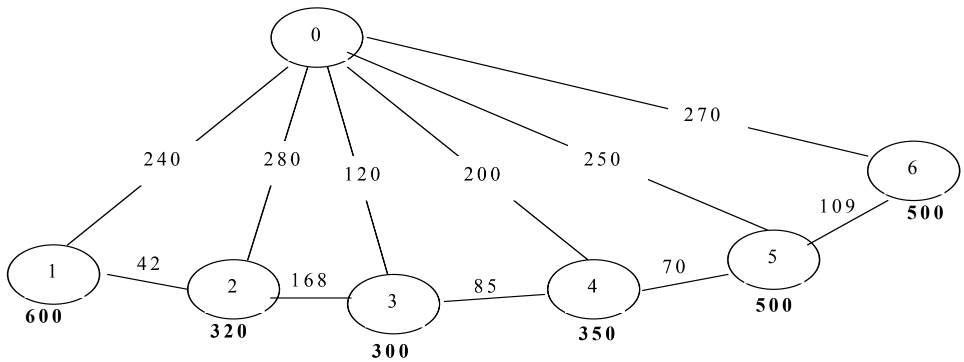

Figure 2. A network with 6 customer DCs and a given route <DC_1, DC_2,…,DC_6>.

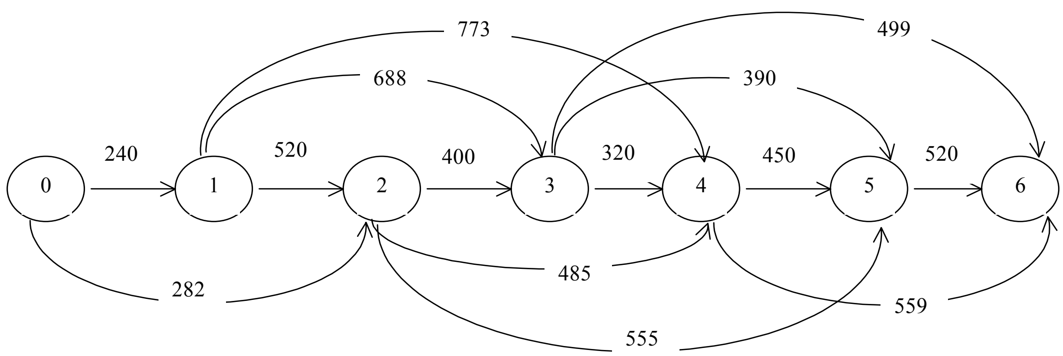

Figure 3. The directed acyclic graph $G(V, E)$ for the network in Figure 2.

Proposition 3. If each DC can be visited only once, then the shortest path on G(V,E) solves $\mathbf{S_2}$ optimally in $O(H^3)$.
Proof. The shortest path on G(V,E) minimizes the total cost while satisfies the demand and transporter capacity constraints. Since G(V,E) is acyclic and contains at most $O(H^2)$ arcs and H+1 nodes, the shortest path on G(V,E) can be determined in $O(H^3)$ according to Ahuja, Magnanti and Orlin (1993). ◊

The EOP procedure is used as a subroutine in our proposed heuristic routing algorithm (for heterogeneous capacitated transporters) in Phase II, called the *Load Consolidation (LC)* algorithm. This algorithm requires, for each period t, a predetermined tour that connects all DCs in set H(t), ^{0}, and a predetermined tour for each transporter, (v), $1 \le v \le V$, that connects all DCs with LTLs assigned to transporter v from the Phase I solution. Given such tours, the proposed LC algorithm calls two heuristic procedures, H_1 and H_2, each is able to generate a feasible solution to P_2. The best feasible solution by H_1 and H_2 is then used as the solution of the LC algorithm.

Let H(t) be a union of all the DCs that has a LTL delivery in period t (regardless by which transporter), the shortest path covering (v) be $P(\pi(v))$, and $_v$(t) be the total available time that transporter v has for performing LTL assignments for the time period. For each given period t, heuristic H_1 calls the EOP procedure to identify P((v)), $\forall v \in V$. Since (v) consists of feasible direct shipment assignments to transporter v from the Phase I solution, the length of any path on (v), including that of P((v)), are no more than $_v$(t). Given $P(\pi(v))$, $\forall v \in V$, heuristic H_1 evaluates the potential savings (in terms of transportation cost) of shifting node j, $j \in \pi(v)$, from P((v)) to P((v')), $v' \neq v$, subject to $_v$(t) and $_{v'}$(t). For any given path $P(\pi(v'))$, $v' \neq v$, if taking node j does not violate $_{v'}$(t), then the net saving is equal to the savings obtained by eliminating j from path P((v)) minus the cost of adding j to $P(\pi(v'))$. Otherwise, this net saving is equal to that achieved by exchanging node j on $P(\pi(v))$ with some node j' on $P(\pi(v'))$. By comparing the resulting net savings from all $P(\pi(v'))$, $v' \neq v$, the path that yields the maximum net savings is selected to take node j (and node j is then permanently eliminated from path $P(\pi(v))$). Heuristic H_1 repeats this process until all nodes j, $j \in H(t)$, are evaluated.

Heuristic H_2 applies two transporter ranking criteria, the *fastest transporter first* and the *least cost-per-mile-per-ton transporter first,* and starts with tour π^0 that connects all the DCs in H(t). For each given criterion, H_2 selects the highest-ranking transporter and applies the EOP procedure to π^0 to construct the shortest path for the selected transporter while relaxing $_v$(t). If the resulting length of the shortest path exceeds $_v$(t) for the transporter being considered, then either a selected trip (when transporters are identical) or a selected node (when transporters are heterogeneous) is removed until the resulting length of the shortest path satisfies $_v$(t). Let H'(t) be the set of removed nodes, procedure H_2 then repeats the same procedure to the next highest-ranking transporter until H'(t) becomes empty.

Proposition 4. For $\mathbf{P_2}$ with two heterogeneous transporter case, if the transporters must travel along a given tour π^0 and the assignments to each transporter must be based on consecutive DCs along π^0, then $\mathbf{P_2}$ is solvable in $O(H^4)$.

Proof. Let π^0=<DC_1, DC_2…., DC_i,… DC_H> and let i, $1 \le i \le H$, be the DC that partitions π^0 into two subsequences of DCs:

σ_1=<DC_1, DC_2…., DC_i > and σ_2=<DC_{i+1}, DC_{i+2}…. DC_H>.

Assign transporter 1 to σ_1, transporter 2 to σ_2, and apply the EOP procedure to solve for the shortest path on σ_1, and σ_2, respectively. Let $L(P(\sigma_1))$ and $L(P(\sigma_2))$ be the resulting path lengths. Partition i is feasible iff $L(P(\sigma_1)) \le \tau_1(t)$ and $L(P(\sigma_2)) \le \tau_2(t)$. Let $G(i,1,2) = L(P(\sigma_1)) + L(P(\sigma_2))$ be the resulting length/cost if partition i is feasible or $G(i,1,2) = \infty$ otherwise. Reverse the assignment of transporters (i.e., assign transporter 2 to σ_1 and transporter 1 to σ_2), apply the above process to obtain G(i,2,1). Let G(i)=min{G(i,1,2), G(i,2,1)} and i* be the partition that $G(i^*) = \min\{G(i) \mid \forall i\}$. The transporter trips under G(i*) gives the optimal solution to the problem. Since there are at most H alternative values for parameter i, and G(i) can be determined in $O(H^3)$ for each give partition i, the total computational time is thus bounded by $O(H^4)$. ◊

The two heuristics, H_1 and H_2, are formally described in the Appendix.

3 Performance Evaluation

In this section we present empirical results for evaluating the computational performance of the LC algorithm and the two-phase approach which uses CPLEX MIP solver to solve $\mathbf{P_1}$ and the LC algorithm to solve $\mathbf{P_2}$. The LC algorithm was coded in C and run on a 750Mhz Pentium III computer. The CPLEX MIP solver was run on a 1.2GMhz Dell Latitude.

The performance of the LC algorithm was evaluated based on two sets of test problems, where set 1 contains 32 relatively small problems and set 2 contains 24 relatively large test problems. Tables 1 and 2 summarize the parameters used to generate these test problems, where ||J|| stands for the number of DCs, C refers to the transporter capacity, c refers to the unit shipping cost ($/mile), and τ_v stands for the maximum traveling time available for a transporter to perform LTL assignments in Phase II. In Table 2, the instances from G-p3 to G-p6 are based on the test problems from Golden et al (1984), and the instances from T-p13 to T-p16 are based on the benchmarking problems from Taillard (1996). Since the intention of using these test data in this study differs from that in Golden et al (1984) and Taillard (1996), we removed the transporter fixed cost and added the maximum transporter available time for LTL assignments (τ_v), while all the other parameters (include node locations) from Golden et al (1984) and Taillard (1996) remain unchanged. For all the remaining test problems, we sampled the distance data from the study by Christofides and Eilon (1969) (i.e., the data for our 8-node problems are based on those for the first 8 nodes in their 100-node problem, and the data for our 12-node problems are based on their first 12-node problems, etc.).

Table 1 Parameters of test problems in data set 1 for the LC algorithm

Test cases	$\|J\|$	Transporter 1		Transporter 2		Transporter 3		Transporter 4		Transporter 5		Available time (τ_V)
		C	c	C	c	C	c	C	c	C	c	
P1	8	20	1	30	1.4	40	1.6	70	2.5	120	5.0	80
P2	8	60	1	80	1.4	150	2.6					140
P3	8	20	1	30	1.4	40	1.6	70	2.5	120	5.0	140
P4	8	60	1	80	1.4	150	2.6					200
P1-1	8	20	1	30	1	40	1	70	1	120	1	80
P2-1	8	60	1	80	1	150	1					140
P3-1	8	20	1	30	1	40	1	70	1	120	1	140
P4-1	8	60	1	80	1	150	1					200
P1-2	8	20	1	30	1	40	1	70	1	120	1	160
P2-2	8	60	1	80	1	150	1					220
P3-2	8	20	1	30	1	40	1	70	1	120	1	160
P4-2	8	60	1	80	1	150	1					220
P1-3	8	20	1	30	1.4	40	1.6	70	2.5	120	5.0	160
P2-3	8	60	1	80	1.4	150	2.6					220
P3-3	8	20	1	30	1.4	40	1.6	70	2.5	120	5.0	160
P4-3	8	60	1	80	1.4	150	2.6					220
P5	12	20	1	30	1.4	40	1.6	70	2.5	120	5.0	100
P6	12	60	1	80	1.4	150	2.6					180
P7	12	20	1	30	1.4	40	1.6	70	2.5	120	5.0	160

P8	12	60	1	80	1.4	150	2.6					280
P5-1	12	20	1	30	1	40	1	70	1	120	1	100
P6-1	12	60	1	80	1	150	1					180
P7-1	12	20	1	30	1	40	1	70	1	120	1	160
P8-1	12	60	1	80	1	150	1					280
P5-2	12	20	1	30	1	40	1	70	1	120	1	240
P6-2	12	60	1	80	1	150	1					280
P7-2	12	20	1	30	1	40	1	70	1	120	1	240
P8-2	12	60	1	80	1	150	1					280
P5-3	12	20	1	30	1.4	40	1.6	70	2.5	120	5.0	240
P6-3	12	60	1	80	1.4	150	2.6					280
P7-3	12	20	1	30	1.4	40	1.6	70	2.5	120	5.0	240
P8-3	12	60	1	80	1.4	150	2.6					280

Table 2. Parameters of test problems in data set 2 for the LC algorithm

Test cases	$\|J\|$	Transporter 1		Transporter 2		Transporter 3		Transporter 4		Transporter 5		Transporter 6		Avail. time
		C	c	C	c	C	c	C	c	C	c	C	C	
P9	16	20	1	30	1.4	40	1.6	70	2.5	120	5.0			150
P10	16	60	1	80	1.4	150	2.6							220
P11	16	20	1	30	1.4	40	1.6	70	2.5	120	5.0			200
P12	16	60	1	80	1.4	150	2.6							320
G-p3	20	20	1	30	1	40	1	70	1	120	1			200
G-p4	20	60	1	80	1	150	1							300
G-p5	20	20	1	30	1	40	1	70	1	120	1			250
G-p6	20	60	1	80	1	150	1							400
P13	25	20	1	40	1.1	50	1.2	70	1.7	120	3.0	200	5.2	200
P14	25	120	1	160	1.1	300	2.4							400
P15	25	50	1	100	1.8	160	3.2							400
P16	25	40	1	80	1.6	140	2.1							400
P17	30	20	1	40	1.1	50	1.2	70	1.7	120	3.0	200	5.2	220
P18	30	120	1	160	1.1	300	2.4							440
P19	30	50	1	100	1.8	160	3.2							450
P20	30	40	1	80	1.6	140	2.1							450
P21	40	20	1	40	1.1	50	1.2	70	1.7	120	3.0	200	5.2	400
P22	40	120	1	160	1.1	300	2.4							640
P23	40	50	1	100	1.8	160	3.2							650
P24	40	40	1	80	1.6	140	2.1							650
T-p13	50	20	1	30	1.1	40	1.2	70	1.7	120	2.5	200	3.2	400
T-p14	50	120	1	160	1.1	300	1.4							900
T-p15	50	50	1	100	1.6	160	2.0							800
T-p16	50	40	1	80	1.6	140	2.1							800

Table 3. The LC Algorithm solutions vs. CPLEX solutions for data set 1

Test cases	$\|J\|$	**LC Algorithm Solution** (All in less than 0.2 s)	**CPLEX**		$[(G_{LC}-G_{CPLEX})/G_{CPLEX}]\%$
			Solution	CPU Time	
P1	8	370.1	370.1	7s	0.00%
P2	8	203.4	203.4	10s	0.00%
P3	8	608.6	557	120s	9.26%
P4	8	299	270.2	64s	10.81%
P1-1	8	183	183	26s	0.00%
P2-1	8	156	154	1.2s	1.30%
P3-1	8	213	213	22s	0.00%
P4-1	8	146	146	0.13s	0.00%

Test cases	$\|J\|$	**LC Algorithm Solution** (All in less than 0.2 s)	**CPLEX**		$[(G_{LC}-G_{CPLEX})/G_{CPLEX}]\%$
			Solution	CPU Time	
P1-2	8	159	154	0.07s	3.25%
P2-2	8	145	145	0.05s	0.00%
P3-2	8	188	185	1.32s	1.62%
P4-2	8	146	146	0.15s	0.00%
P1-3	8	337.4	337.4	9.68s	0.00%
P2-3	8	183	183	2.36s	0.00%
P3-3	8	561.6	557	95.2s	0.83%
P4-3	8	270.2	270.2	120s	0.00%
P5	12	502.8	483.1	2hrs	4.08%
P6	12	260.2	260.2	780s	0.00%
P7	12	734.3	731.3	2hrs	0.41%
P8	12	353	348.8	6100s	1.20%
P5-1	12	257	250	1000s	2.80%
P6-1	12	200	198	5.26s	1.01%
P7-1	12	280	262	850s	6.87%

Tables 3 and 4 compare the solutions of the LC algorithm with that by CPLEX MIP Solver in two-hour CPU time. For relatively small problems ($8 \le N \le 12$), the average deviation from the CPLEX solutions is 1.98% with the largest deviation of 10.81%. For larger problems ($16 \le N \le 50$), the LC algorithm found the same or better solutions, in 13 out of 16 cases, than the best effort by CPLEX MIP Solver in 2 hours. For all the 56 test problems, the LC algorithm terminates in less than 0.2 second.

Table 4. LC Algorithm solutions vs. CPLEX solutions for data set 2

Test cases	$\|J\|$	**LC Algorithm Solution** (All in less than 0.2s)	**CPLEX Solution** (in 2 hours)	$[(G_{LC}-G_{CPLEX})/ G_{CPLEX}]\%$
P9	16	682.4	713.2	-4.3%
P10	16	381.4	351.6	+8.48%
P11	16	947.8	951.7	-0.41%
P12	16	433.4	453.6	-4.5%
G-P3	20	673.7	742.9	-9.3%
G-P4	20	311	295	+5.42%
G-P5	20	857.5	867.3	-1.1%
G-P6	20	344	344	0.00%
P13	25	931.6	--	--
P14	25	413.9	415.4	-0.4%
P15	25	874	911.6	-0.41%
P16	25	818.9	736.8	+11.14%
P17	30	1153.6	--	--
P18	30	463.1	481.8	-3.9%
P19	30	1056.4	--	--
P20	30	1008.7	1062.7	-5.1%
P21	40	1501.7	--	--
P22	40	569.5	838	-32%
P23	40	1034	--	--
P24	40	1073.5	--	--

Test cases	$\|J\|$	**LC Algorithm Solution** (All in less than 0.2s)	**CPLEX Solution** (in 2 hours)	$[(G_{LC}-G_{CPLEX})/G_{CPLEX}]\%$
T-P13	50	1846.8	--	--
T-P14	50	683.2	847	-19.3%
T-P15	50	1135	1335	-15%
T-P16	50	1264.2	--	--

--: No feasible solution within 2 hours by CPLEX.

We randomly generated 48 test problems for comparing the solutions of the proposed two-phase approach with that by applying CPLEX MIP solver to solve the original model (**P)** directly. For the proposed two-phase approach, the optimization problem in phase I ($\mathbf{P_1}$) was solved by CPLEX MIP solver and the LTL consolidation problem in phase II ($\mathbf{P_2}$) was solved by the LC algorithm.

To generate the test problem, the distances between DCs and the distance between each DC and the (single) plant were randomly sampled from a uniform distribution over [0,100]. The travel cost between each pair of locations was given by a constant (c) multiplied by the respective distance. We used a fleet of two heterogeneous transporters with same traveling speed but different loading capacity, $C_i, i = 1,2,$ randomly sampled from uniform [8, 16]. The number of time periods in planning horizons ranged from 2 to 4, and parameter $\mu = (C_1 + C_2)/2$ was used to generate the period DC demand from uniform [1μ, 1.5μ], [1.5μ, 2μ], [2μ, 2.5μ], and [2.5μ, 3μ]. The safety stock level at each DC was set to be one third of the maximum period demand over all time periods in a 12-period planning horizon, and the storage capacity was set to be the maximum period demand over the planning horizon. The starting inventory at each DC had two levels. The lower level was set equal to the DC safety stock requirement, while the high level was set to be the average of safety stock requirement and DC storage capacity. For all the test problems, we set c=2, a=3, and h=0.50. Among the 48 test problems, problems 1 to 16 had $\|J\|=5$ and $T_v(t)=360$, problems 17 to 32 had $\|J\|=6$ and $T_v(t)=640$, problems 33 to 40 had $\|J\|=10$ and $T_v(t)=1280$, and problems 41 to 48 had $\|J\|=12$ and $T_v(t)=1600$, for all transporters v.

Table 5 compares solutions of the two-phase approach versus that obtained by applying CPLEX MIP Solver to model **P** directly. As we can see, in 34 out of 48 cases, the two-phase approach found, in much less time, either the same/better solution than the best solution by the CPLEX MIP solver in 4-hour CPU time, or a feasible solution that CPLEX MIP solver cannot find within the given time limit. We also see that, in 14 out of 48 cases, the two-phase approach failed to get a better solution. The largest gap from the CPLEX MIP solution is 10.22%. Nevertheless, the performance of the two-phase approach seems promising and requires less than 1 minute of CPU time for most test problems.

Table 5. Two-Phase Approach Solutions vs. CPLEX MIP solutions

Test cases	$G^*(\lfloor P_1 \rfloor)$	$G(P_2)$	Two-Phase		CPLEX Solution[a] for P (Best in 4 hrs)	$[(G_{LC}-G_{CPLEX})/G_{CPLEX}]\%$
			Solution	Time (in seconds)		
1	1249.25	202	1451.25	0.3	1451.25	0.00%
2	1466.25	628	2094.25	0.4	2051.75	2.07%
3	2286.25	404	2690.25	0.4	2690.25	0.00%
4	3255.75	280	3535.75	0.4	3536.25	-0.01%
5	704.5	488	1192.5	0.4	1192.5	0.00%
6	1060	618	1678	0.4	1678	0.00%
7	1763.5	490	2253.5	0.4	2224	1.33%
8	2387.5	348	2735.5	0.5	2713	0.83%
9	2290.25	622	2912.25	0.6	2912.25	0.00%
10	3443.25	444	3887.25	0.6	3891.25	-0.10%
11	4239.25	888	5127.25	16.5	5147.75	-0.40%
12	6105.25	220	6325.25	13.3	6327.25	-0.03%
13	1869.25	608	2477.25	0.6	2477.25	0.00%
14	2726.75	914	3640.75	2600.4	3635.75	0.14%
15	4066.75	444	4510.75	1.3	4531.75	-0.46%
16	5202.25	444	5646.25	56.3	5649.25	-0.05%
17	1348.75	950	2298.75	0.3	2298.75	0.00%
18	2354.75	980	3334.75	0.4	3269.75	1.99%
19	2706.25	1504	4210.25	0.4	4216.25	-0.14%
20	4428.75	828	5256.75	0.3	5202.75	1.04%
21	515.75	1504	2019.75	0.4	1970.75	2.49%
22	1649.25	886	2535.25	0.4	2300.25	10.22%
23	2694.25	658	3352.25	0.4	3352.75	-0.01%
24	3498.75	676	4174.75	0.3	4181.25	-0.16%
25	2636.5	1990	4626.5	1.0	4533	2.06%
26	5214	1228	6442	110.2	6463	-0.32%
27	7356.5	772	8128.5	28.3	8313.5	-2.23%
28	9414	676	10090	730.2	10233	-1.40%
29	2266.5	1916	4182.5	0.5	4033.5	3.69%
30	4779	850	5629	12.4	5603.5	0.46%
31	5613	1836	7449	2500.3	7461.5	-0.17%
32	8313	676	8989	770.2	9001	-0.13%
33	3619.75	874	4493.75	0.07	4661.75	-3.60%
34	4292.25	2242	6534.25	0.09	--	--
35	6360.25	1556	7916.25	0.36	8266.75	-4.24%
36	9146.75	576	9722.75	5.93	--	--
37	2105.25	1596	3701.25	0.07	3494.75	5.91%
38	3388.75	1842	5230.75	0.08	5964.75	-12.31%
39	5332.75	1090	6422.75	0.09	7417.25	-13.41%
40	6753.75	1080	7833.75	0.09	8027.25	-2.41%
41	4733.25	912	5645.25	0.09	5712.25	-1.17%
42	5578.75	2780	8358.75	0.12	9760.75	-14.36%
43	7904.75	2168	10072.75	0.12	--	--
44	10584.75	1616	12200.75	0.14	11430.25	6.74%
45	2547.75	2216	4763.75	0.09	4448.75	7.08%
46	4138.75	2558	6696.75	0.10	--	--
47	6795.25	1286	8081.25	0.11	--	--
48	8090.25	2116	10206.25	0.10	--	--

[a]: Best CPLEX solution in four (4) hours of CPU time.

4 An application of the two-phase approach[1]

We have applied the two-phase approach to a real-life supply network planning problem encountered at a leading chemical company that produces various highway/road maintenance chemicals at its multiple plant sites, and distributes finished goods to a large number of DCs in North America. Due to steady increases in customer demand, the company has been continuously expanding the market and the distribution network by adding new distribution channels. The transportation between plants and DCs is by heterogeneous ocean ships and water barges. Due to high marine transportation costs, any inefficiency in scheduling may easily lead to a significant increase (in the magnitude of millions of dollars) in the operation costs.

The specific data set[2] we used for this section includes 12 time periods, 2 plants (Michigan plant and Ontario plant), 13 DCs, and 3 heterogeneous vessels with loading capacities and traveling speed of (9050 tons, 7 knots), (8000 tons, 13 knots), and (9050 tons, 7 knots), respectively. In addition to transporting finished goods to DCs, the vessels are also used to transport raw material from the Michigan plant to the Ontario plant. Whenever the company faces a shortage in vessel capacity, chartered vessels are available but at a higher cost. In addition to those standard constraints modeled in **P**, additional constraints that ensure the practical feasibility/implementation of our solution were also included. First, each vessel requires a one-month maintenance time in each planning year. To do so, we expanded model $\mathbf{P_1}$ by introducing binary (logical) variables

$$\delta^{v}(t) = \begin{cases} 1 & \text{if vessel v is available in month t;} \\ 0 & \text{if vessel v undergoes maintenance in month t} \end{cases}$$

and represented the maintenance requirement as

$$\sum_{t=1}^{12} \delta^{v}(t) = 11.$$

To limit the number of trips in month t, we added constraints

$$y_{i,j}^{v}(t) \le M\delta^{v}(t),$$

where M is a upper bound on the maximum number of trips that vessel v can make in month t. Second, no vessel may travel northeast of Cleveland in winter, because the chemicals may freeze during transportation and the lake ways may shrink during winter season. This is enforced by specifying a set of DCs that cannot be accessed during the winter season (December, January, February and March), J', and then setting a priori the respective trip variables $y_{i,j}^{v}(t) = 0$, $\forall v \in V(i),\ t \in \{1,2,3,12\},\ i \in I, j \in J'$. By applying the CPLEX MIP Solver to solve model $\mathbf{P_1}$ and then the LC algorithm to solve model $\mathbf{P_2}$, our two-phase approach found the solution to the following issues:

(a) Monthly production schedule and ending inventory plans at each plant;
(b) Monthly raw material transportation plan;
(c) Distribution plans and vessel monthly routing schedules;

[1] The original application of the two-phase approach to this company was selected as a semi-finalist for the 2001 INFORMS Edelman Award Competition.
[2] Some of the data used in this section were modified for confidentiality.

(d) Monthly ending inventory plan for each DC; and
(e) Recommended annual vessel maintenance schedules.

Table 6. Itemized Annual Operation Cost Projection

Total Cost	$ 11,440,587.00	100.00%
Transportation	$ 6,697,141.00	62.58%
Chartered Transportation	$ 0.00	0.00%
Production Cost	$ 4,056,680.00	32.18%
Inventory	$ 686,766.00	5.23%

Table 7. Twelve-Month Production Plan for the Ontario Plant

	Production quantity (in tons)	Product ending inventory (in tons)	Raw material ending inventory (in tons)
Initial		81087	40000
July 2001	46330.8	81086.6	40000
August 2001	7699.2	81086.6	40000
September 2001	24758.4	99929.7	50000
October 2001	10984.6	87872.8	50000
November 2001	36582.0	78863.9	50000
December 2001	8136.5	20271.6	26855
January 2002	34096.9	20271.6	26324
February 2002	37554.0	40543.3	26258
March 2002	24764.6	40543.3	32024
April 2002	5252.15	42791.2	50000
May 2002	10790.5	50208.5	50000
June 2002	41745.7	81086.6	40000

Tables 6, 7 and 8 report the itemized annual cost projections, facility production/inventory plans, and vessel routing schedules constructed by the two-phase approach. Table 6 indicates that the vessel transportation cost is the major cost component in company's annual operation budget (62.58% of the annual operation cost). With the support of optimization tools, the company is able to fully rely on its own fleet for distributing the product in the planning horizon without the need for any chartered service. Tables 7 and 8 present the resulting production/inventory plans and vessel routing schedules that together minimize the annual production, inventory and transportation cost.

Table 8. Sampled Vessel Routing Schedules (Vessel #2 from the Ontario Plant)

July. 2001	5 direct shipment trips: Ontario ⇔ Michigan
August 2001	Consolidated trip: Ontario ⇒ DC#11 ⇒ DC#13 ⇒ Ontario
September 2001	3 direct shipment trips: Ontario ⇔ Michigan, 1 direct shipment trip: Ontario ⇔ DC#5, 1 direct shipment trip: Ontario ⇔ DC#7
October 2001	4 direct shipment trips: Ontario ⇔ Michigan
November 2001	4 direct shipment trips: Ontario ⇔ Michigan
December 2001	4 direct shipment trips: Ontario ⇔ Michigan, 1 direct shipment trip: Ontario ⇔ DC#5
January 2002	4 direct shipment trips: Ontario ⇔ Michigan
February 2002	1 direct shipment trip: Ontario ⇔ DC#5
March 2002	3 direct shipment trips: Ontario ⇔ Michigan, 1 direct shipment trip: Ontario ⇔ DC#5
April 2002	3 direct shipment trips: Ontario ⇔ Michigan, 1 direct shipment trip: Ontario ⇔ DC#7
May 2002	5 direct shipment trips: Ontario ⇔ Michigan
June 2002	Consolidated trip: Ontario ⇒ DC#7 ⇒ DC#12 ⇒ Ontario, 1 direct shipment trip: Ontario ⇔ DC#5

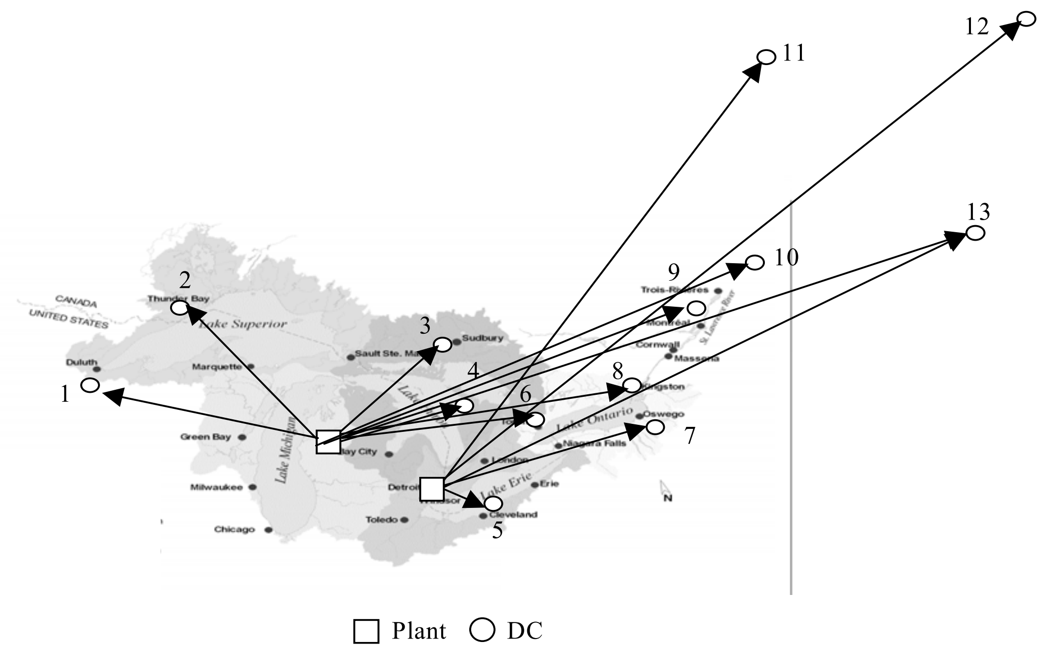

Figure 4: Strategic relations between production facilities and DCs.

Figure 4 shows the strategic plant-DC relations recommended by the two-phase approach. As we can see from the graph, nine DCs (including three DCs near the Lake Superior and Lake Huron areas) should be strategically supplied by the Michigan plant, and five DCs (which are near and to the east of Lake Erie and Ontario areas) should be served by the Ontario plant. One particular DC (DC #13) should be supplied jointly by both plants. In addition to the operation plans, the solution by the two-phase approach also offers the top management of the company valuable information on building direct and long term relationships between plants and customers at the DCs and improving the forecasting accuracy because of the known target customers.

The solution by the two-phase approach to this application was obtained in twenty minutes CPU time of a laptop with 512 MB RAM and 900MHz CPU. For most other applications of the two-phase approach (i.e., either as a decision support tool for operations planning or as a tool for answering what-if type questions) at the company, feasible solutions started to stabilize (i.e., the gap between solutions from consecutive iterations remains within 0.1-0.5 %) within one hour.

5 Conclusion and Future Studies

We have proposed a two-phase solution approach to the integrated production, inventory and distribution problem where the transporter routing must be optimized together with the production lot sizes and the inventory policies. The phase I model is solved as a mixed integer programming problem subject to all the constraints in the original model except that the transporter routings are restricted to direct shipment. The resulting Phase I solution is always feasible to the original problem and determines the production quantity for each DC, the ending inventory for both plants and DCs, and the quantity and trips by each transporter from each plant to each DC in each time period. To handle the potential inefficiency of the direct shipment, Phase II applies a heuristic procedure (the LC algorithm) to solve an associated consolidation problem. The associated delivery consolidation problem is formulated as a capacitated transportation problem with additional constraints. Computational performance of this proposed two-phase approach and its application to a real life supply network are reported.

There are several potential extensions from this work. First, from a practical point of view, models that allow the DC demands to be random variables and some DCs to be used as transshipments point could be of a great value to the real world needs. Second, from an academic research point of view, new algorithms that can effectively solve the integrated production, inventory and distribution routing problem subject to direct shipment for Phase I, other than using the CPLEX MIP solver to solve $\mathbf{P_1}$ directly, will be of interest. For a noticeable number of test cases we experienced, the time required for CPLEX to verify the optimal solution to $\mathbf{P_1}$ was excessive (see test problems 14, 31 and 32 in Table 5). One possible approach in this regard is to use the Lagrangian Relaxation. Third, we assumed in this study that each plant owns a fixed fleet of heterogeneous transporters. Relaxing this assumption and allowing the assignment of transporters to plants to be optimized are likely to lead to a better solution. Finally, there have been a vast amount of research results available for capacitated vehicle routing. A comparative study of the LC algorithm used in Phase II of this study with existing heuristics in the literature results has a potential to further improve the solution quality of the approaches to the integrated production, inventory and distribution routing problems.

Reference

Ahuja, R. K., T. L. Magnanti, and J. B. Orlin. 1990. *Network Flows*. North-Holland, Amsterdam.

Altinkemer, K., and B. Gavish. 1987. Heuristics for unequal weight delivery problems with fixed error guarantee. *Operations Research Letters* **6** 149-158.

Anily, S. and A. Federgruen. 1993. Two-echelon distribution systems with vehicle routing costs and central inventories. *Operations Research* **41** 37-48.

Baita, F., W. Ukovich, R. Pesenti, and D. Favaretto. 1998. Dynamic routing-and-inventory problems: a review. *Transportation Research Part A: Policy and Practice* **32** (8) 585-598.

Beasley, J. 1983. Route first – cluster second methods for vehicle routing. *Omega* **11** 403-408.

Bhatnangar, R., P. Chandra, and S. K. Goyal. 1993. Models for multi-plant coordination. *European Journal of Operational Research* **67** (2) 141-160.

Blumenfeld, D.E., L.D. Burns, and C.F. Daganzo. 1991. Synchronizing production and transportation schedules. *Transportation Science* **25B** 23-37.

Brown, G., J. Keegan, B. Vigus, and K. Wood. 2001. The Kellogg company optimizes production, inventory, and distribution. *INTERFACES* **31** (6) 1-15.

Chandra, P. 1993. A dynamic distribution model with warehouse and customer replenishment requirements. *Journal of the Operational Research Society* **44** (7) 681-692.

Chandra, P., and M. L. Fisher 1994. Coordination of production and distribution planning. *European Journal of Operational Research* **72** (3) 503-517.

Christofides, N., and S. Eilon. 1969. AN algorithm for the vehicle dispatching problem. *Operations Research Quarterly* **20** 309-318.

Cohen, M. A., and H. L. Lee. 1988. Strategic analysis of integrated production-distribution systems: models and methods. *Operation Research* **36** (2) 216-228.

Cohen, M. A., and H. L. Lee. 1989. Resource deployment analysis of global manufacturing and distribution networks. *Journal of Manufacturing and Operations Management* **2** 81-104.

Bloomquist D., D. Graziosi, L. Lei, A. Ruszczynski, S. Liu, and H. Zhong. 2002. Practice abstracts: optimizing production, inventory, and distribution for General Chemical Group. *Interfaces* **32** (4) 67-68.

Dror, M., and M. Ball. 1987. Inventory/routing: reduction from an annual to a short period. *Naval Research Logistics* **34** 891-905.

Erengüç, S. S., N. C. Simpson, and A. J. Vakharia. 1999. Integrated production/distribution planning in supply chains: *An invited review. European Journal of Operational Research* **115** (2) 219-236.

Federgruen, A. and D. Simchi-Levi. (Forthcoming). Analytical analysis of vehicle routing and inventory routing problems. *Handbooks on Operations Research and Management Science* (eds. M. Ball, T. Magnanti, C. Monma and G. Nemhauser).

Fumero, F., and C. Vercellis. 1997. Integrating distribution, machine assignment and lot-sizing via lagrangean relaxation. *International Journal of Production Economics* **49** 45-54.

Fumero, F., and C. Vercellis. 1999. Synchronized development of production, inventory, and distribution schedules. *Transportation Science* **33** (3) 330-340.

Golden, B., and A. Assad (editors). 1988. Vehicle routing: methods and studies. Studies in Management Science and Systems, Vol. 16. Elsevier Science Publishers, North-Holland, Amsterdam.

Golden, B., A. Assad, L. Levy, and F. Gheysens. 1984. The fleet size and mix vehicle routing problem. *Computers and Operations Research* **11** (1) 49-66.

Gupta, V., E. Peters, and T. Miller. 2002. Implementing a distribution-network decision-support system at Pfizer/Warner-Lambert. *INTERFACES* **32** (4) 28-45.

Harps, L. H. 2003. The Nature of Change, *Inbound Logistics* **23** (1) 76-132.

Lee, H. L. and S. Whang. 2001. Winning the Last Mile of E-Commerce. *MIT Sloan Management Review* **42** (4) 54-62.

Li, C. L., and D. Simchi-Levi. 1990. Analysis of heuristics for the multi-depot capacitated vehicle routing problems. *ORSA Journal on Computing* **2** 64-73.

Özdamar, L., and T. Yazgac. 1999. A hierarchical planning approach for a production-distribution system. *International Journal of Production Research* **37** (16) 3759-3772.

Pyke, D.F., and M. A. Cohen. 1993. Performance characteristics of stochastic integrated production-distribution systems. *European Journal of Operational Research* **68** 23-48.

Pyke, D.F., and Cohen, M. A. 1994. Multiproduct integrated production-distribution systems. *European Journal of Operational Research* **74** 18-49.

Slats, P. A., B. Bhola, J. M. Evers, and G. Dijkhuizen. 1995. Logistic chain modeling. *European Journal of Operational Research* **87** 1-20.

Taillard, E. D., G. Laporte, and M. Gendreau, 1996. Vehicle routing with multiple use of vehicles. *Journal of the Operations Research Society* **47** 1065-1070.

Thomas, D. J., and P. M. Griffin. 1996. Coordinated supply chain management. *European Journal of Operational Research* **94** 1-15.

Van Buer, M. G., D. L. Woodruff, and R.T. Olson. 1999. Solving the medium newspaper production/distribution problem. *European Journal of Operational Research* **115** (2) 237-253.

Vidal, C. J., and M. Goetschalckx. 1997. Strategic production-distribution models: A critical review with emphasis on global supply chain models. *European Journal of Operational Research* **98** (1) 1-18.

Williams, J. F. 1981. Heuristic Techniques for simultaneous routing of production and distribution in multi-echelon structures: theory and empirical comparisons. *Management Science* **27** (3) 336-352.

APPENDIX

To state the proposed heuristics (H_1 and H_2) formally, we define the following parameters.

$\pi(v)$	The set of LTL direct shipments assigned to transporter v in Phase I;
$H(t)$	Union of all DCs that has a LTL delivery in period t;
$h_j(v)$	The saving of eliminating node j from $P(\pi(v))$;
$c(v)$	Mileage cost of transporter v ($/mile);
C_v	Capacity of transporter v;
e_v	Efficiency factor for transporter v, where $e_v = c(v)/C_v$;
τ_v	Available time of transporter v to perform LTL assignment;
d_j	Quantity (load) of the LTL assignment originally planned for DC_j in Phase I solution;
$P(\pi(v))$	Shortest path covering $\pi(v)$ subject to C_v;
$L(\pi(v))$	The length of $P(\pi(v))$, $v=1,2,\ldots,\|V\|$;

Procedure H_1

Construct $P(\pi(v))$ using EOP procedure for all transporter v.

Repeat

a. Remove a node from H(t) and label it as node j. Let v be the transporter so that $j \in \pi(v)$;

b. Evaluate the saving of eliminating j from $P(\pi(v))$. Let $\Delta_j^-(v) = h_j(v) \cdot c(v)$ denote the resulting saving, where $h_j(v)$ is determined according to the following 4-case analysis:

if $j^- \neq 0,\ j^+ \neq 0, \quad h_j(v) = t_{j^-,j} + t_{j,j^+} - t_{j^-,j^+}$,

else if $j^- = 0,\ j^+ = 0, \quad h_j(v) = t_{0,j} + t_{j,0}$,

else if $j^- = 0,\ j^+ \neq 0, \quad h_j(v) = t_{0,j} + t_{j,j^+} - t_{0,j^+}$,

else if $j^- \neq 0,\ j^+ = 0, \quad h_j(v) = t_{j^-,j} + t_{j,0} - t_{j^-,0}$;

c. For each of the other transporters, v', where $v' \neq v$ and $d_j \leq C_{V'}$, compute the net saving of adding node j to $\pi(v')$, $\Delta_j(v')$, as follows:

If $L(\pi(v') \cup \{j\}) \leq \tau_{V'}$, let $\Delta_j(v') = \Delta_j^-(v) - c(v') \cdot [L(\pi(v') \cup \{j\}) - L(\pi(v'))]$;

Otherwise, let $\Delta_j(v')$ be the net saving of exchanging node j, $j \in P(\pi(v))$, and node j', $j' \in P(\pi(v'))$. If exchanging j and j' still violates $\tau_{v'}$, or if adding j' violates τ_v, then $\Delta_j(v') = -\infty$.

d. Let v* be the transporter that $\Delta_j(v^*) = \max\ \{\Delta_j(v') \mid \forall v' \neq v\}$. If $\Delta_j(v^*) > 0$, update $\pi(v)$, $\pi(v^*)$, $P(\pi(v))$, and $P(\pi(v^*))$. Otherwise, node j stays with $\pi(v)$.

Until H(t) = O;

For each node j, step b computes the savings of removing j from $P(\pi(v))$. As shown in step c, node j can be a candidate reassigned to another transporter (v') only if d_j is less than or equal to $C_{v'}$. Whenever we need to exchange node j, $j \in P(\pi(v))$, and node j', $j' \in P(\pi(v'))$, we only consider such node j' that was originally assigned to transporter v' and that is located nearest to node j in the given tour.

Procedure H_2:

{Input: a TSP tour connecting all the DCs in H(t), and a ranking of transporters according to a given transporter selection criterion. Without loss of generality, we assume transporter 1 is the highest-ranking transporter.}

Let Π= H(t), and v=0;

Repeat

a. let v <= v+1;

b. Let $\Pi_v = \Pi$, Π=O;

c. Apply EOP procedure to determine the shortest path based on Π_v, P(v), while ignoring the time constraint of transporter v.

d. While $L(P(v)) > \tau_V$, do

If the transporters are identical,

Remove a trip (any trip) from P(v) and add all the DCs on that trip to Π;
Else if the transporters are heterogeneous,
Remove a node from P(v) with the greatest saving based on the four-case analysis and add it to Π;
Until $L(P(v)) \le \tau_V$;
Until Π=O;

For each given transporter, H_2 searches for the greediest feasible routing assignment that covers as many DCs, from the shortest path containing all the LTL assignments, as possible. Note that for the heterogeneous transporter case, we guarantee the solution feasibility by removing only the nodes that are not originally assigned to the transporter under consideration.