RUTCOR RESEARCH REPORTS, 2010

RUTCOR – Rutgers Center for Operations Research
Busch Campus, Rutgers University
640 Bartholomew Road, Piscataway, NJ 08854-8003

RRR 1-2010 **SPARSE SIGNOMIAL CLASSIFICATION AND REGRESSION**, Kyungsik Lee, Norman Kim, and Myong K. Jeong

In this paper, we propose the sparse signomial classification and regression (SSCR), a new supervised learning method for classification and regression. SSCR seeks a sparse signomial function by solving a linear program to minimize the weighted sum of the ℓ_1-norm of the coefficient vector of the function and the ℓ_1-norm of violation (or loss) caused by the function. SSCR gives an explicit description of the resulting function in the original input space, which can be used for prediction purposes as well as interpretation purposes. Moreover, this method can explore very high dimensional feature spaces with less sensitivity to numerical values or numeric ranges of the given data. We present a practical implementation of SSCR based on the column generation. Computational study shows that SSCR gives competitive or even better performance compared to other widely used learning methods for classification and regression.

RRR 2-2010 **BOOLEAN FUNCTIONS WITH A SIMPLE CERTIFICATE FOR CNF COMPLEXITY**, Ondřej Čepek, Petr Kučera, and Petr Savický

In this paper we study relationships between CNF representations of a given Boolean function f and essential sets of implicates of f. It is known that every CNF representation and every essential set must intersect. Therefore the maximum number of pairwise disjoint essential sets of f provides a lower bound on the size of any CNF representation of f. We are interested in functions, for which this lower bound is tight, and call such functions coverable. We prove that for every coverable function there exists a polynomially verifiable certificate for its minimum CNF size. On the other hand, we show that not all functions are coverable, and construct examples of non-coverable functions. Moreover, we prove that computing the lower bound, i.e. the maximum number of pairwise disjoint essential sets, is NP-hard under various restrictions on the function and on its input representation.

RRR 3-2010 **NETWORK RESILIENCY**, Michael Tortorella

"Network resiliency" has been used in common language for many years in a variety of contexts. The intent of the usage seems to always have included the notions that a resilient network is one that continues to deliver services in a satisfactory fashion despite possible disruptions in the network infrastructure and/or one that returns to normal operation quickly after a disturbance. In this article, we consider the former interpretation in detail. The notion of "satisfactory" obviously involves a matter of degree, so a consistent quantitative framework for network resiliency is proposed for the first interpretation. This article organizes and subsumes current streams of network resiliency thought by providing a generalization of the various notions that have been used in the past to describe network resiliency and using this generalization to create a quantitative framework for resiliency characterization. The key concepts are the notions of a network with associated delivery function and of delivery importance. We discuss various design

principles that promote greater network resiliency in the context of this framework. We introduce a network resiliency optimization scheme and briefly show how the network interdiction problem can be solved using the proposed network resiliency framework.

RRR 4-2010 **IT IS A CONP-COMPLETE PROBLEM TO DECIDE WHETHER A POSITIVE $\vee$-$\wedge$ FORMULA OF DEPTH 3 DEFINES A READ-ONCE OR RESPECTIVELY QUADRATIC BOOLEAN FUNCTION**, Vladimir Gurvich

Let us consider the DNF $D_0 = x_1y_1 \vee \ldots \vee x_ny_n$ and let C be a positive CNF of the same $2n$ variables. It is a coNP-complete problem to verify the inequality $D_0 \geq C$, which is obviously equivalent with the equality $C \vee D_0 = D_0$. Clearly, the problem is in coNP and its polynomial reduction from SAT is straightforward. Nevertheless, this trivial observation has many applications in enumeration, game, and graph theories, as Khachiyan and the author demonstrated in 1995. Since then several other interesting applications were found. In this paper, we show that it is coNP-complete to verify whether the expression $C \vee D_0$ defines a quadratic or, respectively, read-once Boolean function.
Keywords: positive Boolean function or formula, read-once, linear, quadratic, disjunctive and conjunctive normal forms, DNF, CNF, coNP-complete problem

RRR 5-2010 **A POTENTIAL REDUCTION ALGORITHM FOR ERGODIC MEAN PAYOFF STOCHASTIC GAMES WITH PERFECT INFORMATION**, Endre Boros, Khaled Elbassioni,Vladimir Gurvich, and Kazuhisa Makino

In this paper, we consider two-person zero-sum stochastic mean payoff games with perfect information, or BWR-games, given by a digraph $G = (V = V_B \cup V_W \cup V_R, E)$, with local rewards $r : E \to \mathbb{R}$, and three types of vertices: black V_B, white V_W, and random V_R. The game is played by two players, White and Black: When the play is at a white (black) vertex v, White (Black) selects an outgoing arc (v, u). When the play is at a random vertex v, a vertex u is picked with the given probability $p(v, u)$. In all cases, Black pays White the value $r(v, u)$. The play continues forever, and White aims to maximize (Black aims to minimize) the limiting mean (that is, average) payoff. It was recently shown in [BEGM09] that BWR-games are polynomially equivalent with the classical Gillette games, which include many well-known subclasses, such as cyclic games, simple stochastic games (SSGs), stochastic parity games, and Markov decision processes. In this paper, we give a new algorithm for solving BWR-games in the *ergodic case*, that is when the game's value does not depend on the initial position. Our algorithm solves a BWR-game by reducing it, using a potential transformation, to a canonical form in which the optimal strategies of both players and the value for every initial position are obvious, since a locally optimal move in it is optimal in the whole game. We show that this algorithm is pseudo-polynomial when the number of random nodes is constant. We also provide an almost matching lower bound on its running time and show that this bound holds for a wider class of algorithms. Let us add that the general (non-ergodic) case is at least as hard as SSGs, for which no pseudo-polynomial algorithm is known.

Keywords: mean payoff games, local reward, Gillette model, perfect information, potential, stochastic games

RRR 6-2010 **UNIFORM QUASI-CONCAVITY IN PROBABILISTIC CONSTRAINED STOCHASTIC PROGRAMMING**, András Prékopa, Kunikazu Yoda, and Mine Subasi

A probabilistic constrained stochastic programming problem is considered, where the underlying problem has linear constraints with random technology matrix. The rows of the matrix are assumed to be stochastically independent and normally distributed. For the convexity of the problem the quasi-concavity of the constraining function is needed that is ensured if the factors are uniformly quasi-concave. In the paper a necessary and sufficient condition is given for that property to hold. It is also shown, through numerical examples, that such a special problem still has practical application in optimal portfolio construction.

RRR 7-2010 **METRIC AND ULTRAMETRIC SPACES OF RESISTANCES**, Vladimir Gurvich

Given an electrical circuit each edge e of which is an isotropic conductor with a monomial conductivity function $y_e^* = y_e^r/\mu_e^s$. In this formula, y_e is the potential difference and y_e^* current in e, while μ_e is the resistance of e; furthermore, r and s are two strictly positive real parameters common for all edges. In particular, the case $r = s = 1$ corresponds to the standard Ohm law.

In 1987, Gvishiani and Gurvich [Russ. Math. Surveys, 42:6(258) (1987) 235–236] proved that, for every two nodes a, b of the circuit, the effective resistance $\mu_{a,b}$ is well-defined and for every three nodes a, b, c the inequality $\mu_{a,b}^{s/r} \leq \mu_{a,c}^{s/r} + \mu_{c,b}^{s/r}$ holds. It obviously implies the standard triangle inequality $\mu_{a,b} \leq \mu_{a,c} + \mu_{c,b}$ when $s \geq r$. For the case $s = r = 1$, these results were rediscovered in 1990s. Now, in 23 years, I venture to reproduce the proof of the original result for the following reasons:

- It is more general than just the case $r = s = 1$ and one can get several interesting metric and ultrametric spaces playing with parameters r and s.

  In particular, (i) the effective Ohm resistance, (ii) the length of a shortest path, (iii) the inverse width of a bottleneck path, and (iv) the inverse capacity (maximum flow per unit time) between any pair of terminals a and b provide four examples of the resistance distances $\mu_{a,b}$ that can be obtained from the above model by the following limit transitions: (i) $r(t) = s(t) \equiv 1$,
  (ii) $r(t) = s(t) \to \infty$, (iii) $r(t) \equiv 1, s(t) \to \infty$, and (iv) $r(t) \to 0, s(t) \equiv 1$,

  as $t \to \infty$. In all four cases the limits $\mu_{a,b} = \lim_{t\to\infty} \mu_{a,b}(t)$ exist for all pairs a, b and the metric inequality $\mu_{a,b} \leq \mu_{a,c} + \mu_{c,b}$ holds for all triplets a, b, c, since $s(t) \geq r(t)$ for any sufficiently large t. Moreover, the stronger ultrametric inequality $\mu_{a,b} \leq \max(\mu_{a,c}, \mu_{c,b})$ holds for all triplets a, b, c in examples (iii) and (iv), since in these two cases $s(t)/r(t) \to \infty$, as $t \to \infty$.

- Communications of the Moscow Math. Soc. in Russ. Math. Surveys were (and still are) strictly limited to two pages; the present paper is much more detailed. Although translation in English of the Russ. Math. Surveys is available, it is not free in the web and not that easy to find out.

- The last but not least: priority.

Key words: distance, metric, ultrametric; potential, voltage, current; Ohm law, Joule-Lenz heat, Maxwell principle; maximum flow, shortest path, bottleneck path

RRR 8-2010 **EMPIRICAL ANALYSIS OF POLYNOMIAL BASES ON THE NUMERICAL SOULUTION OF THE MULTIVARIATE DISCRETE MOMENT PROBLEM**, Gergely Mádi-Nagy

The multivariate discrete moment problem (MDMP) has been introduced by Prékopa. The objective of the MDMP is to find the minimum and/or maximum of the expected value of a function of a random vector with a discrete finite support where the probability distribution is unknown, but some of the moments are given. The MDMP can be formulated as a linear programming problem, however, the coefficient matrix is very ill-conditioned. Hence, the LP problem usually cannot be solved in a regular way. In the univariate case Prékopa developed a numerically stable dual method for the solution. It is based on the knowledge of all dual feasible bases under some conditions on the objective function. In the multidimensional case the recent results are also about the dual feasible basis structures. Unfortunately, at higher dimensions, the whole structure could not be found under any circumstances. This means that a dual method, similar to Prékopa's, cannot be developed. Only bounds on the objective function value are given, which can be far from the optimum. This paper introduces a different approach in order to treat the numerical difficulties. The method is based on multivariate polynomial bases. Our algorithm, in most cases, yields the optimum of the MDMP without any assumption on the objective function. The efficiency of the method is tested on several numerical examples.
Keywords: Discrete moment problem, Multivariate Lagrange interpolation, Linear programming, Expectation bounds, Probability bounds
AMS: 62H99, 90C05, 65D05

RRR 9-2010 **SPARSE WEIGHTED VOTING CLASSIFIER SELECTION AND ITS LP RELAXATIONS**, Noam Goldberg and Jonathan Eckstein

We consider a combinatorial optimization problem that generalizes the minimum disagreement halfspace problem; we seek to minimize the number of misclassifications of a weighted voting classifier, plus a penalty proportional to the density of the vector of weights. To justify the use of this optimization problem, we investigate its relation to the minimum description length principle and statistical learning theory generalization bounds. We prove that the optimum is at least as hard to approximately compute as minimum disagreement halfspace for a large class of penalty parameters. After formulating the problem as a mixed integer program (MIP), we show that common "soft margin" linear programming formulations for constructing weighted voting classsifiers are equivalent to an LP relaxation of our formulation. We illustrate that the LP relaxation can be very weak, with an exponential lower bound on the potential intergrality gap. We then prove that augmenting the optimization problem with certain simple valid inequalities tightens the relaxation considerably, yielding a linear upper bound on the gap for all values of the penalty parameter that exceed a sensible lower bound.

RRR 10-2010 **ON EXACT BLOCKERS AND ANTI-BLOCKERS, Δ-CONJECTURE, AND RELATED PROBLEMS**, Vladimir Gurvich

Let us consider two binary systems of inequalities (i) $Cx \geq e$ and (ii) $Cx \leq e$, where $C \in \{0,1\}^{m \times n}$ is an $m \times n$ $(0,1)$-matrix, $x \in \{0,1\}^n$, and e is the vector of m ones. The set of all support-minimal (respectively, support-maximal) solutions x to (i) (respec-

tively, to (ii)) is called the *blocker* (respectively, *anti-blocker*). A blocker $\mathcal{B}$ (respectively, anti-blocker $\mathcal{A}$) is called *exact* if $Cx = e$ for every $x \in \mathcal{B}$ (respectively, $x \in \mathcal{A}$). Exact blockers can be completely characterized. There is a one-to-one correspondence between them and P_4-free graphs (along with a well known one-to-one correspondence between the latter and the so-called read-once Boolean functions). However, the class of exact anti-blockers is wider and more sophisticated. We demonstrate that it is closely related to the so-called CIS graphs, more general ℓ-CIS d-graphs, and Δ-conjecture.

Key words: blocker, anti-blocker, exact blocker, exact anti-blocker, read-once Boolean function, CIS graph, CIS d-graph, Δ-conjecture, box-partition, solid box-partition.

RRR 11-2010 **A PRACTICAL RELATIVE ERROR CRITERION FOR AUGMENTED LAGRANGIANS**, Jonathan Eckstein and Paulo J. S. Silva

This paper develops a new error criterion for the approximate minimization of augmented Lagrangian subproblems. This criterion is practical in the sense that it requires only information that is ordinarily readily available, such as the gradient (or a subgradient) of the augmented Lagrangian. It is also "relative" in the sense of relative error criteria for proximal point algorithms, in that it is based on the relative magnitude of two quantities and requires only a single parameter, not the choice of an theoretically infinite sequences of parameters. It involves a novel auxiliary sequence that appears only in the approximation criterion, and not in the augmented Lagrangian minimand, nor in the multiplier update. We give a proof of the global convergence of our method in the setting of the abstract convex duality framework of Rockafellar, along with some more concrete applications. The dual convergence result is slightly weaker than usually obtained for multiplier methods, but may be strengthened by enforcing an additional condition in the algorithm. We give some computational results drawn from the CUTE test set, indicating that our approach works well in practice.

RRR 12-2010 **OPTIMAL PORTFOLIO SELECTION BASED ON MULTIPLE VALUE AT RISK CONSTRAINTS**, Kunikazu Yoda and András Prékopa

A variant of Kataoka's portfolio selection model is formulated in which lower bounds are imposed on several VaR values, where the bounds are taken from a reference probability distribution. Under mild assumptions, the problems are formulated as convex nonlinear programming problems, so that the global optimal solution can be found with a nonlinear programming solver. The numerical solution technique will be discussed and numerical examples will be presented.

RRR 13-2010 **SCENARIO DECOMPOSITION OF RISK-AVERSE MULTISTAGE STOCHASTIC PROGRAMMING PROBLEMS**, Ricardo A. Collado, Dávid Papp, and Andrzej Ruszczyński

For a risk-averse multistage stochastic optimization problem with a finite scenario tree, we introduce a new scenario decomposition method and we prove its convergence. The method is applied to a risk-averse inventory and assembly problem. In addition, we develop a partially regularized bundle method for nonsmooth optimization.

RRR 14-2010 **ON NASH EQUILIBRIA AND IMPROVEMENT CYCLES IN PURE POSITIONAL STRATEGIES FOR CHESS-LIKE AND BACKGAMMON-LIKE N-PERSON GAMES**, Endre Boros, Khaled Elbassioni, Vladimir Gurvich, and Kazuhisa Makino

We consider n-person positional games with perfect information modeled by finite directed graphs that may have directed cycles, assuming that all infinite plays form a single outcome a_∞, in addition to the standard outcomes $a_1, \ldots, a_m$ formed by the terminal positions. (For example, in case of Chess or Backgammon $n = 2$ and a_∞ is a draw.) These $m+1$ outcomes are ranked arbitrarily by n players. We study existence of (subgame perfect) Nash equilibria and improvement cycles in pure positional strategies and provide a systematic case analysis in terms of the following conditions:

(i) there are no directed cycles;

(ii) there are no random positions;

(iii) the "infinite outcome" c is ranked as the worst one by all n players;

(iv) $n = 2$;

(v) $n = 2$ and the payoff is zero-sum.

Key words: Back Gammon, Nash equilibrium, subgame perfect, improvement cycle, best reply, stochastic game, perfect information, position, move, random move

RRR 15-2010 **MORE ABOUT SCARF AND SPERNER OIKS**, Jack Edmonds, Stephane Gaubert, Vladimir Gurvich, and Matthew Oster

The Lemke-Howson exchange algorithm for finding a Nash equilibrium in bimatrix games, as well as the classical algorithm for finding the properly colored facet in Sperner's Lemma generalize and abstract to pure combinatorics.

In particular, the idea of Lemke pivoting is extended to an arbitrary family of oiks (*Euler complexes*). Given a room-partition, the corresponding algorithm finds another (distinct) room-partition by traversing an exchange graph.

In this paper we show that each family of k oiks $\mathcal{O} = \{\mathcal{O}_1, \ldots, \mathcal{O}_k\}$ can be reduced to a pair of oiks $\mathcal{O}' = \{\mathcal{O}_1 + \ldots + \mathcal{O}_k, \mathcal{O}_0\}$ (one of which, $\mathcal{O}_0$, is a Sperner oik) such that the exchange graphs for $\mathcal{O}$ and $\mathcal{O}'$ are isomorphic. Numerous application of Sperner's Lemma in combinatorial topology are well known.

We also formulate the famous Scarf Lemma in terms of oiks. This Lemma has two fundamental applications in game and graph theories. In 1967, Scarf derived from it core-solvability of balanced cooperative games. In 1996, it was shown that kernel-solvability of perfect graphs also results from this Lemma.

We show that Scarf's combinatorially defined oiks are in fact realized by polytopes. We also demonstrate that the pivoting path between room-partitions can be exponentially long in d already for two equal d-dimensional Scarf oiks on $2d$ vertices. A similar example is constructed for a pair of d-dimensional Scarf and Sperner oiks.

Keywords: Euler complex (oik), room, wall, manifold, cyclic polytope, matroid; exchange algorithm, pivot; bimatrix game, Nash equilibrium, Lemke-Howson; Sperner Lemma, Brouwer Fixed Point Theorem, KKM-Theorem; core, core-solvability, Scarf Lemma, balanced games; kernel, kernel-solvability, perfect graph

RRR 16-2010 **FURTHER GENERALIZATIONS OF WYTHOFF'S GAME AND MINIMUM EXCLUDANT FUNCTION**, Vladimir Gurvich

Given non-negative integer a and b, let us consider the following game $WYT(a,b)$. Two piles contain x and y matches. Two players take turns. By one move, it is allowed to take x' and y' matches from these piles such that

$0 \le x' \le x,\ 0 \le y' \le y,\ 0 < x' + y'$, and $[\min(x', y') < b \ \text{ or } \ |x' - y'| < a]$.

The player who takes the last match is the winner (respectively, loser) in the normal (respectively, misere) version of the game.

It is easy to verify that cases $(a = 0, b = 1), (a = b = 1)$, and $(b = 1, \forall\, a)$ correspond to the two-pile NIM, Wythoff, and Fraenkel games, respectively. The concept of the minimum excludant function mex is known to be instrumental in solving the last two games. We generalize this concept by introducing a function mex_b such that $mex = mex_1$ and solve the normal and misere versions of game $WYT(a,b)$.

Keywords: combinatorial games, NIM, Wythoff game, Fraenkel game, minimal excludant, normal and misere versions, Sprague-Grundy function

RRR 17-2010 **THE DISCRETE MOMENT METHOD FOR THE NUMERICAL INTEGRATION OF PIECEWISE HIGHER ORDER CONVEX FUNCTIONS**, Andras Prekopa, Mariya Naumova, and Linchun Gao

A new numerical integration method, termed Discrete Moment Method (DMM), is proposed for univariate functions that are piecewise higher order convex. This means that the interval where the function is defined can be subdivided into non-overlapping subintervals such that in each interval all divided differences of given orders, do not change the sign. The new method uses piecewise polynomial lower and upper bounds on the function, created in connection with suitable dual feasible bases in the univariate discrete moment problem and the integral of the function is approximated by tight lower and upper bounds on them. Numerical illustrations are presented for the cases of the normal, exponential, gamma and Weibull probability density functions.

RRR 18-2010 **A FOUR PARAMETRIC GENERALIZATION OF THE WYTHOFF NIM AND ITS RECURSIVE SOLUTION**, Vladimir Gurvich

Given positive integer a, b and p, q, we will consider the following game $\mathrm{NIM}^{p,q}_{a,b}$. Two piles contain x and y matches. Two players take turns. By one move, it is allowed to take x' and y' matches from these two piles such that

$0 \le x' \le x,\ 0 \le y' \le y,\ 0 < x' + y'$, and $[(A)\ \ |x' - y'| < a \ \text{ or } \ (B)\ \ \min(x', y') < b]$.

Furthermore, each player after a move is allowed to block off up to $p - 1$ opponent's moves of type (A) and $2(q - 1)$ moves of type (B), more precisely, at most $q - 1$ moves in each case, when the minimum is realized by x' and y'.

The player who takes the last match is the winner.

Games $\mathrm{NIM}^{1,1}_{1,1}$, $\mathrm{NIM}^{1,1}_{a,1}$, $\mathrm{NIM}^{p,1}_{a,1}$, and $\mathrm{NIM}^{1,1}_{a,b}$. were considered by Wythoff, Fraenkel, Larsson, and the author in 1907, 1982, 2009, and 2010, respectively.

We obtain a simple arithmetic recursion solving $\mathrm{NIM}^{p,q}_{a,b}$ when $p = 1$ or $q = 1$ and get partial results for the general case. The recursion is of a "standard type"

$$x_n = mex^q_b\{x_i, y_i \mid 0 \le i < n\},\ \ y_n = x_n + a\lfloor n/p \rfloor;\ \ n \ge 0,$$

where mex_b^q is a two-parametric generalization of the minimum excludant mex.

Keywords: combinatorial games, NIM, Wythoff's NIM, minimum excludant

RRR 20-2010 **NETWORK SUPPLY SYSTEMS, STABLE FAMILIES OF COALITIONS FOR SUPERADDITIVE TU-GAMES AND BERGE'S NORMAL HYPERGRAPHS**, Vladimir Gurvich and Sergei Schreider

The very common question appearing in resource management is: what is the optimal way of behaviour of the agents and distribution of limited resources. Is any form of cooperation more preferable strategy then pure competition? How cooperation can be treated in the game theoretic framework: just as one of a set of Pareto optimal solutions or cooperative game theory is a more promising approach? This research is based on results proving the existence of a non-empty $\mathcal{K}$-core, that is, the set of allocations acceptable for the family $\mathcal{K}$ of all feasible coalitions, for the case when this family is a set of subtrees of a tree. A wide range of real situations in resource management, which include optimal water, gas and electricity allocation problems can be modeled using this class of games. Thus, the present research is pursuing two goals: 1. optimality and 2. stability. Firstly, we suggest to players to unify their resources and then we optimize the total payoff using some standard LP technique. The same unification and optimization can be done for any coalition of players, not only for the total one. However players may object unification of resources. It may happen when a feasible coalition can guarantee a better result for every coalitionist. Here we obtain some stability conditions which ensure that this cannot happen for some family $\mathcal{K}$. Such families were characterized in Boros et al. (1997) as Berge's normal hypergraphs. Thus, we obtain a solution which is optimal and stable. From practical point of view, we suggest a distribution of profit that will not cause the conflict between players.

RRR 21-2010 **PROOF OF LOGCONCAVITY OF SOME COMPOUND POISSON AND RELATED DISTRIBUTIONS**, Anh Ninh and András Prékopa

Compound Poisson distributions play important role in many applications (telecommunication, hydrology, insurance, etc.). In this paper, we prove that some of the compound Poisson distributions have the logconcavity property that makes them applicable in stochastic programming problems. The proofs are based on classical Turan types theorem and orthogonal polynomials.

RRR 22-2010 **A LOWER BOUND FOR DISCOUNTING ALGORITHMS SOLVING TWO-PERSON ZERO-SUM LIMIT AVERAGE PAYOFF STOCHASTIC GAMES**, Endre Boros, Khaled Elbassioni, Vladimir Gurvich, and Kazuhisa Makino

It is shown that the discount factor needed to solve an undiscounted mean payoff stochastic game to optimality is exponentially close to 1, even in games with a single random node and polynomially bounded rewards and transition probabilities.

RRR 23-2010 **OCCUPATION GAMES ON GRAPHS IN WHICH THE SECOND PLAYER TAKES ALMOST ALL VERTICES**, Alexander Shapovalov

Given a connected graph $G = (V, E)$, two players take turns occupying vertices $v \in V$ by putting black and white tokens so that the current vertex sets $B, W \subseteq V$ are disjoint,

$B \cap W = \emptyset$, and the corresponding induced subgraphs $G[B]$ and $G[W]$ are connected any time. A player must pass whenever (s)he has no legal move. (Obviously, after this, the opponent will take all remaining vertices, since G is assumed connected.) The game is over when all vertices are taken, $V = B^* \cup W^*$. Then, Black and White get $b = |B^*|/|V|$ and $w = |W^*|/|V|$, respectively. Thus, the occupation game is one-sum, $b + w = 1$, and we could easily reduce it to a zero-sum game by simply shifting the payoffs, $b' = b - 1/2, w' = w - 1/2$. Let us also notice that $b \geq 0$ and $w \geq 0$; moreover, $b > 0$ and $w > 0$ whenever $|V| > 1$.

[Let us remark that the so-called Chinese rules define similar payoffs for the classic game of GO, yet, the legal moves are defined in GO differently.]

Like in GO, we assume that Black begins. It is easy to construct graphs in which Black can take almost all vertices, more precisely, for each $\varepsilon > 0$ there is a graph G for which $b > 1 - \varepsilon$. In this paper we show that, somewhat surprisingly, there are also graphs in which White can take almost all vertices.

Keywords: occupation games, GO

RRR 24-2010 **ROBUST CUTPOINTS IN THE LOGICAL ANALYSIS OF NUMERICAL DATA**, Martin Anthonya and Joel Ratsaby

Techniques for the logical analysis of binary data have successfully been applied to non-binary data which has been 'binarized' by means of cutpoints; see [7, 8]. In this paper, we analyse the predictive performance of such techniques and, in particular, we derive generalization error bounds that depend on how 'robust' the cutpoints are.

RRR 25-2010 **ON CUMULATIVE JUMP RANDOM VARIABLES**, Michael Tortorella

Stochastic models for phenomena that can exhibit sudden changes involve the use of processes whose sample functions may have discontinuities. This report provides some tools for working with such processes. We develop a sample path formula for the cumulative jump height over a given time interval. From this formula an expression for the expected value of the cumulative jump random variable is developed under reasonable conditions. The results are applied to finding the expected number of failures in the separate maintenance model over a stated time interval and to the expected number of occurrences of a regenerative event over a stated time interval.

RRR 26-2010 **THE BRANCH AND BOUND METHOD**, Béla Vizvári

This paper is a preliminary version of a chapter on the branch and bound method which will be published in a textbook on algorithms of informatics.

RRR 27-2010 **CONVERGENCE OF COMPOSITION OF MARKOV RISK MEASURES**, Vincent Leclére

Our goal in this paper is to show how one could compose some measures of risks, and more precisely Markovian measures of risk, over a Markov chain. We mainly show the convergence of the composition to a limit independent of the starting point, under ergodicity condition of the Markov chain, and risk averse behaviour of the measure.