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# Flow-based Capacity Allocation in the CEE Region: Sensitivity Analysis, Multiple Optima, Real Income

Ákos Füzi Gergely Mádi-Nagy

**Abstract.** The paper introduces the mechanism of the Flow-based Capacity Allocation (FBA) method of the Central-Eastern Europe (CEE) Region. It reveals the properties of the underlying linear programming problem and discusses their practical consequences. A non-standard sensitivity analysis method of the market spread auction is developed. Finally, the objective and the real overall income of the auction are compared by the aid of a global optimization problem. Several numerical examples and results of practical test problems are presented.

**Keywords:** Linear programming, Sensitivity analysis, Global optimization

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# 1 Introduction

The European electricity market is under substantial transformations. After the liberalization, the next step is that the domestic markets are cooperating and developing regional markets. One crucial point of this process is the allocation of cross-border transmission capacities.

The paper focuses on the Central-Eastern Europe (CEE) region, which consists of Austria, the Czech Republic, Germany, Hungary, Poland, Slovakia and Slovenia. The demand for cross-border electricity exchange between the CEE Transmission System Operators (TSOs) is usually higher than the available electricity transmission capacities at interconnectors. According to the European Community Regulations 1228/2003/EC and 714/2009/EC, a market-based mechanism has to be adopted to allocate the capacities. It has the following requirements:

- prevention of overloading,
- efficient deals with interdependent physical flows,
- avoidance of discrimination in allocating electricity transmission capacities.

Hence, instead of the former OTC bilateral contracts, CEE TSOs intend to introduce stepwise a coordinated flow-based allocation of cross-border electricity transmission capacities at interconnections between CEE TSOs. The steps are

1. *Coordinated Net Transfer Capacity (NTC) assessment method.* It is an explicit coordinated auction, however, only the cross-border capacities are taken into account. This model is applied now.
2. *Coordinated flow-based allocation (FBA) method.* It is planned to be introduced soon. This model takes into account the properties of the whole electricity network.

There is a wide range of literature on congestion management methods as well as on their proposals for the European market. Regulation 1228/2003 is analyzed by Boucher and Smeers (2002) and Ehrenmann and Smeers (2005). The investigated FBA method is based on zonal pricing: the properties of nodal, uniform or zonal pricing are discussed in Ding and Fuller (2005). Within the sectors uniform pricing is applied, however, e.g., the model of nodal pricing can be found in Stigler, Heinz, Todem and Christian (2005) (as regards Austria) and in Leuthold, Weigt and von Hirschhausen (2008) (as regards Germany). The publications, proposals of ETSOE (2012) should also be taken into account.

The approach of our paper is different to the above. While most literature analyzed the methods theoretically, our discussion is based on practice: the solutions of the FBA model have been implemented and used in energy-trading Informatics Platforms of IP Systems LTD. On one hand, we would like to share the experiences of the implementation and tests of the method, focusing on multiple optima and sensitivity analysis.

On the other hand, we would like to demonstrate the difference between the theoretical objective function value and the real overall income of the TSOs: the two values are usually different and their maxima are achieved at different auction prices, as well.

The paper is organized as follows. In Section 2 the mechanism of the FBA method is introduced. In Section 3 the typical physical and business properties of the auction parameters are presented. In Section 4 the cases of multiple optima and a particular sensitivity analysis are discussed. In Section 5 a global optimization problem is introduced in order to maximize the overall income of the TSOs. Here some comparisons with the results of original FBA auction are presented. Section 6 concludes the paper.

# 2 Mechanism of the FBA method

## 2.1 The auction

The coordinator of the auction is the *Central Allocation Office (CAO)*, which is founded and owned by the TSOs of the CEE region. The detailed rules of the market can be found on the CAO website (CAO 2011). In order to make the paper self-contained, the auction rules are summarized in this section.

There are eight TSOs (APG, CEPS, ELES, TENNET, MAVIR, PSEO, SEPS, a.s. and 50HzT), however, in the market APG, TENNET and 50HzT constitute one zone. Hence, the trade is possible among five zones. Regarding the type of the auctions there are yearly, monthly and daily auctions. In case of daily auction, capacities can be purchased separately for each hour of the day.

The auction rules are defined in the following sections.

## 2.2 Bids of the participants

Every bid consists of

- the *source-sink pair*, i.e. from which zone to which zone the capacity is required,
- the *required* quantity of *capacity* (in $MW$),
- the offered *bid price* for one unit capacity (in $EUR/MWh$).

## 2.3 Characterization of the network

The network is depicted in the following way. Initially, the critical lines of the network are considered. On one hand, a loading plan on the network is published: i.e., how many $MWh$s load on each line in case of transmission of one $MWh$ from a given zone to another given zone. On the other hand, the maximum capacities of each line are given.

### 2.3.1 The PDTF (Power Transfer Distribution Factors)

The PTDF matrix is a set of PTDFs expressing the influence of commercial exchanges between all source-sink pairs to all critical lines (so called outage combination). I.e.,

- the value of
$$PTDF(Zone_x, Zone_y, line_k)$$
represents that *how much capacity is used on* $line_k$ *in case of transmission of* $1MW$ *from* $Zone_x$ *to* $Zone_y$.

- *Each line has a direction.* If the actual Source-Sink pair uses the line in this direction, then the PTDF is positive, in case of opposite direction it is negative.

- The actual PTDF is published on the CAO ePortal (2011).

### 2.3.2 The AMF (Available Maximum Flow) matrix

The AMF values give the maximum capacities of the lines. I.e.

- $AMF^+(line_k)$ is the maximum capacity in the direction of $line_k$,

- $AMF^-(line_k)$ is the maximum capacity in the opposite direction.

The reason of the two different capacities regarding the directions is that they are remaining capacities after long-term contracts and previous auctions.

### 2.3.3 The outlook of PTDF published by CAO ePortal

Upper left corner of the first sheet of the PTDF of Daily Auction.

| Technical Parameters for Test 2 Daily Auction 7A-7B | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Parameters (1008) | | | | | | | | |
| Critical Branch | Case | Source | Sink | TMF | AMF+ | AMF- | MAVIR->PSEO | MAVIR->ELES |
| LINE_00001 | n-0 | APG | APG | 305 | 373.9 | 166 | 0.0076 | -0.0313 |
| LINE_00001 | n-1 LINE_00002 | APG | APG | 305 | 413.1 | 126.9 | 0.0104 | -0.0433 |
| LINE_00001 | n-1 LINE_00003 | APG | APG | 305 | 413.2 | 126.8 | 0.0105 | -0.0433 |
| LINE_00001 | n-1 LINE_00004 | APG | APG | 305 | 363.9 | 176 | 0.0058 | -0.0299 |
| LINE_00001 | n-1 LINE_00005 | APG | APG | 305 | 357.4 | 182.5 | 0.0059 | -0.0273 |
| LINE_00001 | n-1 LINE_00006 | APG | APG | 305 | 387.2 | 152.7 | 0.0108 | -0.0207 |
| LINE_00007 | n-0 | APG | APG | 303 | 163.8 | 371.7 | -0.0076 | 0.0313 |
| LINE_00007 | n-1 LINE_00002 | APG | APG | 303 | 124.7 | 410.8 | -0.0104 | 0.0433 |
| LINE_00007 | n-1 LINE_00003 | APG | APG | 303 | 124.5 | 410.9 | -0.0105 | 0.0433 |

The PTDF matrix has $5 \times 4 = 20$ columns corresponding to the Source-Sink Pairs (5 Zones), and it has about 1000 rows (critical lines or branches).

## 2.4 The allocation method

The allocation method should find the most effective usage of the whole network subject to properties and limits of the whole transmission network and all bids together. The auction is modelled by a *linear programming (LP) problem*, detailed below, where the objective function represents the economic effectivity and the constraints describe the properties of the network.

### 2.4.1 The objective function

The so-called *social welfare* will be maximized:

$$F = \sum_{x,y \in Zones,\ b \in Bids} p_b(x,y,b) \cdot d_a(x,y,b) \tag{1}$$

where

| | |
|---|---|
| $Zones$ | is the set of zones, |
| $Bids$ | is the set of offered bids, |
| $x$ | is a source zone, |
| $y$ | is a sink zone, |
| $b$ | is a bid |
| $p_b(x,y,b)$ | is the bid price of bid $b$ (on the source-sink $x \to y$), |
| $d_a(x,y,b)$ | is the allocated quantity for bid $b$. |

The decision variables of the LP will be: $d_a(x,y,b),\ x,y \in Zones,\ b \in Bids$. The objective function is the total (theoretical) income divided by the number of hours of the time-length of capacity. Hence, the objective is maximizing the total income.

### 2.4.2 Network constraints

The network constraints represent the PTDF and AMF structures:

$$\sum_{x,y \in Zones} \left( \max(0, PTDF(x,y,k)) \sum_{b \in Bids} d_a(x,y,b) \right) \leq AMF^+(k) \tag{2}$$

$$\sum_{x,y \in Zones} \left( \max(0, -PTDF(x,y,k)) \sum_{b \in Bids} d_a(x,y,b) \right) \leq AMF^-(k) \tag{3}$$

$k \in Lines$. Where

| | |
|---|---|
| $Lines$ | is the set of the critical lines, |
| $k$ | is a line, |
| $PTDF(x,y,k)$ | how much capacity is used on $k$ in case of transmission of $1MW$ from $x$ to $y$, |
| $AMF^+(k)$ | is the maximum capacity in the direction of $k$, |
| $AMF^-(k)$ | is the maximum capacity in the opposite direction of $k$. |

### 2.4.3 Bid constraints

The allocated capacities cannot be negative or greater than the requested capacities. I.e.,

$$d_a(x,y,b) \leq d_b(x,y,b) \quad x,y \in Zones,\ b \in Bids \tag{4}$$

$$d_a(x,y,b) \geq 0 \quad x,y \in Zones,\ b \in Bids \tag{5}$$

where $d_b(x,y,b)$ is the requested capacity for bid $b$.

## 2.5 Auction prices

In the original objective function the social welfare is calculated by the use of the *offered bid prices*. However, at the end of the auction *uniform auction prices* are announced *to each source-sink zone pair*.

The optimal solution of the LP also yields *Shadow Prices* (dual solutions) corresponding to the constraints. E.g.,

$$SP(AMF^{+}(k)), \quad , SP(AMF^{-}(k)), \quad k \in Lines,$$

denote the shadow prices of lines. They represent an increase in the value of the objective function connected with the marginal increase of the corresponding $AMF$. Intuitively: *the worth of one unit of capacity of the given direction of the line, regarding the optimal allocation.*

The *Auction Price* of source-sink pair from zone $x$ to zone $y$ is given by the formula:

$$\begin{aligned} AP(x,y) &= \sum_{k \in Lines} \left[\max\left(0, PTDF(x,y,k)\right) \cdot SP(AMF^{+}(k))\right] \\ &+ \sum_{k \in Lines} \left[\max\left(0, -PTDF(x,y,k)\right) \cdot SP(AMF^{-}(k))\right]. \end{aligned}$$

Intuitively, this means that the auction price is the worth of one unit capacity on the certain source-sink pair: i.e. the sum of the usage percentage of each (directed) line multiplied by the worth of the line.

Then, the *capacity allocation based on Auction Prices* is the following:

- if the bid price is greater than the auction price then the whole requested quantity of the capacity is allocated,
- if the bid price is less than the auction price then zero capacity allocated,
- if the bid price equals the auction price then the requested capacity is partially satisfied until the limits of the network constraints.

From the *Complementary Slackness Theorem* follows that an optimal allocation of the LP satisfies the above rules and vice versa.

# 3 Physical and business properties

When CAO published the PTDF and AMF matrices corresponding to the auction, the participants of the market try to get a first impression on the network before the bids are submitted. Their main interests are the following.

## 3.1 Physical properties

### 3.1.1 $MTSF(x, y)$ (Maximum Theoretical Single Flow)

The $MTSF(x, y)$ means the maximum amount of electricity which can be flown from Source $x$ to Source $y$. It can be simulated by the following auction. Only one bid $b$ is submitted with parameters

$$p_b(x, y, b) = 1,\ d_b(x, y, b) = +\infty$$

Certainly, $MTSF(x, y)$ simply can be calculated as a bottleneck from Zone $x$ to Zone $y$, by the following formula:

$$MTSF(x, y) = \min\Bigg( \min_{k \in Lines, PTDF(x,y,k)>0} (AMF^+(k)/PTDF(x, y, k)), \min_{k \in Lines, PTDF(x,y,k)<0} (AMF^-(k)/(-PTDF(x, y, k)))\Bigg).$$

### 3.1.2 $MTEX(x)$ (Maximum Theoretical Export)

$MTEX(x)$ is the maximum amount of electricity which can be flown from Source $x$ (to any zone). It can be simulated by the following auction. One bid from Zone $x$ to each zone is submitted with parameters

$$p_b(x, y, b) = 1,\ d_b(x, y, b) = +\infty,\ \text{for all } y \in Zones,\ y \neq x.$$

### 3.1.3 $MTIM(y)$ (Maximum Theoretical Import)

$MTIM(y)$ is the maximum amount of electricity which can be flown to Source $y$ (from any zone). It can be simulated by the following auction. One bid from each zone to Zone $y$ is submitted with parameters

$$p_b(x, y, b) = 1,\ d_b(x, y, b) = +\infty,\ \text{for all } x \in Zones,\ x \neq y.$$

## 3.2 Business properties

The following auction takes into account not only the physical possibilities, but the economic circumstances in calculation of maximal flows.

### 3.2.1 Market Spread Auction

The reason of cross-border transmission is that the price of electricity is cheaper on the market of the source zone than on the market of the sink zone. Assume that the auction participants have some forecast or estimation on the zone prices. Let us use the following notations:

$$p_{bid}(x),\ p_{ask}(x)$$

are the bid and ask price of one unit electricity at Zone $x$. Then assume that all bid prices are their reasonable upper limit. The question is that how much capacity is allocated on each sink-source pair. I.e., the following auction is considered: one bid for each source-sink pair is submitted with parameters

$$p_b(x, y, b) = p_{bid}(y) - p_{ask}(x),\ d_b(x, y, b) = +\infty,$$

$$\text{for all } x, y \in Zones.$$

# 4 Multiple optima, sensitivity analysis

## 4.1 Multiple optima

Multiple optima can be arisen regarding the allocation (primal variables) as well as regarding the auction prices (derived from the dual variables). As it will be seen, the primal case is regulated well by the auction rules while in the dual case the multiple optima do not occur in practice.

## 4.2 Alternative optimal allocations

The following situations can happen:

1. there are two bids with same Source-Sink Pair and Bid Price and at least one of their requests is just partially satisfied,

2. there is a bid with zero bid price which can be (at least partially) satisfied,

3. there are structurally different optimal solutions.

Let us consider some examples on the following simple PTDF matrix:

| Technical Parameters for Test | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Parameters (1008) | | | | | | | | |
| Critical Branch | Case | Source | Sink | TMF | AMF+ | AMF- | MAVIR->PSEO | MAVIR->ELES |
| LINE_00001 | n-0 | APG | APG | 305 | **30** | 166 | **0.5** | **0.6** |
| LINE_00001 | n-1 LINE_00002 | APG | APG | 305 | **20** | 126.9 | **0.1** | **0.2** |

Example for Phenomenon 1:

| Requested Bids | | | | | Awarded Bids | |
|---|---|---|---|---|---|---|
| Product | Source | Sink | Requested Capacity [MW] | Bid Price [EUR/MWh] | Awarded Capacity [MW] | Auction Price [EUR/MWh] |
| H01 | MAVIR | PSEO | 70 | 1 | 0<=x<=60 | 1 |
| H01 | MAVIR | PSEO | 60 | 1 | 60-x | 1 |

Example for Phenomenon 2:

| Requested Bids | | | | | Awarded Bids | |
|---|---|---|---|---|---|---|
| Product | Source | Sink | Requested Capacity [MW] | Bid Price [EUR/MWh] | Awarded Capacity [MW] | Auction Price [EUR/MWh] |
| H01 | MAVIR | PSEO | 20 | 1 | 20 | 0 |
| H01 | MAVIR | PSEO | 10 | 0 | 0<=x<=10 | 0 |

Example for Phenomenon 3:

| Requested Bids | | | | | Awarded Bids | |
|---|---|---|---|---|---|---|
| Product | Source | Sink | Requested Capacity [MW] | Bid Price [EUR/MWh] | Awarded Capacity [MW] | Auction Price [EUR/MWh] |
| H01 | MAVIR | PSEO | 20 | 10 | 20 | 5 |
| H01 | MAVIR | ELES | 20 | 12 | 20 | 6 |
| H01 | MAVIR | PSEO | 20 | 5 | 0<= x <=16 | 5 |
| H01 | MAVIR | ELES | 20 | 6 | 80/6-5/6*x | 6 |

The first two situations are managed by Annex 5 of the auction rules of CAO (2011) :

- As regards Situation 1: The "First-Come-First-Serve Principle." has to be used. I.e., the bid submitted earlier must be preferred.
- Regarding Situation 2: the auction rules say "the Bid Price is replaced for optimisation algorithm purposes with a very small number not influencing the Auction Price calculation.". I.e., if any unused capacity remains in the network, it must be allocated among the submitted zero-priced bids.

Multiple optimal allocations can be arisen in case of multiple optimal (primal) bases. In case of Phenomenon 1 at least two columns of the coefficient matrix (and the corresponding objective function coefficients) of the LP were identical. Practically, this situation can happen only in case of bids with identical source-sink pairs and bid prices.

In all other cases, the bid prices are exactly at the value, where the multiple optimal bases exist. If we perturbate the bid prices, then the optimal basis and solution become unique. This happens, if in Phenomenon 2 the zero bid prices are substituted by a small number.

The existence of multiple optimal bases can happen naturally, only in case of zero-priced bids. The zero-priced bid case follows from the structure of the LP problem, however, there are no other simple connections between the PTDF and bid prices that can explain any other (structurally different) multiple optima. (E.g. the simple connection could be that all PTDF from Zone $x$ to Zone $y$ are twice the PTDF from Zone $x$ to Zone $z$, hence double bid price on the later source-sink pair can cause alternative allocations.) I.e. structurally different alternative allocations can be produced only by the precise (artificial) setting of the bid prices. Hence, *structurally different multiple allocations do not arise practically.* This was confirmed by the results of CAO FBA Dry Run II (2009).

## 4.3 Alternative auction prices

As regards the auction prices, they can be indefinite in case of multiple optima of the dual of the (auction) LP problem. Beside complex situations, this can arise in the following natural way. Consider the following simple example. The PTDF matrix is

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Technical Parameters for Test** | | | | | | | |
| Parameters (1008) | | | | | | | |
| Critical Branch | Case | Source | Sink | TMF | AMF+ | AMF- | MAVIR->PSEO |
| LINE_00001 | n-0 | APG | APG | 305,2 | **30** | 166 | **0,5** |

The submitted bid and result is

| Requested Bids | | | | | Awarded Bids | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Product | Source | Sink | Requested Capacity [MW] | Bid Price [EUR/MWh] | Product | Source | Sink | Awarded Capacity [MW] | Auction Price [EUR/MWh] |
| H01 | MAVIR | PSEO | 60 | 10 | H01 (1h) | MAVIR | PSEO | 60 | 0<= x <=10 |

The reason of multiple dual optima is that in the LP model the (AMF+) constraint (2) of LINE_00001 and the (requested capacity) constraint (4) of the bid are fulfilled by equality at the same time. This means that the values of the corresponding (primal) slack variables are zero. Those variables represent the optimality conditions of the dual problem, hence, the only dual basis variable can correspond to the constraint of LINE_00001 or to the bid constraint. This means two optimal dual bases and infinite optimal solutions. Indeed, if we perturbate the requested capacities:

- in case of e.g., $59MW$ requested capacity, exactly the bid constraint will be the basis and the auction price will be zero (the awarded capacity will be 59),
- in case of e.g., $61MW$ requested capacity, exactly the AMF+ constraint will be the basis and the auction price will be 10 (the awarded capacity will be 60).

The existence of multiple dual solutions can always be eliminated by perturbation. *Among the practical tests of CAO FBA Dry Run II (2009) there is no example for alternative auction prices.*

## 4.4 Sensitivity analysis of Market Spread Auction

In this section a Market Spread Auction, defined in Section 3.2.1, is considered. Assume that the bid and ask price of electricity at a given Zone $k$ changes by the same value $\Delta p$. Let the changed solution be marked by "′". The questions are

1. The *sensitivity of Auction Prices.* i.e.

$$\frac{\Delta AP(x,y)}{\Delta p} = \frac{AP'(x,y) - AP(x,y)}{\Delta p}, \text{ for all } x, y \in Zones, x \neq y.$$

2. The *sensitivity intervals.* I.e., the intervals, where

$$\frac{\Delta AP(x,y)}{\Delta p}$$

as well as the accepted capacities

$$d'_a(x,y,b)$$

for all $x, y \in Zones$, $x \neq y$ remain constant.

Let us illustrate the above by the following

**Example 4.1** *Consider a Market Spread Auction with PTDF matrix "Test 2 Daily Auction 7AB" in CAO FBA Dry Run II (2009) and Zone prices*

$$p_{bid}(CZ\_SK) = p_{ask}(CZ\_SK) = 43;$$

$$p_{bid}(PSEO) = p_{ask}(PSEO) = 43;$$

$$p_{bid}(MAVIR) = p_{ask}(MAVIR) = 54;$$

$$p_{bid}(ELES) = p_{ask}(ELES) = 34.$$

*Assume that*

$$p_{bid}(DE\_AT) = p_{ask}(DE\_AT).$$

*The Auction Prices depending on the price of Zone DE_AT are represented on Figure 1. The Awarded Capacities depending on the price of Zone DE_AT are depicted of Figure 2. It is easy to see that both types of sensitivity are constant on certain intervals. As it will be shown, the endpoints of those intervals correspond to the changes of optimal bases of the LP problem. This means that the changes of the two types of sensitivity have the same origin. This can be illustrated by the mixed chart of Figure 3, where the intervals are changing at the same points.*

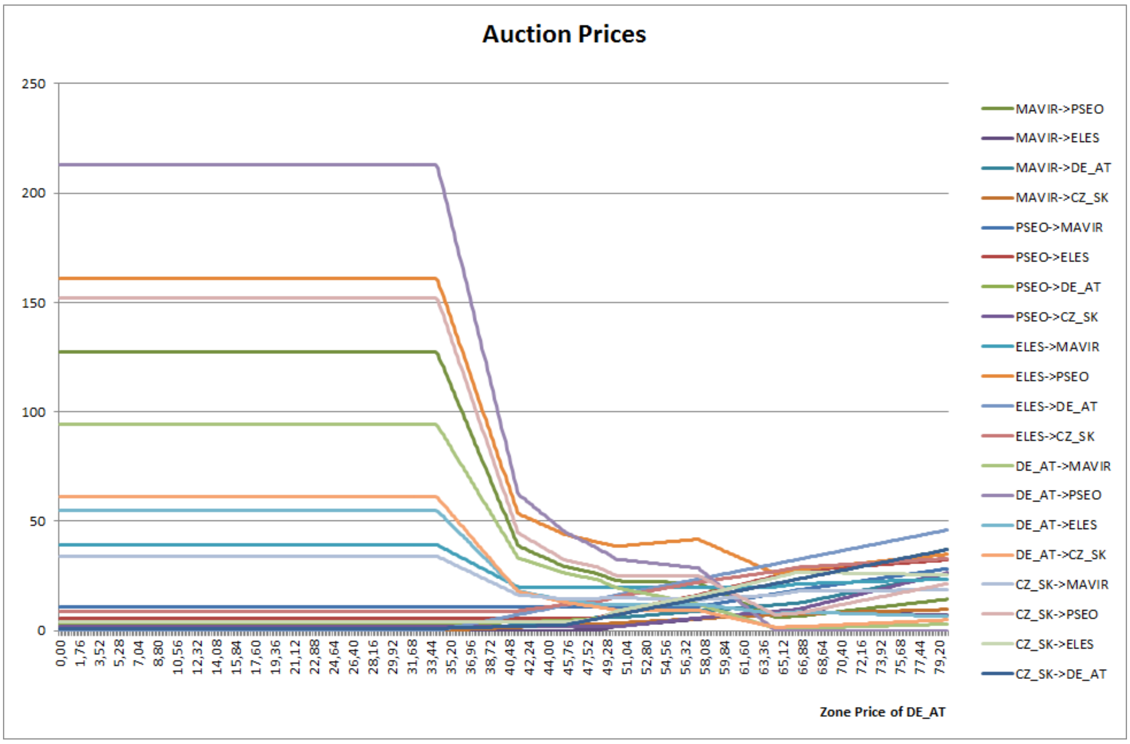


Figure 1: Auction Prices depending on the price of Zone DE_AT

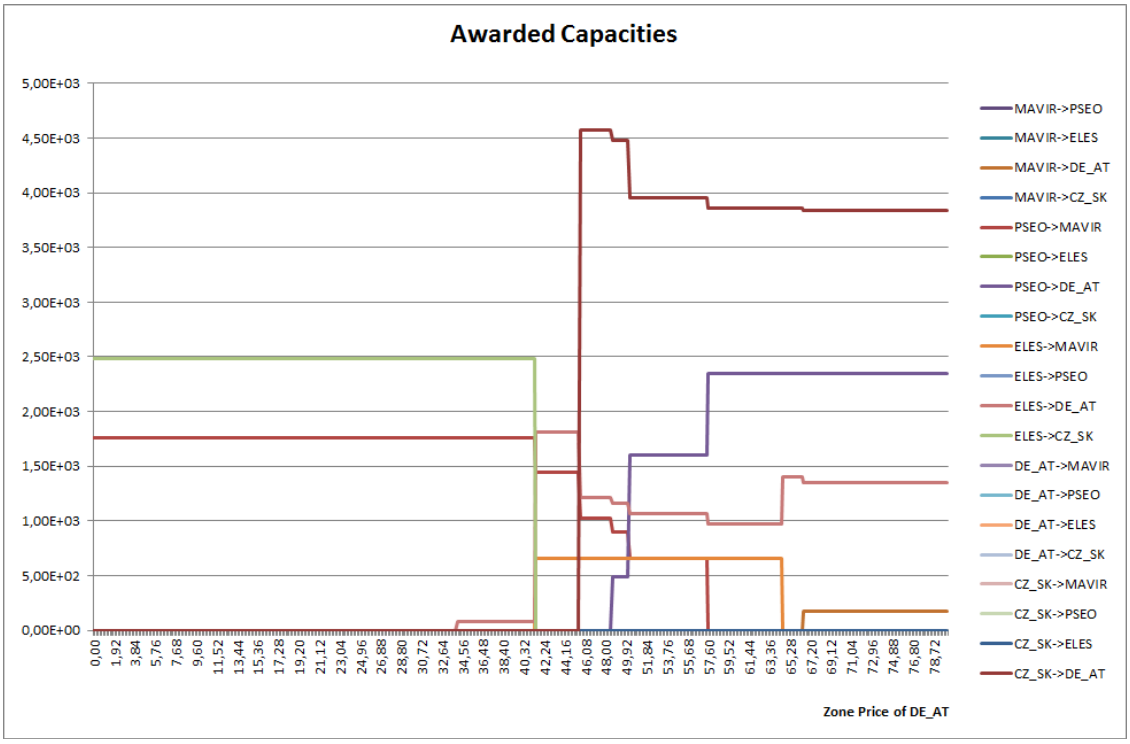


Figure 2: Awarded Capacities depending on the price of Zone DE_AT

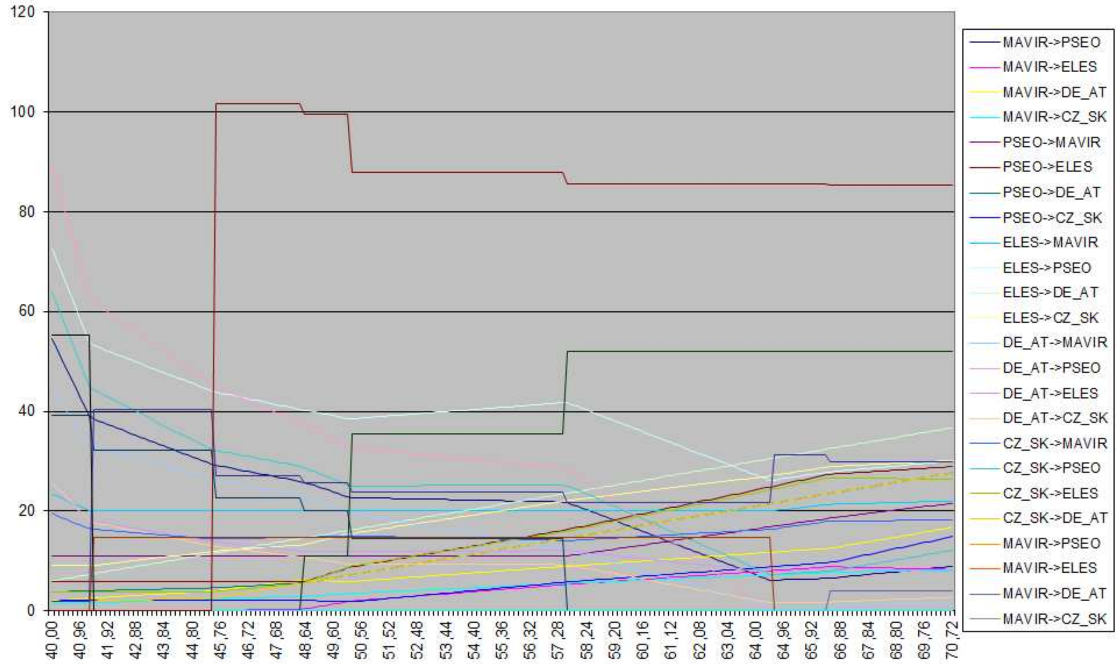


Figure 3: Endpoints of sensitivity intervals are at the changes of optimal bases

In order to calculate the endpoints of the sensitivity intervals a special sensitivity analysis should be applied because *more than one parameter of the LP is changing.* I.e., if the price of Zone $k$ changes by $\Delta p$ then the new bid prices will be

$$
\begin{aligned}
p'(k,y,b) &= p_{bid}(y) - (p_{ask}(k) + \Delta p) = p(k,y,b) - \Delta p, \\
p'(x,k,b) &= (p_{bid}(k) + \Delta p) - p_{ask}(x) = p(k,y,b) + \Delta p, \\
p'(x,y,b) &= p_{bid}(y) - p_{ask}(x) = p(x,y,b),
\end{aligned}
$$

where $x \neq y$, $x \neq k$, $y \neq k$.

Let us use the compact form of Market Spread Auction problem:

$$
\begin{aligned}
\boldsymbol{p}^T \boldsymbol{d} &\rightarrow \max \\
\text{subject to} & \\
A\boldsymbol{d} &\leq \boldsymbol{b} \qquad (6)\\
\boldsymbol{d} &\geq \boldsymbol{0},
\end{aligned}
$$

where constraints (6) are the network constraints (2), (3) of the auction LP problem. The bid constraints (4) are skipped, because the requested capacities are infinity.

Let $B$ be *the matrix of optimal basis* of the above problem.

The vector of changed bid prices can be written into the form:

$$\boldsymbol{p}' = \boldsymbol{p} + \Delta p \cdot \boldsymbol{u}, \qquad (7)$$

where

$$u(k,y,b) = -1,\ u(x,k,b) = +1,\ u(x,y,b) = 0,\ x \neq y,\ x \neq k,\ y \neq k.$$

Then the optimality conditions of $B$:

$$\boldsymbol{p}'_B{}^T B^{-1} A - \boldsymbol{p}'^T \geq \mathbf{0}^T \text{ and } \boldsymbol{p}'_B{}^T B^{-1} \geq \mathbf{0}^T$$

Substituting (7) the above inequalities can be written as

$$\underbrace{(\boldsymbol{p}_B{}^T B^{-1} A - \boldsymbol{p}^T)}_{\boldsymbol{w}_{opt}{}^T \geq \mathbf{0}^T} + \Delta p \cdot \underbrace{(\boldsymbol{u}_B{}^T B^{-1} A - \boldsymbol{u}^T)}_{\boldsymbol{w}_u{}^T} \geq \mathbf{0}^T$$

and

$$\underbrace{(\boldsymbol{p}_B{}^T B^{-1})}_{\boldsymbol{y}_{opt}{}^T \geq \mathbf{0}^T} + \Delta p \cdot \underbrace{(\boldsymbol{u}_B{}^T B^{-1})}_{\boldsymbol{y}_u{}^T} \geq \mathbf{0}^T$$

Hence, the sensitivity interval is $\Delta p \in (\Delta p_{min}, \Delta p_{max)}$ where

$$\Delta p_{min} = -\min\left(\min_{i\in I} \frac{w_{opt}^{(i)}}{\max(0, w_u^{(i)})},\ \min_{j\in J} \frac{y_{opt}^{(j)}}{\max(0, y_u^{(j)})}\right)$$

and

$$\Delta p_{max} = \min\left(\min_{i\in I} \frac{w_{opt}^{(i)}}{\max(0, -w_u^{(i)})},\ \min_{j\in J} \frac{y_{opt}^{(j)}}{\max(0, -y_u^{(j)})}\right),$$

$$|I| = |Zones| \cdot (|Zones| - 1),\ |J| = 2 \cdot |Lines|.$$

Remark, in case of simultaneous changes of the Zone prices the sensitivity analysis can be solved similarly. The difference is that instead of intervals, the sensitivity remains constant in case of $\Delta\boldsymbol{p}$ is within a certain polyhedron.

# 5 Theoretical social welfare and the real income of the TSOs

In the auction rules the objective function of the LP is calculated by the submitted bid prices while the bidders have to pay only the uniform auction prices of the source-sink pairs. This means on one hand, that the bidders can play somehow by the submitted prices. On the other hand, the income of the TSOs will be usually different to the value of the objective function. In this section this second phenomenon is investigated. I.e., the objective is to maximize

$$F = \sum_{x,y\in Zones,\ b\in Bids} p_b(x,y,b) \cdot d_a(x,y,b),$$

however, the real revenue of CAO (TSOs) is

$$R = \sum_{x,y \in Zones,\ b \in Bids} AP(x,y) \cdot d_a(x,y,b). \quad (8)$$

In order to illustrate the arising questions consider the PTDF matrix:

| Technical Parameters for Test | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Parameters (1008) | | | | | | | | |
| Critical Branch | Case | Source | Sink | TMF | AMF+ | AMF- | MAVIR->PSEO | MAVIR->ELES |
| LINE_00001 | n-0 | APG | APG | 305 | **30** | 166 | **1** | **0** |
| LINE_00001 | n-1 LINE_00002 | APG | APG | 305 | **20** | 126,9 | **1** | **1** |

(9)

**Example 5.1** *Consider the following bids and results:*

| Product | Source | Sink | Requested Capacity [MW] | Bid Price [EUR/MWh] | Source | Sink | Awarded Capacity [MW] | Auction Price [EUR/MWh] |
|---|---|---|---|---|---|---|---|---|
| H01 | MAVIR | PSEO | 19 | 10 | MAVIR | PSEO | 19 | 1 |
| H01 | MAVIR | PSEO | 2 | 1 | MAVIR | PSEO | 1 | 1 |

*Here, the value of the theoretical social welfare is:*

$$F = 10 \cdot 19 + 1 \cdot 1 = 191,$$

*however, the real income is*

$$R = 1 \cdot 19 + 1 \cdot 1 = 20.$$

The solution of the contradiction of Example 5.1 can be twofold:

1. instead of uniform auction prices, the submitted bid prices should be paid,
2. or, if the uniform auction prices have to be used it seems obvious to set the auction prices following the objective to maximize the overall income.

In the following the second case will be discussed: *instead of the auction LP the capacity allocation based on Auction Prices of Section 2.5 is applied and the objective is the maximization of the overall revenue (8).*

E.g., in case of Example 5.1, the income maximization auction price is 10 (see the last two columns below):

| Product | Source | Sink | Requested Capacity [MW] | Bid Price [EUR/MWh] | Awarded Capacity [MW] | Auction Price [EUR/MWh] | Awarded Capacity [MW] | Auction Price [EUR/MWh] |
|---|---|---|---|---|---|---|---|---|
| H01 | MAVIR | PSEO | 19 | 10 | 19 | 1 | 19 | 10 |
| H01 | MAVIR | PSEO | 2 | 1 | 1 | 1 | 0 | 10 |
| | | | | | Revenue | 20 | Revenue | 190 |

In this case the *real* revenue is much greater than the original 20. However, one can tell that there is remaining free capacity and corresponding demand in the system, hence, the allocation cannot be effective.

The next example shows that there are situations where the allocation is effective in the above sense, and results greater income than the revenue corresponding to the auction LP allocation.

**Example 5.2** *Consider the PTDF matrix (9) and the following bids and results:*

| Product | Source | Sink | Requested Capacity [MW] | Bid Price [EUR/MWh] | Awarded Capacity [MW] | Auction Price [EUR/MWh] | Awarded Capacity [MW] | Auction Price [EUR/MWh] |
|---|---|---|---|---|---|---|---|---|
| H01 | MAVIR | PSEO | 19 | 10 | 19 | 1 | 19 | 10 |
| H01 | MAVIR | PSEO | 2 | 1 | 1 | 1 | 0 | 10 |
| H01 | MAVIR | ELES | 1 | 0 | 0 | 0 | 1 | 0 |
| | | | | | Revenue | 20 | Revenue | 190 |

*The* $6^{th}$ *and* $7^{th}$ *columns show the results of auction LP while the last two columns represents the solution of real income maximization.*

## 5.1 Global optimization problem of maximizing the total revenue

In the following, the problem to find the auction prices that maximize the total revenue is formulated. It is a non-concave, nonlinear programming problem.

### 5.1.1 Objective function

The objective function (for maximization):

$$R = \sum_{x,y \in Zones,\ b \in Bids} AP(x,y) \cdot d_a(x,y,b).$$

### 5.1.2 Nonlinear constraints

No allocated capacity under the auction price ("awarded bid constraints"):

$$d_a(x,y,b)(AP(x,y) - p_b(x,y,b)) \leq 0 \quad x,y \in Zones,\ b \in Bids \tag{10}$$

All requested capacities above the auction price are fully satisfied ("full bid constraints"):

$$(d_b(x,y,b) - d_a(x,y,b))(p_b(x,y,b) - AP(x,y)) \leq 0 \quad x,y \in Zones,\ b \in Bids \tag{11}$$

### 5.1.3 Linear constraints

The linear constraints are the same as (2), (3), (4), (5) in the auction LP problem.

### 5.1.4 Variables

Beside the allocation variables:

$$d_a(x, y, b) \;\; x, y \in Zones, \; b \in Bids,$$

the auction prices are represented by further variables

$$AP(x, y) \;\; x, y \in Zones.$$

## 5.2 Illustrative example from the CAO FBA Dry Run

The problem of Section 5.1 cannot be solved by most of the regular global optimization techniques. However, a solution method could be found by the exploitation of the special properties of the problem. It is based on the simple fact, that *the maximum is attained at auction prices that equal to one of the submitted bid prices.* The detailed development of the method is out of the scope of this paper and will be the part of future research.

Here, only one illustrative example is presented in order to show, that based on the capacity allocation of the original linear programming auction method, higher income can be realized by setting the auction prices independently from the shadow prices of the lines.

**Example 5.3** *Consider Hour* 10 *in Test Daily 2 Auction 7A of CAO FBA Dry Run II (2009). Table 1 shows the auction prices of the LP method compared to the highest auction prices resulting the same allocation based on the rules of Section 2.5. The values of the real incomes are also presented.*

*Remark that higher real income can happen in case of other capacity allocations (however, in order to find this a special solver is needed).*

## 5.3 Experiences

Examples 5.1-5.2 show that

- the LP auction method of CAO Rules (2011) does not maximize the real income,
- the maximal real income can be attained at other allocation and auction prices.

Certainly, the maximal real income is not equivalent to maximal social welfare or most effective capacity allocation. However, objective function (1) does not really reflect the social welfare either.

Example 5.3 represents that

- still in case of the same capacity allocation, the use of auction prices independent from the shadow prices can result higher income.

| Aggregated Auction Results | | | | | | |
|---|---|---|---|---|---|---|
| Product | Source | Sink | Requested Capacity [MW] | Awarded Capacity [MW] | Auction Price (based on shadow prices) [EUR/MWh] | Auction Price (independent from shadow prices) [EUR/MWh] |
| H10 | APG | CEPS | 101 | 0 | 111,45 | 1 |
| H10 | APG | ELES | 90 | 0 | 200 | 51 |
| H10 | APG | MAVIR | 156 | 0 | 142,32 | 50 |
| H10 | CEPS | APG | 99 | 99 | 0 | 0 |
| H10 | CEPS | ELES | 240 | 200 | 103,06 | 200 |
| H10 | CEPS | MAVIR | 70 | 10 | 32,53 | 50 |
| H10 | CEPS | TPS | 135 | 135 | 0 | 0 |
| H10 | CEPS | VET | 135 | 135 | 0 | 0 |
| H10 | ELES | APG | 87 | 87 | 0 | 0,5 |
| H10 | ELES | CEPS | 200 | 200 | 14,25 | 200 |
| H10 | ELES | MAVIR | 270 | 200 | 47,05 | 200 |
| H10 | ELES | PSEO | 200 | 0 | 220,45 | 201 |
| H10 | ELES | TPS | 200 | 200 | 0 | 200 |
| H10 | ELES | VET | 200 | 200 | 0 | 200 |
| H10 | MAVIR | APG | 105 | 105 | 0 | 0 |
| H10 | MAVIR | ELES | 250 | 200 | 104,73 | 200 |
| H10 | MAVIR | PSEO | 100 | 58 | 200 | 200 |
| H10 | MAVIR | SEPS | 180 | 50 | 1,66 | 3 |
| H10 | PSEO | ELES | 200 | 200 | 78,96 | 200 |
| H10 | PSEO | SEPS | 10 | 10 | 0 | 0,14 |
| H10 | PSEO | VET | 35 | 35 | 0 | 30 |
| H10 | SEPS | ELES | 20 | 0 | 103,06 | 51 |
| H10 | SEPS | MAVIR | 80 | 10 | 32,53 | 50 |
| H10 | TPS | CEPS | 315 | 0 | 111,45 | 2 |
| H10 | TPS | ELES | 200 | 0 | 200 | 200 |
| H10 | VET | CEPS | 115 | 0 | 111,45 | 1 |
| H10 | VET | ELES | 200 | 0 | 200 | 200 |
| | | | | **Real Income:** | ***81943,6*** | ***293844,9*** |

Table 1: Auction prices based on shadow prices vs. based on bid prices with the same capacity allocation

One reason for the use of auction prices based on shadow prices is that shadow prices play important rule in some possible distribution methods of the income, see Leuthod and Todem (2007). However, the shadow prices are calculated on the base of the submitted prices, and does not reflect the auction prices.

The above experiences have exploited some important properties of the auction method and can help to develop better trading strategies.

# 6 Conclusion

The paper represents an introduction of the mathematical part of the FBA auction rules with a more detailed discussion than the rules of CAO (2011). This, completed with the examples and numerical experiences, can make the paper a useful guide to learn about the FBA auction mechanism.

From the mathematical point of view, on one hand, the description of sensitivity analysis of zone prices can help in its implementation to programmers and/or business analysts. On the other hand, the formalization of the revenue maximization problem, beside its illustrative role, can be the first step to develop better bidding strategies.

Remark that all methods and experiences of the paper can be applied to the present NTC allocation mechanism described in Annex 6 of the rules of CAO (2011) as well.

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