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Abstract. We study the computational complexity of the dominating set problem on graphs of bounded vertex degree. In general, this problem is NP-hard. However, under certain restrictions it becomes polynomial-time solvable. In this paper we identify two graph parameters to which the complexity of the problem is sensible.

Keywords: Dominating set problem; Hereditary class of graphs; Treewidth

1 Introduction

A *dominating set* in a graph is a subset of vertices such that every vertex outside the subset has a neighbor in it. The DOMINATING SET problem is that of finding in a graph a dominating set of minimum cardinality. The size of a minimum dominating set in a graph G is called the *domination number* of G and is denoted $\gamma(G)$. We say that G is a *weighted* graph if each vertex of G is assigned a positive integer, the *weight* of the vertex. In case of weighted graphs, the problem consists in finding a dominating set of minimum total weight, and we refer to it as the WEIGHTED DOMINATING SET problem.

DOMINATING SET is one of the central problems of algorithmic graph theory [11] with numerous applications [7, 12]. It is also one of the most difficult problems (we refer the reader to [10] for some recent developments on the complexity of the problem). Moreover, the problem remains difficult in many restricted graph families such as planar [9], bipartite [17], split [13], line graphs [18] or graphs of bounded vertex degree. The latter restriction is in the focus of the present paper.

We study the dominating set problem on graphs in which no vertex degree exceeds a certain constant Δ. For any $\Delta \geq 3$, the problem is known to be NP-hard. Moreover, under this restriction it is APX-hard [1], which means that the problem admits no polynomial-time approximation scheme, unless P=NP. However, further restrictions on input graphs may lead to efficient solutions of the problem. This is the case, for instance, when we additionally bound the *chordality*, i.e., the size of a largest chordless cycle in a graph.

Which other graph parameters are crucial for the complexity of the problem in bounded degree graphs? Trying to answer this question, we focus on graph properties that are *hereditary*, in the sense that whenever a graph possesses a certain property, the property is inherited by all induced subgraphs of the graph. Any such property can be described by a unique set of minimal graphs that do not possess the property, so-called *forbidden induced subgraphs*. The class of graphs containing no induced subgraphs in a set M will be denoted $Free(M)$. For instance, if M consists of all graphs obtained from graphs on $\Delta + 1$ vertices by adding a dominating vertex, then $Free(M)$ is the class of graphs of vertex degree at most Δ. Also, if M consists of all chordless cycles of length more than c, then $Free(M)$ is the class of graphs of chordality at most c. Observe that in the former case the set of forbidden induced subgraphs M is finite, while in the latter one it is infinite. Our objective in this paper is to reveal restrictions on the set M that would imply polynomial-time solvability of the problem. To this end, we first identify in Section 2 two types of restrictions that do not simplify the problem. Then in Section 3 by violating those restrictions we discover several areas where the problem can be solved in polynomial time.

Most notations we use are customary: $V(G)$ and $E(G)$ denote the vertex set and the edge set of a graph G, respectively. The degree of a vertex v is the number of edges incident with v. By $\Delta(G)$ we denote the maximum vertex degree in G. The *girth* of G is the size of a smallest cycle in the graph. Given a subset of vertices $U \in V(G)$, we denote by $G - U$ the subgraph induced by $V(G) - U$. As usual, C_n and P_n denote the chordless cycle and the chordless path on n vertices, respectively. Subdivision of an edge is the operation of creation of a new vertex on the edge.

2 Hardness results

We start by revealing some restrictions that *do not* simplify the problem in question. First, we prove two auxiliary results.

Lemma 1. *If a graph G' is obtained from a graph G by triple subdivision of an edge, then $\gamma(G') = \gamma(G) + 1$.*

Proof. Assume that G' has been obtained from G by introducing three new vertices a, b, c on an edge (x, y). Let W be a minimum dominating set in G. Define a set $W' \subseteq V(G')$ as follows:

$$W' = \begin{cases} W \cup \{c\}, & \text{if } W \cap \{x, y\} = \{x\}; \\ W \cup \{a\}, & \text{if } W \cap \{x, y\} = \{y\}; \\ W \cup \{b\}, & \text{otherwise.} \end{cases}$$

It is straightforward to verify that W' is a dominating set in G'. Therefore, $\gamma(G') \leq \gamma(G)+1$.

Now we show that $\gamma(G) \leq \gamma(G') - 1$. Let W' be a minimum dominating set in G' and $W'' := W' \cap \{a, b, c\}$. As W' is dominating, W'' is not empty. If $x \in W'$ or $y \in W'$, then clearly $W' \setminus W''$ is a dominating set in G. If $x, y \notin W'$, then either $|W''| \geq 2$ or $W'' = \{b\}$, and we can find a dominating set W in G with $|W| \leq |W'| - 1$ as follows:

$$W = \begin{cases} W' \setminus \{b\}, & \text{if } W'' = \{b\}; \\ (W' \setminus W'') \cup \{x\}, & \text{if } a \in W''; \\ (W' \setminus W'') \cup \{y\}, & \text{if } c \in W''. \end{cases}$$

□

To prove the second auxiliary result let us introduce the following notation. Denote by H_i the graph on the left of Figure 1 and let $\mathcal{S}_k$ be the class of all $(C_3, \ldots, C_k, H_1, \ldots, H_k)$-free graphs of vertex degree at most 3.

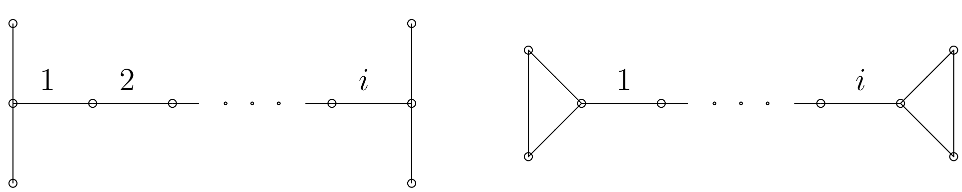

Figure 1: Graphs H_i (left) and L_i (right)

Lemma 2. *For any $k \geq 3$, the dominating set problem is NP-hard for graphs in $\mathcal{S}_k$.*

Proof. To prove the lemma, we use a reduction from the same problem in the class of graphs of vertex degree at most 3.

Let G be a graph with $\Delta(G) \leq 3$. By Lemma 1, triple subdivision of an edge increases the domination number of the graph by exactly one. Moreover, this operation preserves the maximum vertex degree (except for the trivial case when the maximum degree is one). Therefore, if G' is the graph obtained by subdividing each edge of G three times, then $\Delta(G') \leq 3$ and $\gamma(G') = \gamma(G) + |E(G)|$. Clearly the girth of G' is strictly greater than that of G. Analogously, triple subdivision of all edges of G strictly increases the size of a smallest induced subgraph of the form H_i. In other words, by means of triple subdivisions of edges G can be transformed into a graph in the class $\mathcal{S}_k$ for any $k \geq 3$. Since k is fixed, this transformation can be implemented in polynomial time. Moreover, a minimum dominating set of G can be easily derived from a minimum dominating set of the transformed graph and vice versa. Therefore, we have proved that for every particular value of k, the dominating set problem is NP-hard in the class $\mathcal{S}_k$. □

To state the main result of this section, let us associate with every graph G the following parameter: $\kappa(G)$ is the maximum k such that $G \in \mathcal{S}_k$. If G belongs to no class $\mathcal{S}_k$, we define $\kappa(G)$ to be 0, and if G belongs to all classes $\mathcal{S}_k$, then $\kappa(G)$ is defined to be ∞. Also, for a set of graphs M, we define $\kappa(M) = \sup\{\kappa(G) \ : \ G \in M\}$. With these definitions in mind, we can now prove the following hardness result.

Theorem 1. *Let X be a class of M-free graphs of vertex degree at most 3. If $\kappa(M) < \infty$, then the dominating set problem is NP-hard for graphs in X.*

Proof. To prove the theorem, we will show that there is a k such that $\mathcal{S}_k \subseteq X$. Denote $k := \kappa(M) + 1$ and let G belong to $\mathcal{S}_k$. Assume that G does not belong to X. Then G contains a graph $A \in M$ as an induced subgraph. From the choice of G we know that A belongs to $\mathcal{S}_k$, but then $k \leq \kappa(A) \leq \kappa(M) < k$, a contradiction. Therefore, $G \in X$ and hence, $\mathcal{S}_k \subseteq X$. □

In what follows, we present another theorem of a similar nature. To this end, let us introduce a related problem. An *edge dominating set* in a graph $G = (V, E)$ is a subset of edges $E' \subseteq E$ such that every edge of G outside E' has a vertex in common with some edge in E'. The EDGE DOMINATING SET problem is that of finding in a given graph an edge dominating set of minimum cardinality. The size of a minimum edge dominating set in G is called the *edge domination number* of G and is denoted $\beta(G)$. The relation between the two problems, DOMINATING SET and EDGE DOMINATING SET, can be seen through the notion of a line graph. For a graph $G = (V, E)$, the *line graph* of G, denoted $L(G)$, is the graph whose vertex set is E, and whose two vertices are adjacent if and only if they share a common vertex as edges of G. Therefore, finding a minimum edge dominating set in G is the same as finding a minimum dominating set in $L(G)$. Now we use the relationship between the problems in order to derive a result analogous to Theorem 1. First, we state a lemma on edge domination numbers, which can be proved in a similar way as Lemma 1.

Lemma 3. *If a graph G' is obtained from a graph G by triple subdivision of an edge, then $\beta(G') = \beta(G) + 1$.*

The EDGE DOMINATING SET problem is known to be NP-hard for graphs of vertex degree at most 3 [18]. Therefore, similarly as for the dominating set problem, we can apply Lemma 3 to conclude that the edge dominating set problem is NP-hard in the class $\mathcal{S}_k$ for any fixed k. Translating this proposition in terms of the dominating set problem, we obtain the following conclusion.

Lemma 4. *For any $k \geq 3$, the dominating set problem is NP-hard in the class of line graphs of graphs in $\mathcal{S}_k$.*

Let us denote the class of line graphs of graphs in $\mathcal{S}_k$ by $\mathcal{T}_k$. By analogy with the parameter $\kappa(G)$, we define one more parameter $\lambda(G)$, as follows: $\lambda(G)$ is the maximum k such that $G \in \mathcal{T}_k$. If G belongs to no $\mathcal{T}_k$, then $\lambda(G) := 0$, and if G belongs to every $\mathcal{T}_k$, then $\lambda(G) := \infty$. For a set of graphs M, we define $\lambda(M) = \sup\{\lambda(G) \ : \ G \in M\}$. The following theorem is a direct analogue of Theorem 1.

Theorem 2. *Let X be the class of M-free graphs of vertex degree at most 3. If $\lambda(M) < \infty$, then the dominating set problem is NP-hard for graphs in X.*

Remark. We conclude the section with an observation that both Theorems 1 and 2 can be strengthened to an inapproximability result: if $\kappa(M) < \infty$ or $\lambda(M) < \infty$, then the dominating set problem is APX-hard for M-free graphs of vertex degree at most 3. Since our transformations from Lemmas 1 and 3 are L-reductions, this conclusion follows immediately from the fact that the DOMINATING SET and the EDGE DOMINATING SET problems are APX-hard on graphs of degree at most 3 [1, 6].

3 Polynomial results

Let $\Delta \geq 3$ be a fixed integer and let M be a set of graphs. Unless $P = NP$, the results of the previous section suggest that the dominating set problem is solvable in polynomial time for M-free graphs of vertex degree at most Δ only if

$$\kappa(M) \text{ is unbounded and } \lambda(M) \text{ is unbounded.} \tag{1}$$

In the present section, we identify several areas where condition (1) is sufficient for polynomial-time solvability of the problem. First of all, let us reveal three major ways to push $\kappa(M)$ to infinity.

One of the possible ways to unbound $\kappa(M)$ is to include in M a graph G with $\kappa(G) = \infty$. According to the definition, in order $\kappa(G)$ to be infinite, G must belong to every class $\mathcal{S}_k$. It is not difficult to see that this is possible only if every connected component of G is of the form $S_{i,j,k}$ represented in Figure 2 (left). Let us denote the class of all such graphs by $\mathcal{S}$. More formally, $\mathcal{S} := \bigcap_{k\geq 3} \mathcal{S}_k$. Any other way to push $\kappa(M)$ to infinity requires the inclusion in M of infinitely many graphs. We distinguish two particular ways of achieving this: $M \supseteq \{H_k, H_{k+1}, \ldots\}$ and $M \supseteq \{C_k, C_{k+1}, \ldots\}$ for a constant k.

Translating the above three conditions to the language of line graphs, we obtain three respective ways to unbound $\lambda(M)$. In the first one, we include in M a graph G with $\lambda(G) = \infty$, i.e., the line graph of a graph in $\mathcal{S}$. Let us denote the class of all such graphs by $\mathcal{T}$. In other words, $\mathcal{T}$ is the class of graphs every connected component of which has the form $T_{i,j,k}$ represented in Figure 2, or equivalently, $\mathcal{T} = \bigcap_{k\geq 3} \mathcal{T}_k$. Also, $\lambda(M)$ is unbounded if $M \supseteq \{L_k, L_{k+1}, \ldots\}$, where L_i is the line graph of H_{i+1} (see Figure 1). Finally, $\lambda(M)$ is unbounded if $M \supseteq \{C_k, C_{k+1}, \ldots\}$, since the line graph of a cycle is the cycle itself.

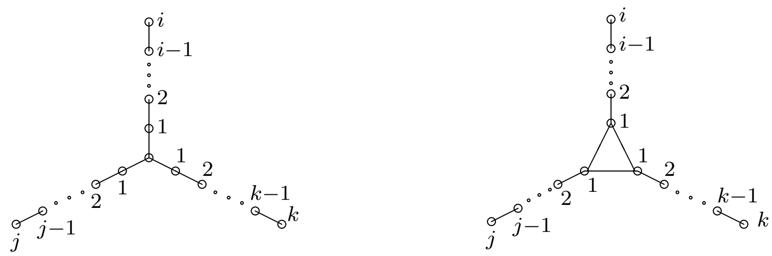

Figure 2: Graphs $S_{i,j,k}$ (left) and $T_{i,j,k}$ (right)

In the rest of this section, we consider all possible combinations of the above ways to unbound $\kappa(M)$ and $\lambda(M)$, and show that each of the combinations leads to a polynomially solvable case for the dominating set problem. Moreover, all the results below are valid for the weighted version of the problem. This is due to the fact that finding a dominating set of minimum weight can be performed in polynomial time on graphs of bounded treewidth [2].

(1) Excluding a graph from $\mathcal{S}$ and a graph from $\mathcal{T}$. It was shown in [15] that exclusion of a graph from $\mathcal{S}$ and a graph from $\mathcal{T}$ results in a class in which every graph of bounded vertex degree has bounded treewidth. Since we will make use of this result later on in this section, we restate it here.

Theorem 3 ([15]). *For any positive integer Δ and any two graphs $S \in \mathcal{S}$ and $T \in \mathcal{T}$ there is an integer N such that every graph of vertex degree at most Δ with no induced subgraphs isomorphic to S or T has treewidth at most N.*

(2) Excluding large cycles. Observe that by excluding from $Free(M)$ all cycles of length more than k, for a constant k, we simultaneously push both parameters $\kappa(M)$ and $\lambda(M)$ to infinity. This immediately bounds the treewidth of graphs in $Free(M)$, as the treewidth of a graph is upper-bounded by a function of its maximum degree and chordality [5].

(3) Excluding large graphs of the form H_i and L_i. For a fixed positive integer k, let $\mathcal{C}_k$ denote the class of $(H_k, L_k, H_{k+1}, L_{k+1}, \ldots)$-free graphs. These graphs form a generalization of graphs without long induced paths, for which many structural results are known, see for instance [3, 8, 14]. An obvious property of connected graphs without long induced paths is that they have small diameter.[1]

We will show that connected graphs in $\mathcal{C}_k$ are not far from having small diameter. More precisely, we will prove that any connected graph G from $\mathcal{C}_k$ can be transformed into a graph $\tilde{G} \in \mathcal{C}_{k+1}$ of bounded diameter. Informally, our transformation is inverse to a sequence of edge subdivisions. To describe it formally, we introduce the notion of P_4-handles. A P_4-*handle* in a graph G is an induced P_4 whose two midpoints have degree two in G. We will call a graph P_4-*handle-free* if it has no P_4-handles. Our transformation simply consists of contracting middle edges of P_4-handles as long as possible. (Figure 3 shows one such contraction.) Clearly, it can be applied to any graph G, and it results in a P_4-handle-free graph $\tilde{G}$. Note that up to isomorphism, $\tilde{G}$ is uniquely defined, and the graph G can be obtained from $\tilde{G}$ by applying a sequence of edge subdivisions.

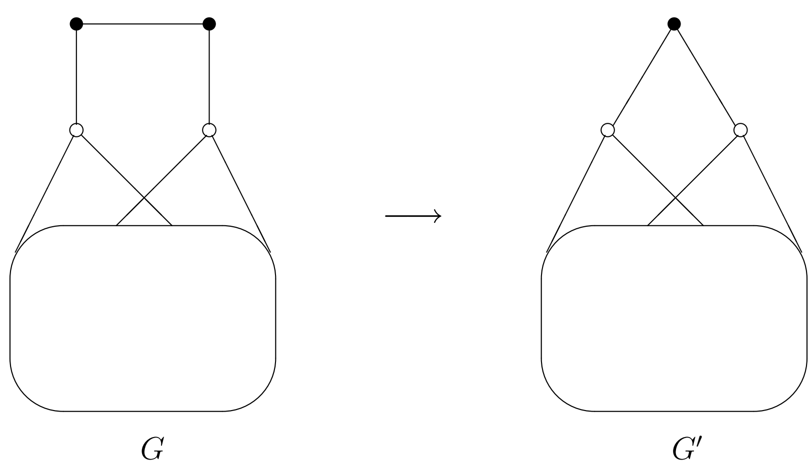

Figure 3: Breaking a P_4-handle

We now show that the diameter of P_4-handle-free graphs in our class is bounded.

Lemma 5. *Let $G \in \mathcal{C}_k$ be a connected P_4-handle-free graph. Then diam$(G) \leq 2k + 18$.*

Proof. The proof is by contradiction. Assume that G contains two vertices a and b of distance at least $2k + 19$, and let $P = (v_1, \ldots, v_s)$ be a shortest a-b path. By assumption, P has at least $2k + 20$ vertices.

[1] The *diameter* of a connected graph G is the maximum length in a shortest path connecting two vertices of G.

As G has no P_4 handles, at least one of v_2 and v_3 has a neighbor, say x, outside P. Similarly, at least one of v_{k+10} and v_{k+11} has a neighbor y outside P, and at least one of v_{2k+18} and v_{2k+19} has a neighbor z outside P.

Since P is a shortest a-b path, the neighborhood of x on P is contained in the set $\{v_1, \ldots, v_5\}$. Similarly, the neighborhood of y on P is contained in the set $\{v_{k+8}, \ldots, v_{k+13}\}$, and the neighborhood of z on P is contained in the set $\{v_{2k+16}, \ldots, v_{2k+20}\}$. In particular, this implies that the vertices x, y and z are distinct. By the minimality of P, the vertices x, y and z are pairwise nonadjacent.

Let v_r be the neighbor of x on P with the largest value of r, and let v_l be the neighbor of z on P with the smallest value of l. To avoid a large graph isomorphic to an H_i or L_i, we conclude without loss of generality that x is adjacent to v_{r-1} and z is nonadjacent to v_{l+1}. Similarly, let us denote by v_R the neighbor of y on P with the largest value of R, and by v_L the neighbor of y on P with the smallest value of L. Then, it follows that y is nonadjacent to v_{L+1}, and adjacent to v_{R-1}. However, this implies that $R - L \geq 3$, which contradicts the minimality of P. □

With these result in mind, it is now easy to prove the following theorem.

Theorem 4. *For any positive integers k and Δ, there is an N such that the treewidth of every $(H_k, L_k, H_{k+1}, L_{k+1}, \ldots)$-free graph of maximum degree at most Δ is at most N.*

Proof. It is known (see e.g. [16]) that subdivision of an edge does not change the treewidth of the graph. Therefore, the treewidth of $\tilde{G}$ equals to that of G, and hence, to prove the theorem it is enough to show that the treewidth is bounded in the class $X_{k,\Delta} := \{\tilde{G} : G \in \mathcal{C}_k : \Delta(G) \leq \Delta\}$.

It is not difficult to see that $X_{k,\Delta}$ is a subclass of $\mathcal{C}_{k+1}$, and that vertex degrees of graphs in $X_{k,\Delta}$ are also bounded by Δ. Since all graphs $\tilde{G}$ are P_4-handle-free, it follows from Lemma 5 that the diameter of connected graphs in $X_{k,\Delta}$ does not exceed $2k + 20$. As there are only finitely many connected graphs of bounded degree and bounded diameter, the treewidth of graphs in $X_{k,\Delta}$ is bounded by a constant depending only on k and Δ. This concludes the proof. □

(4) **Excluding a graph from $\mathcal{S}$, and large graphs of the form L_i.** We start with an auxiliary lemma.

Lemma 6. *For any positive integers k and Δ, there is an integer $\rho = \rho(k, \Delta)$ such that any connected $(L_k, L_{k+1}, \ldots)$-free graph G of maximum vertex degree at most Δ contains an induced triangle-free subgraph with at least $|V(G)| - \rho$ vertices.*

Proof. We first show that for any two induced copies T and T' of a triangle in G, the distance between them does not exceed k. Suppose by contradiction that a shortest path P joining a triangle T to another triangle T' consists of $r \geq k + 1$ edges. Let us write $P = (v_0, v_1, \ldots, v_{r-1}, v_r)$ where $v_0 \in V(T), v_r \in V(T')$.

Observe that the vertex v_1 may belong to another triangle induced by $\{v_0, v_1\}$ and another vertex of T, in which case we denote this triangle by $\widetilde{T}$; otherwise let $\widetilde{T} := T$. Analogously, by $\widetilde{T}'$ we denote either a triangle induced by $\{v_{r-1}, v_r\}$ and another vertex of T' (if such a triangle exists) or T' otherwise. But now the two triangles $\widetilde{T}$ and $\widetilde{T}'$ together with the vertices of P connecting them induce a graph L_i with $i \geq k$. Contradiction.

To conclude the proof, assume that G contains an induced triangle T, and let v be a vertex of T. According to the above discussion, the distance from v to a vertex of any other triangle in G (if any) is at most k. Since G is a connected graph of maximum degree at most Δ, there is a constant $\rho = \rho(k, \Delta)$ bounding the number of vertices of G of distance at most k from v. Deletion of all these vertices leaves an induced subgraph of G which is triangle-free. □

We also recall the following result from [15].

Lemma 7 ([15]). *For a class of graphs X and an integer ρ, let $[X]_\rho$ denote the class of graphs G such that $G - U$ belongs to X for some subset $U \subseteq V(G)$ of cardinality at most ρ. If X is a class of graphs of bounded treewidth, then so is $[X]_\rho$.*

Combining Lemma 6 with Lemma 7 and Theorem 3, we obtain the following conclusion.

Theorem 5. *For any positive integers k and Δ and any graph $S \in \mathcal{S}$, there is an integer N such that the treewidth of every $(S, L_k, L_{k+1}, \ldots)$-free graph of maximum degree at most Δ is at most N.*

(5) **Excluding a graph from $\mathcal{T}$, and large graphs of the form H_i.** This case is similar to the previous one. In particular, the following lemma can be proved by analogy with Lemma 6.

Lemma 8. *For any positive integers k and Δ, there is an integer $\rho = \rho(k, \Delta)$ such that any connected $(H_k, H_{k+1}, \ldots)$-free graph G of maximum vertex degree at most Δ contains an induced claw-free subgraph with at least $|V(G)| - \rho$ vertices.*

Combining Lemma 8 with Lemma 7 and Theorem 3, we obtain the following conclusion.

Theorem 6. *For any positive integers k and Δ and any graph $T \in \mathcal{T}$ there is an integer N such that the treewidth of every $(T, H_k, H_{k+1}, \ldots)$-free graph of maximum degree at most Δ is at most N.*

The results of this section are summarized in the following theorem.

Theorem 7. *Let Δ be a positive integer and let X be a class of M-free graphs of vertex degree at most Δ. If at least one of the following conditions holds:*

(1) there is a $k \geq 3$ such that $M \supseteq \{C_k, C_{k+1}, \ldots\}$, or

(2) there is a $k \geq 1$ *such that* $M \supseteq \{H_k, L_k, H_{k+1}, L_{k+1}, \ldots\}$*, or*

(3) $M \cap \mathcal{S} \neq \emptyset$ *and there is a* $k \geq 1$ *such that* $M \supseteq \{L_k, L_{k+1}, \ldots\}$*, or*

(4) $M \cap \mathcal{T} \neq \emptyset$ *and there is a* $k \geq 1$ *such that* $M \supseteq \{H_k, H_{k+1}, \ldots\}$*, or*

(5) $M \cap \mathcal{S} \neq \emptyset$ *and* $M \cap \mathcal{T} \neq \emptyset$*,*

then the treewidth of graphs in X *is bounded.*

Corollary 1. *Let* Δ *be a positive integer and let* X *be a class of* M*-free graphs of vertex degree at most* Δ*. If at least one of the conditions stated in Theorem 7 holds, then the weighted dominating set problem is solvable in polynomial time for graphs in* X*.*

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