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RRR 3 – 2006, March 2006

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Maximum k-regular induced subgraphs

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Abstract. Independent sets, induced matchings and cliques are examples of regular induced subgraphs in a graph. In this paper, we prove that finding a maximum cardinality k-regular induced subgraph is an NP-hard problem for any value of k. We propose a convex quadratic upper bound on the size of a k-regular induced subgraph and characterize those graphs for which this bound is attained. Finally, we extend the Hoffman bound on the size of a maximum 0-regular subgraph (the independence number) from $k = 0$ to larger values of k.

Acknowledgements: The first author's research was supported by *Centre for Research on Optimization and Control (CEOC)* from the "Fundação para a Ciência e a Tecnologia" FCT, cofinanced by the European Community Fund FEDER/POCTI.

1 Introduction

Several fundamental graph problems consist in finding a maximum induced subgraph possessing some regularity. In this paper, we study the problem of finding a maximum induced subgraph in which every vertex has degree k. For $k = 0$, this is equivalent to the MAXIMUM INDEPENDENT SET problem and for $k = 1$, to the MAXIMUM INDUCED MATCHING problem. Both problems are known to be NP-hard. We show that, not surprisingly, the same conclusion holds for any value of k.

The NP-hardness of the problem in question suggests to look for polynomially computable upper bounds on the maximum size of a k-regular induced subgraph. Many such bounds have been obtained for the independence number (the case of $k = 0$) and one of the best known is due to Hoffman. In this paper, we extend the Hoffman bound to larger values of k. We also propose a convex quadratic upper bound on the size of a k-regular induced subgraph and give a characterization of those graphs for which this bound is attained.

For a simple graph G we denote its vertex set and edge set by $V(G)$ and $E(G)$, respectively. The neighborhood of a vertex $v \in V(G)$, denoted $N_G(v)$, is the set of vertices adjacent to v, and the degree of v is $d_G(v) := |N_G(v)|$. The average vertex degree of G is $\overline{d_G} = \frac{1}{n}\sum_{v \in V(G)} d_G(v)$. The minimum (maximum) vertex degree of G is denoted by $\delta(G)$ ($\Delta(G)$). If $\delta(G) = \Delta(G) = p$, then we say that G is p-regular. The subgraph of G induced by a vertex subset S is denoted by $G[S]$.

Given a vertex subset $S \subseteq V(G)$, the vector $x \in \{0,1\}^{|V(G)|}$ with $x_v = 1$ if $v \in S$ and $x_v = 0$ if $v \notin S$ is called the *characteristic vector* of S and is denoted $x(S)$. The adjacency matrix of the graph G, denoted $A_G = (a_{ij})_{n \times n}$, is defined by

$$a_{ij} = \begin{cases} 1, & \text{if } ij \in E(G); \\ 0, & \text{otherwise.} \end{cases}$$

The minimum eigenvalue of A_G is denoted $\lambda_{min}(A_G)$.

In a graph G, an *independent set* is a subset of vertices such that no two of them are adjacent, and a *matching* is a subset of edges such that no two of them have a vertex in common. The line graph of G, denoted $L(G)$, is the graph with the vertex set $E(G)$ in which two vertices are adjacent whenever the respective edges of G have a vertex in common. Obviously, a matching in G corresponds to an independent set in $L(G)$ and vice versa.

Clearly, a subset of vertices inducing a 0-regular induced subgraph is independent, and the edges of a 1-regular induced subgraph form a matching. Moreover, this matching possesses the additional property that no two endpoints of different edges are adjacent. Such matchings are called *induced*. If no confusion arises, we use the notions of 1-regular induced subgraph and induced matching interchangeably.

The cardinality of a maximum independent set in a graph G is called the independence number of G and is denoted $\alpha(G)$. Finding an independent set of size $\alpha(G)$ is known to be an NP-hard problem [8]. It is also known that finding a maximum induced matching is NP-hard [2, 13]. The next section generalizes these two results.

2 A complexity result

Generalizing the NP-hardness of the maximum independent set and maximum induced matching problems, in this section we show that the problem of finding a maximum induced k-regular subgraph is NP-hard for any value of k. In fact, we prove even a stronger result stating that the problem remains NP-hard if we restrict ourselves to finding a maximum induced k-regular *bipartite* subgraph.

Theorem 2.1 *For any fixed natural k, the problem of finding a maximum induced k-regular bipartite subgraph is NP-hard.*

Proof. For $k = 0$, the problem coincides with the maximum independent set problem and hence it is NP-hard [8]. For larger values of k, we use a reduction from the case $k = 0$. To this end, we shall need a large k-regular bipartite graph H, which can be constructed, for instance, as follows: for $t \geq k$, we let the vertex set of H be $\{x_1, \ldots, x_t, y_1, \ldots, y_t\}$, where x_i is adjacent to y_j if and only if $0 \leq j - i < k$ (the difference of indices is taken modulo t).

Let G be any graph with vertex set $V = \{v_1, \ldots, v_n\}$ and H be a k-regular bipartite graph with more than kn vertices. We construct an auxiliary graph $G(H)$ by substituting each vertex of G with a copy of H. More formally, $G(H)$ is obtained from the union of n disjoint copies of H, denoted $H_1, \ldots, H_n$, by connecting every vertex of H_i to every vertex of H_j whenever v_i is adjacent to v_j in G. With some abuse of terminology, we shall say that H_i is adjacent to H_j in $G(H)$ if v_i is adjacent to v_j in G.

Suppose that Q is a maximum k-regular bipartite induced subgraph of $G(H)$, and let C be a connected component of Q. We claim that the vertices of C cannot belong to more than one copy of the graph H in $G(H)$. Indeed, suppose C intersects more than one copy of H. Then, due to the connectivity of C, for each such a copy H_i there must exist another such copy which is adjacent to H_i. This implies that H_i contains at most k vertices of C, since otherwise any vertex of C in an adjacent copy of H would have degree more than k in C. Therefore, C has at most kn vertices. If we replace in Q the connected component C by any copy H_i containing vertices of C, we obtain a k-regular bipartite induce subgraph of $G(H)$, which is strictly larger than Q. This contradiction shows that every connected component of Q intersects exactly one copy of the graph H in $G(H)$. Moreover, in order the graph Q to be maximum each component of Q must coincide with the copy of H it intersects. Clearly, the vertices of G corresponding to connected components of Q form an independent set. Moreover, this set is maximum, since otherwise the copies of the graph H in $G(H)$ corresponding to the vertices of any larger independent set in G would create a larger k-regular bipartite induced subgraph of $G(H)$.

We have proved that if Q is a maximum k-regular bipartite induced subgraph of $G(H)$, then each connected component of Q coincides with a copy of the graph H in $G(H)$ and the vertices of G corresponding to connected components of Q form a maximum independent set in G. Similarly, we can prove that if a set of vertices forms a maximum independent set in G, then the union of the respective copies of the graph H in $G(H)$ forms a maximum k-regular bipartite induced subgraph of $G(H)$. Together these two propositions provide a reduction from the maximum independent set problem to the problem of finding a maximum k-regular bipartite induced subgraph. This reduction is polynomial in the size n of the input graph whenever the size of the graph H is bounded by a polynomial in n. ♢

3 A convex quadratic upper bound on the maximum cardinality of a k-regular induced subgraph

Given a graph G with n vertices and at least one edge, let us consider the convex quadratic program introduced in [12]

$$\upsilon(G) \;=\; \max_{x\geq 0}\; 2\hat{e}^T x - x^T\left(\frac{A_G}{-\lambda_{min}(A_G)} + I_n\right)x, \tag{3.1}$$

where $\hat{e}$ denotes the all ones vector and I_n the identity matrix of order n. According to [12], if G is a graph with at least one edge, then $\alpha(G) \leq \upsilon(G)$. Furthermore, $\upsilon(G) = \alpha(G)$ if and only if for any maximum independent set S (and then for all)

$$-\lambda_{min}(A_G) \leq \min\{|N_G(v) \cap S| : v \notin S\}. \tag{3.2}$$

Actually, $\upsilon(G) = \alpha(G)$ if and only if there exists an independent set S satisfying (3.2), in which case the set is maximum. A graph G such that $\upsilon(G) = \alpha(G)$ is called in [3] graph with convex-QP stability number. For instance, according to [3], line graphs of graphs with a perfect matching and line graphs of line graphs with an even number of edges are examples of graphs with convex-QP stability number.

Now let us introduce a family of convex quadratic programming problems depending on a parameter k, where by τ we denote $-\lambda_{min}(A_G)$:

$$\upsilon_k(G) \;=\; \max_{x\geq 0} 2\hat{e}^T x - \frac{\tau}{k+\tau} x^T\left(\frac{A_G}{\tau} + I_n\right)x. \tag{3.3}$$

Notice that $\upsilon_0(G) = \upsilon(G)$. Our first result shows that $\upsilon_k(G)$ is an upper bound on the size of a k-regular induced subgraph of G.

Theorem 3.1 *Let G be a simple graph with at least one edge and k a non-negative integer. If $S \subseteq V(G)$ induces a subgraph of G such that $\overline{d}_{G[S]} = k$, then $|S| \leq \upsilon_k(G)$.*

Proof. Let S be a vertex subset of G inducing a subgraph such that $\overline{d}_{G[S]} = k$ and let $x = x(S)$ be the characteristic vector of S, then $(A_G x)_i = d_{G[S]}(i)$ if $i \in S$ and thus $x^T A_G x = \sum_{i\in S} d_{G[S]}(i) = \overline{d}_{G[S]}|S| = k|S|$. Therefore,

$$\begin{aligned}\upsilon_k(G) &\geq 2\hat{e}^T x - \frac{\tau}{k+\tau} x^T\left(\frac{A_G}{\tau} + I_n\right)x \\ &= 2|S| - \frac{\tau}{k+\tau}\left(\frac{k|S|}{\tau} + |S|\right) \\ &= 2|S| - \frac{\tau}{k+\tau}\frac{k+\tau}{\tau}|S| \\ &= |S|.\end{aligned}$$

♢

For instance, if S_1 is a subset inducing a 1-regular subgraph (induced matching), then $|S_1| \leq \upsilon_1(G)$, and if S_2 is a subset inducing a 2-regular subgraph, then $|S_2| \leq \upsilon_2(G)$. In particular, if G has at least one cycle then $g(G) \leq \upsilon_2(G)$, where $g(G)$ is the girth of G, that is, the cardinality of the vertex subset inducing a minimum size 2-regular subgraph in G.

Our next theorem provides a characterization for those graphs that contain k-regular induced subgraphs of cardinality $\upsilon_k(G)$.

Theorem 3.2 *Let G be a simple graph with at least one edge and let S be a subset of $V(G)$ inducing a k-regular subgraph. Then $|S| = \upsilon_k(G)$ if and only if*

$$\tau + k \leq |N_G(v) \cap S| \;\; \forall v \notin S. \tag{3.4}$$

Proof. According to the Karush-Khun-Tucker conditions, x^* is an optimal solution for (3.3) if and only if $\exists y^* \geq 0$ such that

$$y^{*T} x^* = 0; \tag{3.5}$$

$$A_G x^* = \tau(\hat{e} - x^*) + k\hat{e} + y^*, \tag{3.6}$$

and then $(A_G x^*)_i = |N_G(i) \cap S| = \tau(1 - x_i^*) + k + y_i^*$ and $x_i^* > 0 \Rightarrow y_i^* = 0$. Since $|S| = \upsilon_k(G)$ if and only if the characteristic vector of S, $x = x(S)$, is an optimal solution for (3.3), it follows that $|S| = \upsilon_k(G)$ if and only if

$$|N_G(i) \cap S| = \begin{cases} \tau + k + y_i^*, & i \notin S, \\ k, & i \in S. \end{cases} \tag{3.7}$$

Therefore, $|S| = \upsilon_k(G)$ if and only if

$$\tau + k \leq |N_G(i) \cap S| \;\; \forall i \notin S.$$

$\diamondsuit$

Line graphs of Hamiltonian graphs provide a nice illustration of Theorem 3.2 for $k = 2$. Indeed, let G be a Hamiltonian graph and $L(G)$ its line graph. The edges of a Hamiltonian cycle C in G induce a 2-regular subgraph in $L(G)$. Any other edge of G is adjacent, as a vertex of $L(G)$, to exactly 4 vertices in C. Since $\lambda_{min}(A_{L(G)}) \geq -2$, we conclude that condition (3.4) of Theorem 3.2 holds for C, which means that C induces a maximum 2-regular subgraph in $L(G)$.

As a consequence of Theorem 3.2, we have the following corollary.

Corollary 3.3 *If the convex quadratic programming problem* (3.3) *has an optimal solution x^*, which is binary, then the vertex subset $S \subseteq V(G)$ such that $x^* = x(S)$ induces a maximum k-regular subgraph.*

Proof. Since x^* is an optimal solution for the convex quadratic programming problem (3.3), there exists $y^* \geq 0$ for which the Karush-Khun-Tucker conditions (3.5)-(3.6) hold. Therefore, since x^* is binary, for the set $S \subseteq V(G)$ such that $x^* = x(S)$, the equalities (3.7) are verified and hence S induces a k-regular subgraph. Finally, since $\upsilon_k(G) = |S|$, taking into account Theorem 3.1, it follows that S induces a maximum k-regular subgraph. $\diamondsuit$

4 An extension of the Hoffman bound

In this section we study bounds on the size of k-regular subgraphs of *regular* graphs. For $k = 0$ (i.e. for the independence number), one of the most famous upper bounds was obtained by Hoffman (unpublished) and presented by Lovász in [11] as follows: for any connected regular graph G with n vertices,

$$\alpha(G) \quad \leq \quad n\frac{-\lambda_{min}(A_G)}{\lambda_{max}(A_G) - \lambda_{min}(A_G)}. \tag{4.1}$$

This bound can also be derived from the following inequalities proved in [1] (see also [7]), where G is a p-regular connected graph, S is a subset of $V(G)$ inducing a k-regular subgraph, and $\lambda_2(A_G)$ is the second largest eigenvalue of A_G,

$$\frac{|S|(p - \lambda_{min}(A_G))}{n} + \lambda_{min}(A_G) \leq \overline{d}_{G[S]} \leq \frac{|S|(p - \lambda_2)}{n} + \lambda_2(A_G).$$

Our purpose is to extend the Hoffman bound from $k = 0$ to larger values of k. To this end, we use the notion of a (k, τ)-regular set introduced in [5, 6]: a subset of vertices S in a graph G is called (k, τ)-*regular* if S induces a k-regular subgraph and every vertex outside S has τ neighbors in S. For instance, for the graph represented in Figure 1, the vertex subset $S = \{1, 3, 4, 6\}$ is $(2, 4)$-regular and the vertex subset $T = \{2, 5\}$ is $(0, 2)$-regular.

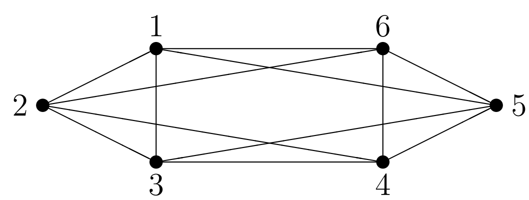

Figure 1: A graph G with the $(2, 4)$-regular set $S_1 = \{1, 3, 4, 6\}$ and the $(0, 2)$-regular set $S_2 = \{2, 5\}$.

According to the definition, the vertex set of a p-regular graph G is (p, τ)-regular for every $\tau \in \mathbb{N}$. In this case, by convention, we shall say that $V(G)$ is $(p, 0)$-regular.

With this terminology, we obtain the following conclusion, which is a direct consequence of Theorem 3.2.

Corollary 4.1 *Let G be a graph with at least one edge and $\tau = -\lambda_{min}(A_G)$. If $S \subset V(G)$ is $(k, k+\tau)$-regular, then S is a maximum cardinality set inducing a k-regular subgraph.*

Thus, the concept of a (k, τ)-regular set provides a simple sufficient condition for a subset of vertices to induce a maximum cardinality k-regular subgraph. For regular graphs, the existance of a (k, τ)-regular set can be verified by the following result, which is due to Thompson: using a different terminology, he proved in [14] that a p-regular graph G has a

(k,τ)-regular set $S \subset V(G)$, with $k < p$, if and only if $k-\tau$ is an adjacency eigenvalue and $(p-k+\tau)x(S) - \tau\hat{e}$ belongs to the $(k-\tau)$-eigenspace. In [4], the following slightly different version of Thompson's result is proved.

Theorem 4.2 *[4] A p-regular graph G has a (k,τ)-regular set, with $k < p$, if and only if $k-\tau$ is an adjacency eigenvalue and there exists $x \in \{0,1\}^{|V(G)|}$, such that $x - \frac{\tau}{p+\tau-k}\hat{e}$ is a $(k-\tau)$-eigenvector. Furthermore, x is the characteristic vector of a (k,τ)-regular set.*

Now let us prove the main result of this section.

Theorem 4.3 *If G is a p-regular graph of order n, with $p > 0$, then*

$$\upsilon_k(G) = n\frac{k - \lambda_{min}(A_G)}{p - \lambda_{min}(A_G)}. \tag{4.2}$$

Furthermore, there exists a vertex subset S inducing a k-regular subgraph such that $|S| = \upsilon_k(G)$ if and only if S is $(k, k+\tau)$-regular, with $\tau = -\lambda_{min}(A_G)$.

Proof. Since G is p-regular, with $p > 0$, the Karush-Khun-Tucker conditions (3.5)-(3.6) are fulfilled by $y^* = 0$ and x^* such that $x_i^* = \frac{k+\tau}{p+\tau}$, for $i = 1,\ldots,n$, with $\tau = -\lambda_{min}(A_G)$, and then (4.2) follows. Additionally, the equality $|S| = \upsilon_k(G)$ holds if and only if $\overline{x} = x(S)$ is an optimal solution for (3.3) and hence if and only if $\overline{x}$ verifies the Karush-Khun-Tucker conditions (3.5)-(3.6), with $y^* = 0$ [1]. Therefore,

$$(A_G\overline{x})_i = |N_G(i) \cap S| = \begin{cases} k, & \text{if } \ i \in S; \\ k+\tau, & \text{if } \ i \notin S; \end{cases}$$

and this is equivalent to say that S is $(k, k+\tau)$-regular. $\diamondsuit$

Taking into account that for a p-regular graph G, $\lambda_{max}(A_G) = p$, we obtain, as an immediate corollary from the above theorem, the following extension of the Hoffman bound.

Corollary 4.4 *Let G be a p-regular graph with n vertices ($p > 0$) and S a subset of $V(G)$ inducing a k-regular subgraph, then*

$$|S| \leq n\frac{k - \lambda_{min}(A_G)}{\lambda_{max}(A_G) - \lambda_{min}(A_G)}.$$

[1] Suppose that the characteristic vector of S, $\overline{x} = x(S)$ verifies the Karush-Khun-Tucker conditions (3.5)-(3.6), with $\overline{y} \geq 0$. Then, considering (3.6) with both optimal solutions, $\overline{x}$ and x^*, such that $x_i^* = \frac{k+\tau}{p+\tau}$, for $i = 1,\ldots,n$, we have the following equations:

$$\begin{aligned} \overline{x}^T A_G x^* &= \tau(|S| - \sum_{j\in S} x_j^*) + k|S|, \\ x^{*T} A_G \overline{x} &= \tau(\sum_j x_j^* - \sum_{j\in S} x_j^*) + k\sum_j x_j^* + \sum_j x_j^*\overline{y}_j. \end{aligned}$$

Therefore, since $\sum_j x_j^* = |S|$, it follows that $\sum_j x_j^*\overline{y}_j = 0$, and then $\overline{y}_j = 0$, for $j = 1,\ldots,n$.

To illustrate the result, consider the 4-regular graph G represented in Figure 1 and the vertex subset set $S = \{1, 3, 4, 6\}$ which induces a 2-regular subgraph. Since $\lambda_{min}(A_G) = -2$, we have

$$4 = |S| \leq n \frac{k - \lambda_{min}(A_G)}{\lambda_{max}(A_G) - \lambda_{min}(A_G)} = 6\frac{2+2}{4+2} = 4$$

and hence S is a maximum cardinality 2-regular induced subgraph of G.

5 Induced matchings and efficient edge dominating sets

In a graph G, an *edge dominating set* is a subset $D \subseteq E(G)$ such that every edge of G has a vertex in common with at least one edge of D. An edge dominating set D is called *efficient* if every edge of G has a vertex in common with exactly one edge of D. Obviously, any graph has an edge dominating set, but not all of them have efficient edge dominating sets. For instance, a cycle on four vertices provides an example of a graph with no efficient edge dominating set, while every chordless cycle with $3k$ edges has an efficient edge dominating set. In [10], it was shown that the problem of determining whether a graph has an efficient edge dominating set is NP-complete.

It is not difficult to see that any efficient edge dominating set is an induced matching. Moreover, as was shown in [9], any efficient edge dominating set is a maximum induced matching. The converse statement is generally not true. However, for regular graphs with at least one efficient edge dominating set, the notions of maximum induced matching and efficient edge dominating set coincide.

In the present paper we use the notion of an efficient edge dominating set to provide a sufficient condition characterizing graphs G for which the bound $\upsilon_1(G)$ is attained.

Theorem 5.1 *Let G be a graph such that $\lambda_{min}(A_G) \geq 1 - \delta(G)$ and G has an efficient edge dominating set $M \subseteq E(G)$. Then $\upsilon_1(G) = |V(M)|$ and the characteristic vector of $V(M)$ is an optimal solution for the convex quadratic program* (3.1).

Proof. Let us denote $V_1 := V(M)$ and $V_0 := V(G) \setminus V_1$. Since M is an efficient edge dominating set, V_0 is a stable set. Therefore,

$$|N_G(i) \cap V_1| = \begin{cases} d_G(i), & i \notin V_1 \\ 1, & i \in V_1 \end{cases}$$

and hence, $x^* = x(V_1)$ fulfills the Karush-Khun-Tucker conditions (3.5)-(3.6), with y^* such that $y_i^* = d_G(i) - 1 + \lambda_{min}(A_G)$, if $i \notin V_1$ and $y_i^* = 0$ otherwise. Note that $\lambda_{min}(A_G) \geq 1 - \delta(G) \Leftrightarrow \delta(G) - 1 + \lambda_{min}(A_G) \geq 0$ and then $d_G(i) - 1 + \lambda_{min}(A_G) \geq 0$. Thus x^* is an optimal solution for (3.3) and $|V_1| = \upsilon_1(G)$. ♢

For the particular case of a p-regular graph G, with $p > 0$, by definition, G has an efficient edge dominating set $M \subseteq V(G)$ if and only if $V(M)$ is $(1, p)$ regular. Hence, by Theorem 4.2, G has an efficient edge dominating set M if and only if $1 - p$ is an adjacency eigenvalue and the vector $\overline{x} - \frac{p}{2p-1}\hat{e}$, where $\overline{x} = x(V(M))$, belongs to the corresponding eigenspace. Additionally, as a direct consequence of Theorems 4.3 and 5.1, we have the following corollary.

Corollary 5.2 *Let G be a p-regular graph, with $p > 0$, and $M \subseteq E(G)$ an efficient edge dominating set. Then $\upsilon_1(G) = |V(M)|$ if and only if $\lambda_{min}(A_G) = 1 - p$.*

Proof. Assuming that $\upsilon_1(G) = |V(M)|$, according to Theorem 4.3, $V(M)$ is $(1, 1+\tau)$-regular, with $\tau = -\lambda_{min}(A_G)$, and then $\lambda_{min}(A_G) = 1 - p$ (since $V(M)$ is $(1, p)$-regular). Conversely, supposing that $\lambda_{min}(A_G) = 1 - p$, by Theorem 5.1, $\upsilon_1(G) = |V(M)|$. ♢

Note that, according to (4.2) and this corollary, if a p-regular graph G such that $\lambda_{min}(A_G) = 1 - p$ has an efficient edge dominating set then $\upsilon_1(G) = \frac{2|E(G)|}{2p-1}$.

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