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MODELING COUNTRY RISK RATINGS USING PARTIAL ORDERS

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Abstract. In order to evaluate the creditworthiness of various countries, a learning model is induced from the 1998 S&P country risk ratings, using the 1998 values of nine economic and three political indicators. This learning model allows the construction of a partially ordered set describing the relative superiority of countries on the basis of their creditworthiness, and it is shown that the Condorcet linear extensions of this poset match closely the S&P ratings. Moreover, the ratings derived from the model correlate highly with those of other rating agencies. The model is shown to provide excellent ratings even when applied to the following years' data or to the ratings of previously unrated countries. Rating changes implemented by S&P in subsequent years resolved most of the (few) discrepancies between the constructed poset and S&P's initial ratings.

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1 Introduction

In the last decades, investors have been experiencing a dramatic expansion and diversification of investment opportunities brought about by the rapid globalization of the economy. Along with new opportunities come new risks, and therefore the necessity of quantifying the advantages and threats related to investments in specific countries. These concerns have led investors, be they private or institutional, to pay special attention to the concept of country risk, and the methodology of country risk ratings. Several of the major credit rating agencies (Moody's, Standard & Poor, Fitch) have started publishing on regular basis their estimates of risk associated with investment in various countries. These estimates are presented in the form of ratings, which are generally viewed as measures of likelihood of future default. Haque et al. (1996) define country credit risk ratings compiled by commercial sources as an attempt "to estimate country-specific risks, particularly the probability that a country will default on its debt-servicing obligations".

The major impact of country risk ratings is due to the fact that they have a major influence on the interest rates at which countries can obtain credit on the international financial markets. Moreover, institutional investors are usually limited to investing only in those countries that are rated above a certain level (Kaminsky and Schmukler, 2002, Larrain et al., 1997). Finally, the country ratings influence the ratings of national banks and companies, and affect their attractiveness to foreign investors (Erb et al., 1996, Jüttner and McCarthy, 2000, Hammer et al., 2004). This is why sovereign ratings are referred to as the "pivot of all other country's ratings" (Ferri et al., 1999).

Numerous aspects of the effectiveness of country risk ratings, and of the ways they are established and used, have been severely criticized for:

- their lack of comprehensibility: the ratings appear as "black boxes", the real meaning of which is not easy to interpret, since rating agencies specify neither the factors which are taken into consideration in determining their ratings, nor the "rules of aggregation" of multiple factors into a single rating;
- their failure in predicting crises (Reinhart, 2002, Levich et al., 2002): criticisms directed towards ratings institutions have been especially intense after the Tequila and the Asian crises;
- their regional bias – there are claims that certain rating agencies favor certain regions: according to Haque et al. (1997), Euromoney usually gives higher ratings to Asian and European countries than to Latin or Caribbean countries, while the Institutional Investor is more generous to Asian and European countries than to African ones;
- their latency: it is claimed that the rating agencies take too much time to react to new facts;
- their over-reactivity: the IMF criticizes rating agencies for their panic-driven reactions during the Asian crisis. After the agencies had failed to predict the Asian crisis, they reacted by harshly downgrading countries such as Thailand or South Korea, thus accelerating the flight of capital. In this and other situations, the rating agencies gave the impression of overreacting instead of being a stabilizing force.

The problems with the objectivity and reliability of country risk ratings outlined above are mainly due to human intervention and to possibly conflicting goals and interests. Therefore, it is important to develop an objective self-contained methodology for estimating country creditworthiness.

Several studies have been recently published in the literature (e.g., Cantor and Packer, 1996, Haque et al., 1998, Monfort and Mulder, 2000, Jüttner and McCarthy, 2000) on the use of econometric tools for estimating country risk ratings. All of these models include directly or indirectly among their independent variables information derived from past Standard & Poor ratings (lagged ratings, rating history). One of the consequence of this limitation, is the impossibility of applying these models to not-yet-rated countries.

In Hammer et al. (2004), we have proposed two objective, non-recursive models (i.e., not relying directly or indirectly on previous years' ratings) for country risk ratings. One of the two models was of a statistical nature (a linear regression model), while the second was based on combinatorial and logical methods (LAD - logical analysis of data); the remarkable result of that paper was an extremely high level of agreement between the ratings provided by these two radically different techniques.

In spite of the strong results reported in that paper, the assumptions on which it relied have to be reexamined. The linear regression model assumed that the ratings can be viewed as numbers, and that the numbers corresponding to the consecutive ratings are equally spaced. There are two weaknesses in these assumptions, since both the hypotheses that every pair of countries have comparable creditworthiness, and that the superiority of a country rated AAA versus one rated AA is the same as that of a country rated BB and another one rated B, are obviously questionable. The latter problem was addressed in the same paper with the development of the LAD-based model, which did not require the assumption that consecutive ratings are equally spaced, but which still relied on the interpretation of ratings as numbers.

In this paper, we derive a new, objective LAD-based model for country risk rating by "learning" from past S&P ratings. This model utilizes the values of nine economic and three political variables associated to a country, but does not use directly or indirectly previous years' ratings. Moreover, this model completely eliminates the need to view the ratings as numbers. Consequently, the result of this model is a partial order in which the relationships between countries are determined by the demonstrable superiority of one's creditworthiness over the other's. It is remarkable that the ratings represented by naturally associated linear preorders extending the partial order developed in this paper are in a very strong agreement with the ratings provided by the major rating agencies, as well as with the results of Hammer et al. (2004).

The paper is structured as follows. Section 2 describes the data considered and selected for use in this paper. After describing the selected independent economic and political variables, and after identifying from which sources the values of these variables for a particular year (1998) have been obtained, we associate to each of the 69 vectors of attribute values of the countries examined their S&P ratings for the same year. All these data form the training set for the learning stage of the LAD method. In Section 3, we briefly describe the methodology of LAD - the logical analysis of data (Hammer, 1986, Boros et al., 2000).

Section 4 is devoted to the development of the combinatorial model proposed in this paper and its cross-validation using the jackknife procedure. First, we build a learning model which analyzes, instead of observations corresponding to individual countries, "pseudo-observations" corresponding to pairs of countries. Second, we use LAD to determine "relative preferences" associated with pseudo-observations, which indicate the relative superiority of a country's creditworthiness compared with that of the other. Third, we develop a procedure for deriving a partial order called the dominance relationship from the matrix of relative preferences. Finally, we propose an optimistic and a pessimistic extension of the dominance relationship, both of which can be viewed as country risk ratings.

The results provided by the model are analyzed in Section 5, where separate evaluations are obtained for the matrix of relative preferences, the dominance relationship, and the optimistic and pessimistic extensions. The results show that the goodness-of-fit measured using various indicators is remarkably high.

In Sections 6 and 7 we apply the LAD model obtained on the basis of the 1998 data, on the one hand to compare the ratings it provides for 1999 data with those given by the rating agencies in 1999, and on the other hand to the ratings of previously unrated countries. We show that the temporal validity of the model is very high, and that the ratings of the previously unrated countries provided by the model are remarkably close to the subsequent ratings of those countries by S&P. Moreover, in cases in which there is a discrepancy between the ratings provided by the model and those of S&P, in ulterior years, S&P's ratings were subsequently modified in the direction suggested by the proposed model. Section 8 provides some concluding remarks.

2 Data

2.1 Data set

We shall use in this study the following twelve economic and political variables:

- nine economic/financial variables: gross domestic product per capita (GDPc), inflation rate (IR), trade balance (TB), international reserves (RES), fiscal balance (FB), exports growth rate (EGR), debt to GDP (DGDP), financial depth and efficiency (FDE), and exchange rate (ER), and
- three political variables: political stability (PS), government effectiveness (GE) and corruption level (COR),

on which -- as was shown in Hammer et al.[2004] -- a highly accurate analytical model for the Standard and Poor country risk ratings can be built.

We have compiled the values taken by these twelve variables at the end of 1998 and the corresponding country risk ratings given by Standard and Poor in December of 1998 for those 69 countries for which all variable values were available. These countries include: 24 developed, 11 Eastern European, 8 Asian, 10 Middle Eastern, 15 Latin American, and 1 African country.

The data for the economic/financial variables used in this paper come from the International Monetary Fund (World Economic Outlook database), from the World Bank (World Development Indicators database) and, for the ratio of debt to gross domestic product, from Moody's publications. Values of political variables are provided by Kaufmann et al. [1999a,b and 2002] in three working papers that are joint products of the Macroeconomics and Growth, Development Research Group and Governance, Regulation and Finance Institutes which are affiliated with the World Bank.

3 Logical Analysis of Data -- An Overview

The logical analysis of data (LAD) is a combinatorics-, optimization-, and Boolean logic-based methodology for analyzing archives of observations. Initially created for the classification of binary data (Hammer 1986), LAD was later extended (Boros et al., 1997) from datasets having only binary variables to dataset which contain numerical variables. LAD distinguishes itself from other classification methods and data mining algorithms by the fact that it generates and analyses exhaustively a significant subset of those combinations of variables which can describe the positive or negative nature of an observation, and uses combinatorial techniques in order to extract models which use a limited number of the patterns generated in this way (Boros et al., 2000).

Each observation is represented by an (n+1)-dimensional vector, the first component of which is the 0 or 1 classification of the observation which specifies its negative or positive nature, while the n other components represent the values of the explanatory variables.

The purpose of LAD is to discover a binary-valued function f depending on the n input variables, which provides a discrimination between positive and negative observations, which closely approximates the actual one. The function f is constructed as a weighed sum of patterns. Positive (negative) patterns are combinatorial rules which impose upper and lower bounds on the values of a subset of input variables, such that:

- a sufficiently high proportion of the positive (negative) observations in the dataset satisfy the conditions imposed by the pattern, and
- a sufficiently high proportion of the negative (positive) observations violate at least one of the conditions of the pattern.

The conditions defining a pattern specify that the values of some of the variables are "large" or are "small"; more precisely, these conditions require these values to be above or below certain specified levels, called cutpoints. By associating an indicator variable to each cutpoint, the dataset is "binarized", i.e., each original numerical variable is replaced by several binary ones.

The following terminology will be useful. The degree of a pattern is the number of variables the values of which are bounded in the definition of the pattern. The prevalence of a positive (negative) pattern is the proportion of positive (negative) observations covered by it. The homogeneity of a positive (negative) pattern is the proportion of positive (negative) observations among those covered by it. Patterns of low degree, high prevalence and high homogeneity have been shown to be the most efficient in LAD applications, e.g., (Boros et al., 2000).

The first step in applying LAD to the dataset is to generate the pandect, i.e., the collection of all patterns in an dataset. The number of patterns contained in the pandect of a dataset of such dimensions can be exponentially large, in the order of hundreds of thousands, possibly millions. Because of the enormous redundancy in this set, we shall impose a number of limitations on the set of patterns to be generated, by restricting their degrees (to low values), their prevalences (to high values), and their homogeneities (to high values); these bounds are known as LAD control parameters. It should be added that the quality of patterns satisfying these conditions is usually much higher than that of patterns having high degrees, or low prevalences, or low homogeneities. Several algorithms have been developed for the efficient generation of substantial subsets of the pandect corresponding to reasonable values of the control parameters (Alexe and Hammer 2001; Hammer et al. 2001; Alexe et al. 2002; Alexe and Hammer 2002).

The substantial redundancy among the patterns of the pandect makes necessary the extraction of (usually small) subsets of positive and negative patterns, sufficient for classifying the observations in the dataset. Such collections of positive and negative patterns are called models. A model is supposed to contain positive (negative) patterns "covering" (i.e., having their conditions satisfied by) each of the positive (negative) observations in the dataset. Furthermore, good models tend to minimize the number of points in the dataset covered simultaneously by both positive and negative patterns in the model.

The way a LAD model can be used for classification is the following. An observation (whether it is contained or not in the given dataset) which satisfies the conditions of some of the positive (negative) patterns in the model, but which does not satisfy the conditions of any of the negative (positive) patterns in the model, is classified as positive (negative). An observation satisfying both positive and negative patterns in the model is classified with the help of a discriminant which assigns specific weights to the patterns in the model (Boros et al., 2000). More precisely, if p and q represent the number of positive and negative patterns in a model, and if h and k represent the numbers of positive, respectively negative patterns in the model covering a new observation , then the value of the discriminant is simply

$$\Delta(\theta) = h/p - k/q, \tag{1}$$

and the corresponding classification is determined by the sign of this expression. Finally, an observation for which is left unclassified, since the model either does not provide enough evidence, or provides conflicting evidence for its classification. Fortunately it has been seen in all the real-life problems considered that the number of unclassified observations is extremely small (usually less than 1%).

We represent the results of classifying the observations in a dataset in the form of a [2x3] *classification matrix* (*Table 1*).

Table 1: Classification Matrix

Observation Classes	**Classification of Observations**		
	Positive	**Negative**	**Unclassified**
Positive	*a*	*c*	*e*
Negative	*b*	*d*	*f*

Here, the value *a* (respectively *d*) represents the percentage of positive (negative) observations that are correctly classified. The value *c* (respectively *b*) is the percentage of positive (negative) observations that are misclassified. The value *e* (respectively *f*) represents the number of positive (negative) observations that remain unclassified. Clearly, *a+c+e=100%* and *b+d+f=100%*. The quality of the classification is defined by:

$$Q = \tfrac{1}{4}[a + d + (100 - b) + (100 - e)] \tag{2}$$

4 Combinatorial Model

Taking into account that the available dataset contains information only about 69 countries, the application of standard econometric methods (e.g. linear regression) for developing a self-contained model of country risk ratings on the basis of 12 explanatory variables is methodologically problematic. An alternative at hand is to examine the relative riskiness of one country compared to another one, rather than modeling the riskiness of each individual country. This approach has the advantage of allowing the modeling to be based on a much richer dataset, which consists of the comparative descriptions of all pairs of countries in the current dataset.

While apparently the transformation of the description of 69 countries into the description of 2346 pairs of countries does not change the amount of information, a powerful argument for this approach is given by Bhatia [2003]: "Although ratings are measures of absolute creditworthiness, in practice, the ratings exercise is highly comparative in nature… On one level, the ratings task is one of continuously sorting the universe of rated sovereigns – assessed under one uniform set of criteria – to ensure that the resulting list of sovereigns presents a meaningful global order of credit standing. On another level, the sorting task is constrained by a parallel need to respect each sovereign's progression over time, such that shifting peer comparisons become a necessary condition -- but not a sufficient condition – for upward or downward ratings action".

Moreover, the appropriateness of country risk modeling on the basis of pairwise country comparisons has been empirically demonstrated by the high accuracy of the logical rating scores developed in Hammer et al. [2004].

4.1 From pairwise country comparisons to pseudo-observations

Let us associate to every country $i \in I = \{1,\ldots,69\}$ considered in this study, the 13-dimensional vector C_i, whose first component is the country risk rating given by Standard and Poor, while the remaining 12 components specify the values of the nine economic/financial and of the three political variables. For every pair of countries $i, j \in I$, we shall construct a *pseudo-observation* P_{ij}, which shall provide in a way specified below a comparative description of the two countries.

The pseudo-observations are also represented as 13-dimensional vectors. The first component is an indicator which takes the value 1 if the country i in the pseudo-observation P_{ij} has a higher rating (i.e., lower risk) than the country j, takes the value –1 if the country j has a higher rating than the country i, and takes the value 0 if the two countries have the same rating. The other components k , $k = 2,\ldots,13$ of the pseudo-observation $P_{ij}[k]$ are obtained simply by taking the differences of the corresponding components of C_i and C_j:

$$P_{ij}[k] = C_i[k] - C_i[k], k = 2,\ldots,13 \tag{3}$$

The fundamental idea of this study is that a rating system can be essentially reconstructed from the knowledge of the relative standings of all pairs of rated countries. In other words, all that matters in a rating is the order relation between countries. Therefore, this study will focus on inducing a model for the order relation between countries.

An additional advantage of transformation (3) is that it allows to avoid the problems related to the small size ($|I|$) of the original dataset. This transformation provides a substantially larger dataset containing $|I|*(|I| - 1)$ pseudo-observations. Clearly, the set of pseudo-observations is not independent, since $P_{hi} + P_{ij} = P_{hj}$. It will be seen below that the non-independence of pseudo-observations does not create any problems for the combinatorial data analysis techniques used here.

In order to illustrate the construction of pseudo-observations, let us consider as an example the case of Japan and Canada.

Table 2 reports the values taken by the twelve economic/financial and political variables, as well as the rating given by Standard & Poor to these countries at the end of December 1998.

Table 2: Examples of Country Observations

	S&P RATING	*FDE*	*RES*	*IR*	*TB*	*EGR*	*GDPc*	*ER*	*FB*	*DGDP*	*PS*	*GE*	*COR*
C_{Japan}	AAA	138.44	5.168	.65	21.7471	-2.54	24314.2	0.839	-7.7	0.47	1.153	0.839	0.724
C_{Canada}	AA+	94.69	1.01964	0.99	55.9177	8.79	24855.7	0.939	0.9	0.5	1.027	1.717	2.055

In *Table 3* below, we construct the pseudo-observation $P_{Japan,Canada}$ from the country observations C_{Japan} and C_{Canada} given in *Table 2*. Since Japan and Canada are rated respectively

AAA and AA+ by Standard & Poor at the end of December 1998, and the rating AAA is superior to the rating AA+, the first component of the pseudo-observation vector takes the value 1. Clearly, the set of pseudo-observations is anti-symmetric: the pseudo-observation $P_{Canada,Japan}$ is anti-symmetric to $P_{Japan,Canada}$, as shown in *Table 3*.

Table 3: Examples of Pseudo-Observations

	Indicator	*FDE*	*RES*	*IR*	*TB*	*EGR*	*GDPc*	*ER*	*FB*	*DGDP*	*PS*	*GE*	*COR*
$P_{Japan,Canada}$	1	43.75	4.15	-0.34	-34.17	-11.33	-541.5	-0.1	-8.6	-0.03	0.126	-0.878	-1.331
$P_{Canada,Japan}$	-1	-43.75	-4.15	0.34	34.17	11.33	541.5	0.1	8.6	0.03	-0.126	0.878	1.331

4.2 From pseudo-observations to relative preferences

In order to "learn" the Standard & Poor rating system, we shall use a large margin classifier called Logical Analysis of Data (LAD). LAD is applied to the set of all those pseudo-observations P_{ij}, which correspond to pairs of countries *i* and *j* having different ratings. Each pseudo-observation P_{ij} is classified as positive or negative, according to the value of the indicator variable, i.e., depending on whether *i* is rated higher than *j*, or vice versa. The application of LAD to the set of pseudo-observations provides a LAD model (constructed as a weighed sum of patterns), derived using only the comparison between the S&P ratings of each of those pairs of countries whose ratings differ.

LAD applied to the 1998 dataset produces a model consisting of 320 patterns. For illustration of the concept of patterns, let us describe two of these patterns below. The interpretation of the positive pattern

$$FDE > 28.82; GDPc > 1539.135 ; GE > 0.553,$$

is the following:

If country *i* is characterized by

- i) a financial depth and efficiency *(FDE)* exceeding that of country *j* by at least 28.82, **and**
- ii) a gross domestic product per capita (*GDPc)* exceeding that of country *j* by at least 1539.135, **and**
- iii) a government efficiency (*GE*) exceeding that of country *j* by at least 0.553,

then country *i* is perceived as more creditworthy than country *j*.

Similarly, the interpretation of the negative pattern

$$GDPc < -4886.96 ; ER < 0.195 ; COR < -0.213$$

is the following:

If country *j* is characterized by

- i) a gross domestic product per capita (*GDPc)* exceeding that of country *i* by 4886.96, **and**
- ii) an exports growth rate (*EGR*) exceeding that of country *i* by 0.195, **and**
- iii) a level of incorruptibility (*CR*) exceeding that of country *i* by 0.213,

then country *i* is perceived as less creditworthy than country *j*.

After having constructed the LAD model, we compute (according to (1)) the discriminant $\Delta(P_{ij})$ for each pseudo-observation P_{ij} $(i \neq j)$. The values $\Delta(P_{ij})$ of the discriminant are called the *relative preferences,* and provide a numerical measure of the "superiority" of the country *i*'s rating over that of country *j*. The [69 x 69]-dimensional anti-symmetric matrix *Δ* having the relative preferences as components is called the *relative preference matrix*. While the LAD model was derived using only those pseudo-observation P_{ij} for which *i* and *j* had different ratings, the components of the discriminant matrix are the values $\Delta(P_{ij})$ $(i \neq j)$ taken by the discriminant for every pair of countries, including those that have the same S&P ratings.

4.3 Classification of pseudo-observations and cross-validation

A naïve approach to deriving country ratings from relative preferences would rely on the direct interpretation of their signs as indicators of rating superiority. More specifically, a positive value $\Delta(P_{i,j})$ could be interpreted as indicating that country *i* is more creditworthy than country *j* , while the opposite conclusion could be drawn from a negative value of the relative preference $\Delta(P_{i,j})$. A value equal to 0 would mean that the evidence for drawing a conclusion about the relative creditworthiness of countries *i* and *j* is either lacking or conflicting.

We shall apply this naïve approach to the classification of the dataset of pseudo-observations which was used to derive the LAD model. This dataset contains 4360 pseudo-observations, with an equal number of positive and negative pseudo-observations, and is anti-symmetric (P_{ij} = - P_{ij}). Among the 2180 positive pseudo-observations, 2047 (93.90%) are correctly classified as positive, 81 (3.72%) are erroneously classified as negative, and 51 (2.38%) are left unclassified by our model. The same results hold for the negative pseudo-observations (see *Table 4*), since the set of patterns in the LAD model is anti-symmetric. The resulting classification quality of the LAD model (according to formula (2)) is 95.425%.

Table 4: Classification Matrix

	Classified as			
	Positive	Negative	Unclassified	Total
Positive Observations	93.90%	3.72%	2.38%	100%
Negative Observations	3.72%	93.90%	2.38%	100%

As any learning method, LAD can be susceptible to overfitting, i.e., it can adapt excessively well to the training data allowing random noise to influence the induced model. In such situations, the resulting model could have an excellent accuracy on the training data, but perform very poorly on new observations. To verify that the high classification quality reported in the previous sub-section is not due to overfitting, we use a statistical technique known as "jackknife" (Quenouille, 1949,1956) or "leave-one-out". The jackknife technique consists of removing from the dataset one observation at a time, learning a model from all the remaining observations, and evaluating the resulting model on the removed observation; the above steps are repeated for each individual observation in the dataset. If on the average the predicted evaluations are "close to" the actual ones, then the model is not affected by overfitting.

A straightforward implementation of the jackknife technique would require the elimination of one pseudo-observation at a time. This approach would not be statistically sound because of the dependencies among pseudo-observations. More specifically, explicitly eliminating only a single pseudo-observation P_{ij} from the dataset would still leave it implicitly in the remaining dataset, in view of the relation $P_{ih} + P_{hj} = P_{ij}$.

In order to overcome this problem, we implement a variant of the above described procedure in which at each step, instead of just removing a single pseudo-observation P_{ij}, we remove all the pseudo-observations which involve a particular country *i*. Then, we derive the LAD discriminant on the basis of the remaining pseudo-observations, and use this discriminant to evaluate the relative preferences for every removed pseudo-observation. This evaluation provides a row of relative preferences of all pseudo-observations P_{ij} which involve the country *i* . After repeating this procedure for every country in the dataset, we combine the obtained rows into a matrix of relative preferences denoted by Δ^{JK}. The correlation level between the matrix Δ^{JK} and the original matrix of relative preferences Δ turns out to be 96.48%, indicating clearly the absence of overfitting.

The direct test for overfitting consists in using the obtained matrix of relative preferences Δ^{JK} for classifying the dataset of 4360 pseudo-observations. Among the 2180 positive pseudo-observations, 2038 (93.49%) are correctly classified as positive, 80 (3.67%) are erroneously classified as negative, and 62 (2.84%) are left unclassified by our model. The same results hold for the negative pseudo-observations (see *Table 5*), since the set of patterns in the LAD model is anti-symmetric. The resulting classification quality of the LAD model (according to formula (2)) is 95.10%. These results are virtually identical to those obtained by using the matrix of relative preferences Δ , thus proving the absence of overfitting.

Table 5: Classification Matrix for Δ^{JK}

	Classified as			
	Positive	Negative	Unclassified	Total
Positive observations	93.49%	3.67%	2.84%	100%
Negative observations	3.67%	93.49%	2.84%	100%

While the naïve approach towards classifying pseudo-observations does not seem to suffer from overfitting, it ignores the potential noise in the data and in relative preferences. The noisiness of relative preferences makes it difficult to transform naïve classifications of pseudo-observations to a consistent ordering of countries by their creditworthiness, as can be demonstrated in the following example (*Table 6*). In this example, we consider the relative preferences of the six pseudo-observations associated with the countries of Japan, Canada, and Belgium. The relationship based on the naïve interpretation of the relative preferences would rate Japan above Canada, Canada above Belgium, and at the same time Belgium above Japan, which contradicts the basic requirement of transitivity of an order relation. Therefore, the relationship based on the naïve classification does not provide a consistent partially ordered set of countries.

Table 6: Relative Preferences for Japan,Canada and Belgium

	Japan	**Canada**	**Belgium**
Japan		0.00625	-0.00625
Canada	-0.00625		0.03125
Belgium	0.00625	-0.03125	

In order to overcome this issue, we shall relax in the following section the overly constrained search for (possibly non-existent) country ratings whose pairwise orderings are in *precise* agreement with the signs of relative preferences, to the more flexible search for *a partial order* on the set of countries, which satisfies the transitivity requirements, and *approximates* well the set of relative preferences.

4.4 From LAD relative preferences to a partial order on the set of countries

A reflexive, anti-symmetric, and transitive binary relation on a set X is called a partial order $\Pi(X)$. Any two distinct elements x and y of X such that $(x,y) \in \Pi(X)$, are said to be comparable; in this case we write $x \succ y$. If neither $(x,y) \in \Pi(X)$, nor $(y,x) \in \Pi(X)$, then x and y are called incomparable; in this case, we write $x \,||\, y$.

In view of the comments made at the end of the previous section, the relation based on the naïve classification approach is not transitive. In order to overcome this difficulty, we shall define a strengthened version of it, to be called the *dominance relationship*. While the relation based on the naïve classification approach only relies on the sign of the relative preference $\Delta(P_{ij})$, the definition of dominance of a country i over another country j takes into account not only the sign of the relative preference $\Delta(P_{ij})$, but also the values of the relative preferences of each of these two countries i and j over every other country k.

The following notation will be needed for defining the dominance relationship. Let

$$S_{ij}(k) = \Delta(P_{ik}) - \Delta(P_{jk}) \tag{4}$$

define the external preference of country i over country j with respect to country k, let

$$S_{ij} = \frac{\sum_{k \in I} S_{ik}(k)}{|I|} \tag{5}$$

define the *average external preference* of i over j , and let

$$\sigma_{ij} = \sqrt{\frac{\sum_{k \in C} [(\Delta(P_{ik}) - \Delta(P_{jk})) - S_{ij}]^2}{|I|}} \tag{6}$$

define the *standard deviation of the external preference of i over j.*

We shall define now the dominance relationship of a country i over another country j using two conditions. The first condition will stipulate as before that the relative preference of i over j shall be positive. A natural second condition would require that the external preference of i over j should be positive for every country k, $k \neq i, j$. It turns out however that such a second condition is so strong that applying it results in a partially ordered set in which the vast majority of the country pairs are incomparable. Therefore, we shall relax the second condition to simply require that, only at a certain confidence level, the external preference of i over j should be positive. The level of confidence will be parameterized by the multiplier η of the standard deviation σ_{ij}. More formally, for a given $\eta > 0$,

- a country i is said to dominate another country j if:

$$\begin{cases} \Delta(P_{ij}) > 0 \\ S_{ij} - \eta\sigma_{ij} > 0 \end{cases} \tag{7}$$

- a country i is said to be dominated by another country j if:

$$\begin{cases} \Delta(P_{ij}) < 0 \\ S_{ij} + \eta\sigma_{ij} < 0 \end{cases} \tag{8}$$

- in all the other cases, countries i and j are said to be not comparable; this happens if either there is not enough evidence, or there is conflicting evidence about the dominance of i over j.

A few observations are in order. First, the larger the value of η is, the stronger the conditions are, and the fewer pairs of countries are comparable. It is also clear that for sufficiently large values of η, the dominance relationship, which may turn out to apply only to very few pairs of countries or even to be empty, is transitive, i.e., it is a partial order. Conversely, for small values of η (and in particular for $\eta = 0$), the dominance relationship applies to a relatively large number of country pairs, but it is not necessarily transitive.

Our objective is to define a "rich" dominance relationship (which applies to as many country pairs as possible) which is transitive. Formally, we shall maximize the number of comparable country pairs subject to the constraint that the dominance relationship shall be transitive.

Let us call the richest dominance relationship defined by the two conditions above ((7) and (8)) in the case of $\eta = 0$ the *base dominance relationship*. Note that for every pair of countries i and j which are comparable in the base dominance relationship one can easily calculate the smallest value of the parameter $\eta_{ij} = | S_{ij} / \sigma_{ij} |$ such that the countries i and j are not comparable in any dominance relationship defined by a parameter value greater than or equal to η_{ij}. For any given value of the parameter η one can easily check in polynomial time [Tarjan,1972] whether the corresponding dominance relationship is transitively closed. Therefore, one can determine in polynomial time the minimum value η^* for which the corresponding dominance relationship is still transitive. The algorithm simply sorts at most $|I|^2$ numbers η_{ij} in ascending order and then checks one by one the transitivity of the corresponding dominance relationships. The algorithm stops as soon as the dominance relationship turns out to be transitive. Then η^* equals the corresponding value of the parameter η_{ij}. In our study, we use this value η^* and the corresponding dominance relationship between countries which we shall call the *logical dominance relationship* and denote by the subscript of LAD (e.g., $\underset{LAD}{\succ}$).

The use of average external preferences in defining the dominance relationship between countries is similar to the so-called "column sum methods" (see, *inter alia*, Lipovetsky and Conklin [2002], Chao and Wendley [2004]) which are commonly applied for reconciling inconsistencies that occur in the application of pairwise comparison matrix methods.

4.5 Extending partially ordered sets to "extreme" linear preorders

While the logical dominance relationship defined in the previous section represents most faithfully the information about country preferences extracted from their economic and political attributes, the large amount of data needed to describe a partial order makes its use impractical. Note that country ratings provide a very compact way of expressing country preferences. This is due to the fact that ratings correspond to a very special type of partial orders called linear preorders.

A partial order $\Pi(X)$ is called a *linear preorder* if there exists a mapping $M: X \rightarrow \{0,1, \dots, k\}$ such that $x \succ y$ if and only if $M(x) > M(y)$. Thus a linear preorder is completely described by specifying its mapping M. Without loss of generality, it is assumed that for every $i \in \{0,1, \ldots , k\}$, there exists $x \in X$ such that $M(x) = i$. We shall say that such a linear preorder has $k+1$ levels.

As a matter of practicality, it is useful to transform the logical dominance relationship defined above into a linear preorder which preserves all the order relations between countries (i.e. is an *extension* of the partial order), and is as close as possible to it. One can extend the logical dominance relationship in a multitude of ways to a variety of linear preorders. Below, we derive two extreme linear preorders to be called respectively the *optimistic* and the *pessimistic* extensions. The former is such that it assigns to each country the highest level it can expect, while the latter assigns to each country the lowest level it can expect:

- To construct the optimistic extension (*OE*), in the first step, those countries that are not dominated by any other country are assigned the highest level, and are then removed from the set of countries under consideration. The process is repeated for the remaining set of countries until a level is assigned to every country. The level assigned to a country *i* in this optimistic extension is denoted by OE_i.
- To construct the pessimistic extension *(PE)*, in the first step, those countries that do not dominate any other country are assigned the lowest level, and are then removed from the set of countries under consideration. The process is repeated for the remaining set of countries until a level is assigned to every country. The level assigned to a country *i* in the pessimistic extension is denoted by PE_i.

The two extensions described above are obtained by using the *Condorcet method*, named after its inventor, the 18th century mathematician and philosopher Marquis de Condorcet. The Condorcet method can be viewed as a specific type of voting system [Gehrlein and Lepelley, 1998], and it is very frequently used to define the winner of an election.

Very generally, the Condorcet winner(s) of an election can be described as the candidate(s) who, when compared in turn with each of the other candidates, is preferred over each of them. The *weak Condorcet winner* (Ng et al., 1996, Martin and Merlin, 2002] is determined by constructing the *Schwartz set*. Given a particular election using preferential votes, the Schwartz set is the union of all possible candidates such that:

- every candidate inside the set is pairwise unbeatable by any other candidate outside the set, i.e. ties are allowed;
- no proper subset of the set satisfies the first property.

If there are weak Condorcet winners, then the Schwartz set consists exactly of all of them. The reverse of the weak Condorcet winner is called the *weak Condorcet loser*, which loses pairwise to each one of the other candidates.

The countries assigned to the highest level in the optimistic extension are the weak Condorcet winners, i.e. the candidates who beat or are incomparable with every other candidate in a pairwise matchup. We extract recursively the highest levels from the remaining set of countries, until every country has been assigned a level.

The countries assigned to the lowest level in the pessimistic extension are the weak Condorcet losers, i.e. the candidates who are defeated by or are incomparable with every other candidate in a pairwise matchup. We extract recursively the lowest levels from the remaining set of countries, until every country has been assigned a level.

If the set of countries is represented by a directed graph, every arc of which corresponds to a pair of comparable countries in the dominance relationship, then the length of the longest directed path in this graph provides a lower bound on the number of levels contained in any linear preorder extending the dominance relationship. One can easily see that the optimistic and the pessimistic extensions have the minimum possible number of levels, which equals the length of the longest path.

5 Model analysis

In this section, we shall evaluate the results obtained with the rating system proposed in this paper. The evaluation of the results involves the analysis of

- the relative preferences,
- the partially ordered set (i.e. the logical dominance relationship), and
- the associated extremal linear extensions

with respect to

- the S&P rating system,
- the rating system of other major rating agencies (Moody's and The Institutional Investor), and
- the logical rating scores (LRS) and the non-recursive regression model (Hammer et al., 2004).

5.1 Canonical relative preferences

In order to evaluate the matrix Δ of relative preferences obtained using LAD, we shall need a comparable point of reference. A natural benchmark of this sort can be associated to any set of numerical scores s_i representing sovereign ratings, by defining the *canonical relative preferences* d_{ij} to be simply the differences $d_{ij} = s_i - s_j$ associated to every pair of countries. In this paper, we shall compare the LAD relative preferences $\Delta(P_{ij})$ with the canonical relative preferences associated to a variety of rating systems. In particular we shall consider the canonical relative preferences $d^{S\&P}_{ij}$, d^{M}_{ij}, d^{II}_{ij}, d^{REG}_{ij}, and d^{LRS}_{ij} associated respectively to the S&P ratings, Moody's ratings, The Institutional Investor's scores, the non-recursive regression model scores and the logical rating scores (Hammer et al., 2004). The corresponding matrices of relative preferences will be denoted $d^{S\&P}$, d^{M}, d^{II}, d^{REG}, and d^{LRS} respectively. We evaluate the proximity between the LAD relative preferences and the canonical relative preferences based on their correlation levels, which are shown in *Table 7*.

Table 7: Correlation Levels between LAD and Canonical Relative Preferences

	Δ	$d^{S\&P}$	d^{M}	d^{II}	d^{REG}	D^{LRS}
Δ	100%	93.21%	92.89%	91.82%	95.91%	97.57%
$d^{S\&P}$	93.21%	100%	98.01%	96.18%	95.73%	95.54%
D^{M}	92.89%	98.01%	100%	96.31%	95.00%	95.20%
D^{II}	91.82%	96.18%	96.31%	100%	94.16%	94.11%
d^{REG}	95.91%	95.73%	95.00%	94.16%	100%	98.29%
d^{LRS}	97.57%	95.54%	95.20%	94.11%	98.29%	100%

The high levels of correlation show that the LAD relative preferences are in a surprising agreement with both the ratings of S&P and those of the other agencies, as well as with the non-recursive regression and logical rating scores. Note that the superiority of the logical rating scores and non-recursive regression scores, compared to the relative preferences given by the LAD discriminant, is due to the significant noise filtering done in the process of deriving the LRS and non-recursive regression scores, as opposed to the potential noisiness of the LAD relative preferences.

5.2 Logical dominance relationship

Before evaluating the logical dominance relationship proposed in this paper, we need to introduce the concept of density of a partially ordered set, which determines the extent to which a partial order on the set of countries discriminates them by their creditworthiness.

The density *D* of a partially ordered set is given by:

$$D = \frac{b-a}{b} \tag{9}$$

where a is the number of incomparable pairs of countries (i.e. pairs *(i,j)* where i neither dominates, nor is dominated by j), and b is the total number of pairs of countries.

As explained above, the construction of the logical dominance relationship involves the determination of the value η^*, which is the smallest positive number for which there is no violation of the transitivity property. The logical dominance relationship is obtained by setting η equal to 0.345, and it has the density equal to 89.685%.

The logical dominance relationship proposed in this paper is compared to the preference orders based on the S&P and Moody's ratings (viewed as partially ordered sets), and the partially ordered sets associated with The Institutional Investor's scores, the non-recursive regression model scores and the logical rating scores.

The partially ordered sets associated with The Institutional Investor's scores, the non-recursive regression model scores and the logical rating scores, are obtained as follows:

- $i \succ j$ if $s_i - s_j > \theta$,
- $i \prec j$ if $s_i - s_j < -\theta$,
- $i \parallel j$ otherwise,

where θ is the positive number chosen so that to obtain a partially ordered set of same density as the dominance relationship, and s_i represents the numerical score given to country i by the respective rating system.

It must be noted that the interpretation of the fact that countries i and j are not comparable differs depending on the partially ordered set considered. In the case of S&P and Moody's, it means that the countries i and j are viewed as equally creditworthy, while for the logical dominance relationship, this means that the evidence is either missing or conflicting.

To assess the extent to which the preference orders agree with each other, we have to introduce the following concepts. Given a pair of countries *(i,j)*, two partially ordered sets are said to be

- in *concordance* if one of the following relations:
$$i \succ j \text{ , } i \prec j \text{ , or } i \parallel j \text{ ,} \tag{10}$$
holds for both partially ordered sets;
- in *discordance* if
$$i \succ j \text{ for one of the partially ordered sets, and}$$
$$i \prec j \text{ for the other partially ordered set;} \tag{11}$$

- *incomparable* otherwise, i.e., if
  $i \parallel j$ for one of the partially ordered sets, and
  either $i \succ j$ or $i \prec j$ for the other partially ordered set. (12)

The *levels of concordance, discordance,* or *incomparability* between two partially ordered sets are defined as the proportions of pairs of countries for which the two partially ordered sets are respectively in concordance, discordance, or incomparable.

Table 8 displays the concordance, discordance and incomparability levels between the logical dominance relationship on one hand and on the other hand the preference orders associated with S&P's and Moody's ratings, The Institutional Investor scores, the logical rating scores and the non-recursive regression scores. *Table 8* shows a remarkable level of agreement between the logical dominance relationship and the logical rating scores, as well as a very high level of agreement with the other preference orders. *Table 8* displays the concordance, discordance and incomparability levels between the S&P ratings on one hand and on the other hand the preference orders associated with Moody's ratings, The Institutional Investor scores, the logical rating scores, the non-recursive regression scores, and the logical dominance relationship. This table indicates that the noisiness of LAD relative preferences has been filtered out in deriving the logical dominance relationship, which is in better agreement with the preference order of S&P than either the regression scores or the logical rating scores. It is also important to note that the level of agreement between the logical dominance relationship and the S&P ratings is comparable with the agreement between the latter and Moody's ratings.

Table 8: *Concordance, Discordance and Incomparability Levels with Logical Dominance*

	Logical Dominance Relationship		
	Concordance	**Incomparability**	**Discordance**
S&P	85.04%	12.79%	2.17%
Moody's	83.55%	13.13%	3.32%
The Institutional Investor	81.93%	14.88%	3.19%
Regression Score	88.15%	11.08%	0.77%
Logical Rating Score	94.29%	5.71%	0.00%

Table 9: *Concordance, Discordance and Incomparability Levels with S&P's Ratings*

	Standard & Poor		
	Concordance	**Incomparability**	**Discordance**
Moody's	90.11%	8.35%	1.54%
The Institutional Investor	87.08%	7.21%	5.71%
Regression Score	83.80%	12.45%	3.75%
Logical Rating Score	83.76%	12.62%	3.62%
Logical Dominance	85.04%	12.79%	2.17%

5.2.1 Discrepancies with S&P

In this section, we analyze the discordance between the logical dominance relationship and the preference order of S&P. A country pair for which the logical dominance relationship and the preference order of S&P are in discordance will be called a *discrepancy*. The 2.17% discordance level between the logical dominance relationship and the preference order of S&P represents 51 discrepancies that are listed in the Appendix (*Table 18*).

We shall determine now the minimum number of countries, for which the S&P ratings must be changed so that the new *adjusted* S&P preference order should have a 0% discordance level with the dominance relationship proposed in this paper.

This can be achieved by solving the following integer program:

$$\min \sum_{i \in I} a_i$$

$$\textit{subject to}$$

$$\left| S_i^* - S_i \right| \le M * a_i, \text{ for all } i \in I \tag{13}$$

$$S_i^* \ge S_j^* \text{ for every pair } (i, j) \text{ such that } i \underset{LAD}{\succ} j$$

$$a_i \in \{0,1\}, S_i^* \varepsilon \{0,1,...,21\} \text{ for all } i \in I$$

where a_i takes the value 1 if the S&P rating of country i must be modified, and the value 0 otherwise; S_i is the original S&P rating of country i; S_i^* is the adjusted S&P rating of country i; and M is a sufficiently large positive number (e.g., $M = 22$).

Solving problem (13) shows that 0% discordance level can be achieved by adjusting the S&P ratings of nine countries: France, India, Japan, Colombia, Latvia, Lithuania, and Croatia must be downgraded, while Iceland and Romania must be upgraded.

To check the relevance of the proposed rating adjustments, it is instructive to observe the ratings published by Standard & Poor subsequent to December 1998. It can be seen that Romania, Japan and Columbia's S&P ratings have been modified in the direction suggested by our model. More precisely, Columbia was downgraded by Standard & Poor twice, moving from BBB- in December 1998 to BB+ in September 1999, and then to BB in March 2000. Japan was downgraded to AA+ in February 2001 and AA in November 2001. Romania was upgraded to B in June 2001. On the other side, Iceland, France, India, Croatia, Latvia, and Lithuania's ratings have remained unchanged.

5.3 Optimistic and pessimistic extensions

The optimistic and pessimistic extensions of the logical dominance relationship both contain 21 levels while the S&P rating system contains 22 rating categories. We report in *Table 17* (in the Appendix) the optimistic and pessimistic extension levels assigned to every country.

Table 10 provides the values of the correlation coefficients between all the ratings (scores) considered in this paper. This analysis reconfirms the high level of agreement between the proposed rating model and that of S&P. Interestingly, the pessimistic extension seems to be in a slightly better agreement with S&P than the optimistic one.

Table 10: Correlation Analysis

	S&P	*Moody*	*II*	*OE*	*PE*	*REG*	*LRS*
S&P	100%	98.01%	96.18%	94.31%	95.40%	95.73%	95.54%
Moody	98.01%	100%	96.31%	94.13%	95.42%	95.00%	95.20%
II	96.18%	96.31%	100%	93.26%	94.62%	94.16%	94.11%
OE	94.31%	94.13%	93.26%	100%	99.15%	97.32%	99.24%
PE	95.40%	95.42%	94.62%	99.15%	100%	98.04%	99.10%
REG	95.73%	95.00%	94.16%	97.32%	98.04%	100%	98.29%
LRS	95.54%	95.20%	94.11%	99.24%	99.10%	98.29%	100%

6 Temporal validity

In order to provide additional validation of the robustness and relevance of the proposed model, we shall apply the so-called "out-of-time" or "walk-forward" testing to our model (Sobehart et al., 2000, Stein, 2002). More specifically, we shall test how well the LAD model derived from the 1998 data (see Section 4.2) performs when applied to the 1999 data. This "temporal validity" of the proposed model will be evaluated by first deriving the LAD relative preferences, second constructing the logical dominance relationship, third calculating the pessimistic and optimistic extensions, and finally comparing these to the S&P rating system, the rating systems of other major rating agencies (Moody's and The Institutional Investor), and the logical rating scores and the non-recursive regression model (see Hammer et al., 2004).

Before presenting the results of the comparison, we note that the classification quality of the 1998 LAD model applied to the 1999 data is equal to 93.33%.

6.1 Relative preferences

We shall evaluate now the relative preferences provided by our LAD model (built on the 1998 S&P ratings) applied to the 1999 data. *Table 11* shows that these relative preferences are highly correlated with those of the S&P rating system, as well as with the logical rating scores, and the non-recursive regression scores, both of which were obtained by applying to the 1999 data the models derived from the 1998 data.

Table 11: Correlation Levels between Relative Preference Matrices

	$d^{S\&P}$	Δ	d^{REG}	d^{LRS}
$d^{S\&P}$	100%	91.70%	94.74%	94.12%
Δ	91.70%	100%	95.00%	96.98%
d^{REG}	94.74%	94.46%	100%	97.50%
d^{LRS}	94.12%	96.98%	97.50%	100%

The main conclusion following from the high levels of pairwise correlations reported in *Table 11* is that the LAD model has a very strong temporal stability, which indicates its high predictive power.

6.2 Logical dominance relationship

We recall that the logical dominance relationship is defined in Section 4.4 using a parameter η. It turns out that the lowest value of η for which the dominance relationship is transitive, when using the 1998 model for the 1999 data, is equal to 0.352. The density of the associated partially ordered set is 87.98%, which is almost as high as the density of the partial order of the 1998 logical dominance relationship, indicating the preservation of the power of the model to differentiate countries by their creditworthiness.

The logical dominance relationship is compared to the 1999 preference order of the S&P, and the partially ordered sets associated with the non-recursive regression model and the logical rating scores. *Table 12* displays the concordance, discordance and incomparability levels between the logical dominance relationship on one hand, and on the other hand the preference order of S&P, and the partial orders associated with the logical rating scores and the non-recursive regression scores. Comparing these results with those presented in *Table 8* clearly demonstrates the temporal stability of the logical dominance relationship. *Table 13* shows the concordance, discordance and incomparability levels between the preference order of S&P on one hand, and on the other hand, the logical dominance relationship and the partial orders associated with the logical rating scores and the non-recursive regression scores. Comparing these results with those presented in *Table 9* provides additional evidence of the temporal stability of the logical dominance relationship.

Table 12: Concordance, Discordance and Incomparability Levels with Dominance Relationship

	Logical Dominance Relationship		
	Concordance	**Incomparability**	**Discordance**
S&P	84.21%	13.15%	2.64%
Logical Rating Score	93.43%	6.49%	0.08%
Regression Score	88.36%	11.38%	0.26%

Table 13: Concordance, Discordance and Incomparability Levels with S&P Ratings

	Standard & Poor		
	Concordance	**Incomparability**	**Discordance**
Logical Dominance	84.21%	13.15%	2.64%
Logical Rating Score	83.46%	12.69%	3.85%
Regression Score	83.42%	12.60%	3.98%

6.2.1 Discrepancies with S&P

In this section, we analyze the discordance between the logical dominance relationship obtained using the 1999 data and the preference order of the 1999 S&P ratings. The 2.64% discordance level reported in *Table 12* represents 62 discrepancies. Using the integer programming model (13), one can see that these discrepancies would disappear if the ratings of the following eight countries were modified:

- downgrades: France, Japan, India, Colombia, Latvia, and Croatia,
- upgrades: Iceland and Hong-Kong.

The relevance of the proposed rating modifications is justified by observing the rating changes published by S&P subsequent to December 1999. The case of Colombia, Croatia, France, Iceland, India, Japan and Latvia was already discussed in Section 5.2.1, while Hong-Kong's rating was upgraded to A+ in February 2001.

6.3 Optimistic and Pessimistic Extensions

The optimistic and pessimistic extensions of the logical dominance relationship obtained using the 1999 data both contain 20 levels, while the S&P rating system contains 22 rating categories. We report in the Appendix (*Table 19*) the optimistic and pessimistic extension levels for every country considered in this study.

Table 14: Correlation analysis

	S&P	*OE*	*PE*	*REG*	*LRS*
S&P	100.00%	95.09%	95.15%	94.74%	94.12%
OE	95.09%	100.00%	99.59%	97.72%	98.43%
PE	95.15%	99.59%	100.00%	97.36%	98.24%
REG	94.74%	97.72%	97.36%	100.00%	97.50%
LRS	94.12%	98.43%	98.24%	97.50%	100.00%

Table 14 provides the correlation levels between the 1999 S&P ratings, the optimistic and pessimistic extension levels, the logical rating scores and the non-recursive regression scores. The high levels of correlation shown in *Table 14* and their comparison with those presented in *Table 10* provides further evidence of the temporal validity of the proposed model.

7 Predicting Creditworthiness of unrated countries

The application of our model to countries not included in the original dataset, and to years subsequent to 1998, constitutes an additional validation procedure, sometimes referred to as "out-of-universe" cross-validation (Sobehart et al., 2000, Stein, 2002). To derive the OE / PE levels of previously unrated countries, one has to use the 1998 LAD model in order to calculate all the relative preferences for all pseudo-observations involving one or two of these four countries. On this basis one can then recalculate the logical dominance relationship, and the optimistic and the pessimistic extensions.

Using the 1998 LAD model, we shall derive in this section ratings for those four countries (Ecuador, Guatemala, Jamaica, Papua New Guinea) that were not rated by S&P in 1998, and for which the values of the economic and political variables used in our model were available from the data sources described in Section 2. The levels assigned to these countries by the recalculated optimistic and pessimistic extensions are shown in *Table 15*.

Table 15: Out-of-Universe Validation

	Optimistic Extension	**Pessimistic Extension**	**First S&P Rating**	**S&P Linear Extension**
Ecuador	3	2	SD (07/2000)	0
Guatemala	5	5	BB (10/2001)	10
Jamaica	3	2	B (11/1999)	7
Papua New Guinea	3	3	B+ (01/1999)	8

It can be seen that:

- Guatemala's OE / PE levels are exactly the same as Morocco's (the only country with OE = 5, PE = 5); Morocco's S&P rating in 1999 was BB, and Guatemala's first S&P rating (in 2001) was also BB.
- Jamaica's OE / PE levels (OE = 3, PE = 2) are exactly the same as those of Paraguay, Brazil, the Dominican Republic and Bolivia, which had 1999 S&P ratings of B, B+, B+ and BB- respectively; Jamaica's first S&P rating (1999) was also B.
- Papua New Guinea's OE / PE levels (OE = 3, PE = 3) are exactly the same as those of Peru and Mexico, which both had 1999 S&P ratings of BB; similarly, the first S&P rating for Papua New Guinea was B+.
- Ecuador's OE / PE levels (OE = 3, PE = 2) are exactly the same as those of Paraguay, Brazil, the Dominican Republic and Bolivia, which as mentioned above had 1999 S&P ratings of B, B+, B+ and BB- respectively; interestingly, while the initial S&P rating of Ecuador was SD (in July 2000), it was upgraded in August 2000 (just one month later) to B.

The remarkable similarity between the initial S&P rating of each of the four countries discussed above, and the S&P ratings of those countries which have the same OE / PE levels as that country, validates the proposed model, indicating at the same time its power to predict the creditworthiness of previously unrated countries.

8 Concluding remarks

This paper develops a new methodology for evaluating the creditworthiness of countries. The proposed methodology is illustrated by inducing a learning model from the 1998 S&P country risk ratings. The model uses the 1998 values of nine economic and three political indicators, and allows the construction of a partially ordered set describing the relative superiority of countries on the basis of their creditworthiness. It is shown that the Condorcet linear extensions of this poset match closely the original S&P ratings. Moreover, the ratings derived from the model correlate highly with those of other rating agencies. The model is shown to provide excellent ratings even when applied to the following years' data, or to the ratings of previously unrated countries. Rating changes implemented by S&P in subsequent years have resolved most of the (few) discrepancies between the constructed poset and S&P's initial ratings.

The domain of applicability of the proposed methodology exceeds by far the specific application described in this paper. While the paper deals with the specific case of country risk ratings, the same methodology is applicable to a wide range of contexts in which rating are relevant (e.g., ratings of hotels and restaurants, universities, bonds, banks and insurance companies). In fact, the proposed methodology can be useful for the general case of inferring an objective rating system from archival data, provided that the rated objects are described by vectors of attributes with numerical values.

Moreover, the techniques presented in the paper can also be generalized. In particular, the specific classification technique of logical analysis of data, which is used in this paper, can be replaced by other "large margin" classification techniques such as discriminant analysis, logistic regression, or probit (see e.g. Bartlett et al., 2000).

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Appendix

Table 16: Standard & Poor's country rating system

	Level	**Description**
INVESTMENT RATING	**AAA**	An obligor rated AAA has extremely strong capacity to meet its financial commitments. AA is the highest issuer credit rating assigned by S&P.
	AA	An obligor rated AA has very strong capacity to meet its financial commitments. It differs from the highest rated obligors only in small degree.
	A	An obligor rated A has strong capacity to meet its financial commitments but is somewhat more susceptible to the adverse effects of changes in circumstances and economic conditions than obligors in higher-rated categories.
	BBB	An obligor rated BBB has adequate capacity to meet its financial commitments. However, adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity of the obligor to meet its financial commitments.
SPECULATIVE RATING	**BB**	An obligor rated BB is less vulnerable in the near term than other lower-rated obligors. However, it faces major ongoing uncertainties and exposure to adverse business, financial, or economic conditions which could lead to its inadequate capacity to meet financial commitments.
	B	An obligor rated B is more vulnerable than the obligors rated BB, but, at the time of the rating, it has the capacity to meet financial commitments. Adverse business, financial, or economic conditions could likely impair its capacity or willingness to meet financial commitments.
DEFAULT RATING	**CCC**	An obligor rated CCC is vulnerable at the time of the rating, and is dependent upon favorable business, financial, and economic conditions to meet financial commitments.
	CC	An obligor rated CC is highly vulnerable at the time of the rating.
	C	An obligor rated C is vulnerable to nonpayment at the time of the rating and is dependent upon favorable business, financial, and economic conditions to meet financial commitments.
	D	An obligator rated D is predicted to default.
	SD	An obligator rated SD (selected default) is presumed to be unwilling to repay.

We have converted the Standard & Poor rating scale (columns 1 and 4) into a numerical scale (columns 2 and 5). Such a conversion is not specific to us. Bouchet et al. (2003), Estrella (2000), Ferri et al.(1999), Kräussl (2000), Monfort and Mulder (2000), Mulder and Perelli (2001), Sy (2003) proceed similarly. Moreover, Bloomberg, a major provider of financial data services, developed a standard cardinal scale for comparing Moody's, S&P and Fitch-BCA ratings (Kaminsky and Schmukler, 2002). A higher numerical value denotes a higher probability of default. The numerical scale is referred to in this paper as Standard & Poor's preorder.

Table 17: 1998 Ratings

Countries	S&P Ratings	S&P Preorder	Optimistic Extension *(OE)*	Pessimistic Extension *(PE)*	LRS Scores	Non-recursive Regression Scores	Moody's Ratings	The Institutional Investor Ratings
Argentina	BB	10	7	6	-0.2768	13.913	9	42.7
Australia	AA	19	16	16	-0.0289	20.117	19	74.3
Austria	AAA	21	17	17	-0.0094	19.655	21	88.7
Belgium	AA+	20	15	14	-0.0476	17.282	20	83.5
Bolivia	BB-	9	3	3	-0.366	9.132	8	28
Brazil	BB-	9	2	2	-0.3744	9.164	7	37.4
Canada	AA+	20	16	16	-0.0241	20.269	20	83
Chile	A-	15	11	11	-0.191	15.285	14	61.8
China	BBB+	14	10	9	-0.2159	13.589	15	57.2
Colombia	BBB-	12	2	2	-0.3854	8.337	12	44.5
Costa Rica	BB	10	7	6	-0.2748	11.586	11	38.4
Croatia	BBB-	12	5	5	-0.297	11.866	12	39.03
Cyprus	A+	17	12	12	-0.1081	17.711	16	57.3
Czech Republic	A-	15	10	10	-0.2088	15.063	14	59.7
Denmark	AA+	20	15	15	-0.048	20.373	20	84.7
Dominican Rep	B+	8	3	2	-0.3568	8.768	10	28.1
Egypt	BBB-	12	6	5	-0.2915	11.247	11	44.4
El Salvador	BB	10	4	4	-0.3379	10.460	12	31.2
Estonia	BBB+	14	9	8	-0.2518	12.486	14	42.8
Finland	AA	19	14	14	-0.064	19.362	21	82.2
France	AAA	21	13	13	-0.0828	18.262	21	90.8
Germany	AAA	21	18	18	-0.001	20.149	21	92.5
Greece	BBB	13	9	9	-0.2255	13.239	14	56.1
Hong-Kong	A	16	17	13	-0.017	18.360	15	61.8
Hungary	BBB	13	9	8	-0.2442	12.623	13	55.9
Iceland	A+	17	15	15	-0.047	20.544	18	67
India	BB	10	1	1	-0.4063	8.548	10	44.5
Indonesia	CCC+	5	0	0	-0.4576	4.821	6	27.9
Ireland	AA+	20	17	16	-0.0179	18.929	21	81.8
Israel	A-	15	11	9	-0.2215	13.934	15	54.3
Italy	AA	19	12	12	-0.1064	17.066	18	79.1
Japan	AAA	21	15	14	-0.0604	19.106	20	86.5
Jordan	BB-	9	4	3	-0.323	10.532	9	37.3
Kazakhstan	B+	8	1	1	-0.4095	7.715	9	27.9
Korea. Rep.	BB+	11	9	6	-0.2649	12.822	11	52.7

Latvia	BBB	13	5	5	-0.3026	11.281	13	38
Lebanon	BB-	9	4	4	-0.3223	10.625	8	31.9
Lithuania	BBB-	12	4	4	-0.3247	11.255	11	36.1
Malaysia	BBB-	12	11	9	-0.1676	13.589	12	51
Malta	A+	17	13	12	-0.0999	16.302	15	61.7
Mexico	BB	10	3	3	-0.3608	9.548	10	46
Morocco	BB	10	6	5	-0.2952	10.587	11	43.2
Netherlands	AAA	21	19	19	0.0251	20.525	21	91.7
New Zealand	AA+	20	18	17	0.0001	19.613	19	73.1
Norway	AAA	21	19	18	0.0125	22.234	21	86.8
Pakistan	CC	2	0	0	-0.4563	5.184	5	20.4
Panama	BB+	11	7	6	-0.2712	11.039	11	39.9
Paraguay	BB-	9	2	2	-0.3865	7.920	7	31.3
Peru	BB	10	3	3	-0.3536	10.386	9	35
Philippines	BB+	11	4	4	-0.3242	9.595	11	41.3
Poland	BBB-	12	7	6	-0.2772	12.718	12	56.7
Portugal	AA	19	14	13	-0.0742	17.556	19	76.1
Romania	B-	6	1	1	-0.3987	8.000	6	31.2
Russia	CCC-	3	0	0	-0.4428	5.161	6	20
Singapore	AAA	21	19	18	0.0073	19.716	20	81.3
Slovak Republic	BB+	11	7	6	-0.2814	11.386	11	41.3
Slovenia	A	16	11	9	-0.1922	14.319	15	58.4
South Africa	BB+	11	8	6	-0.2523	10.920	12	45.8
Spain	AA	19	13	12	-0.0924	17.433	19	80.3
Sweden	AA+	20	18	17	0.0106	19.310	19	79.7
Switzerland	AAA	21	20	20	0.071	23.436	21	92.7
Thailand	BBB-	12	8	8	-0.2452	11.207	11	46.9
Trinidad & Tob	BB+	11	6	6	-0.2824	11.624	11	43.3
Tunisia	BBB-	12	9	8	-0.2488	11.346	12	50.3
Turkey	B	7	0	0	-0.4458	6.388	8	36.9
UK	AAA	21	18	18	-0.0057	20.958	21	90.2
United States	AAA	21	19	19	0.0205	23.005	21	92.2
Uruguay	BBB-	12	7	7	-0.2695	12.516	12	46.5
Venezuela	B+	8	0	0	-0.4444	6.999	7	34.4

Table 18: Discrepancies

Discrepancies	**Higher Rated by S&P's**	**Discrepancies**	**Higher Rated by S&P's**
(Colombia, Panama)	Colombia	(Latvia, Uruguay)	Latvia
(Colombia, South Africa)	Colombia	(Latvia, Tunisia)	Latvia
(Colombia, Korea)	Colombia	(Latvia, Malaysia)	Latvia
(Colombia, Philippines)	Colombia	(Latvia, Panama)	Latvia
(Colombia, Slovak Rep.)	Colombia	(Latvia, Trinidad)	Latvia
(Colombia, Morocco)	Colombia	(Latvia, Slovak Rep.)	Latvia
(Colombia, Trinidad)	Colombia	(Latvia, Poland)	Latvia
(Colombia, Argentina)	Colombia	(India, Brazil)	India
(Colombia, Costa Rica)	Colombia	(India, Lebanon)	India
(Colombia, El Salvador)	Colombia	(India, Bolivia)	India
(Colombia, Jordan)	Colombia	(India, Jordan)	India
(Colombia, Lebanon)	Colombia	(Iceland, Finland)	Finland
(Colombia, Mexico)	Colombia	(Iceland, Italy)	Italy
(Colombia, Peru)	Colombia	(Iceland, Portugal)	Portugal
(Colombia, Bolivia)	Colombia	(Iceland, Spain)	Spain
(France, Belgium)	France	(Romania, Turkey)	Turkey
(France, Canada)	France	(Romania, Venezuela)	Venezuela
(France, Ireland)	France	(Hong-Kong, Malta)	Malta
(France, Denmark)	France	(Lithuania, Panama)	Lithuania
(France, Sweden)	France	(Lithuania, Slovak Rep.)	Lithuania
(France, New Zealand)	France	(Lithuania, Trinidad)	Lithuania
(France, Iceland)	France	(Croatia, Slovak Rep.)	Croatia
(France, Finland)	France	(Croatia, Korea)	Croatia
(Japan, Canada)	Japan	(Croatia, South Africa)	Croatia
(Japan, Ireland)	Japan	(Croatia, Trinidad)	Croatia
(Japan, New Zealand)	Japan		

Table 19: 1999 Ratings

Countries	S&P Ratings	S&P Preorder	Optimistic Extension *(OE)*	Pessimistic Extension *(PE)*	LRS Scores	Non-recursive Regression Scores
Argentina	BB	10	7	6	-0.263	14.186
Australia	AA+	20	16	16	-0.0128	21.369
Austria	AAA	21	17	16	0.0038	20.524
Belgium	AA+	20	15	15	-0.0439	18.443
Bolivia	BB-	9	3	2	-0.3518	8.877
Brazil	B+	8	3	2	-0.4016	9.766
Canada	AA+	20	16	16	-0.0112	20.850
Chile	A-	15	11	11	-0.1841	15.155
China	BBB	13	10	10	-0.224	13.371
Colombia	BB+	11	2	2	-0.3964	8.738
Costa Rica	BB	10	7	6	-0.257	12.418
Croatia	BBB-	12	6	6	-0.3202	11.360
Cyprus	A	16	12	12	-0.1021	18.143
Czech Republic	A-	15	10	10	-0.1904	15.436
Denmark	AA+	20	15	15	-0.0492	20.729
Dominican Rep	B+	8	3	2	-0.3431	9.650
Egypt	BBB-	12	6	6	-0.3067	10.710
El Salvador	BB+	11	5	4	-0.3301	10.482
Estonia	BBB+	14	8	8	-0.245	12.602
Finland	AA+	20	14	13	-0.0458	20.005
France	AAA	21	13	13	-0.0614	19.179
Germany	AAA	21	18	17	0.0126	20.959
Greece	A-	15	9	9	-0.1917	14.452
Hong-Kong	A	16	16	15	0.0213	18.594
Hungary	BBB	13	9	8	-0.247	13.483
Iceland	A+	17	15	14	-0.0378	21.400
India	BB	10	2	1	-0.3994	8.622
Indonesia	CCC+	5	0	0	-0.4316	6.747
Ireland	AA+	20	17	16	-0.0249	20.491
Israel	A-	15	10	9	-0.2189	14.200
Italy	AA	19	12	12	-0.1122	17.362
Japan	AAA	21	15	14	-0.0506	20.079
Jordan	BB-	9	4	3	-0.2818	10.574
Kazakhstan	B+	8	1	1	-0.4048	9.005
Korea. Rep.	BBB	13	10	8	-0.2182	15.386
Latvia	BBB	13	5	5	-0.3039	11.316
Lebanon	BB-	9	4	4	-0.3121	9.342
Lithuania	BBB-	12	4	4	-0.3233	10.621
Malaysia	BBB	13	11	11	-0.1712	14.235
Malta	A	16	12	12	-0.2402	16.131

Mexico	BB	10	3	3	-0.3284	10.951
Morocco	BB	10	5	5	-0.2881	10.198
Netherlands	AAA	21	19	18	0.0337	21.009
New Zealand	AA+	20	17	16	0.0001	19.661
Norway	AAA	21	18	18	-0.0076	22.399
Pakistan	B-	6	0	0	-0.4501	6.236
Panama	BB+	11	6	6	-0.2487	11.522
Paraguay	B	7	3	2	-0.4066	7.481
Peru	BB	10	3	3	-0.3644	10.250
Philippines	BB+	11	4	3	-0.349	9.940
Poland	BBB	13	7	7	-0.2743	12.899
Portugal	AA	19	14	13	-0.0706	17.436
Romania	B-	6	1	1	-0.3942	8.324
Russia	SD	0	0	0	-0.4197	6.489
Singapore	AAA	21	18	18	0.0225	18.543
Slovak Republic	BB+	11	7	6	-0.269	12.736
Slovenia	A	16	10	9	-0.1878	15.275
South Africa	BB+	11	7	6	-0.2386	12.873
Spain	AA+	20	13	13	-0.0798	18.238
Sweden	AA+	20	17	17	0.0143	19.538
Switzerland	AAA	21	19	19	0.0613	23.763
Thailand	BBB-	12	8	8	-0.2383	13.127
Trinidad & Tob	BBB-	12	6	6	-0.248	12.681
Tunisia	BBB-	12	8	8	-0.242	12.920
Turkey	B	7	0	0	-0.4177	6.407
UK	AAA	21	18	18	0.0062	20.769
United States	AAA	21	19	18	0.0264	23.981
Uruguay	BBB-	12	7	7	-0.2409	14.123
Venezuela	B	7	1	0	-0.3921	8.422