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RRR 1-2012 **ON THE REPETITION-FREE SUBTRACTION GAMES AND VILE–DOPEY INTEGERS**, Vladimir Gurvich

Given two integers $n \geq 0$ and $k \geq 3$, two players alternate turns taking stones from a pile of n stones. By one move a player is allowed to take any number of stones $k' \in \{1, \ldots, k-1\}$. In particular, it is forbidden to pass.

Furthermore, it is not allowed to take the same number of stones as the opponent by the previous move. The player who takes the last stone wins in the normal version of the game and loses in the misère version; (s)he also wins, in both cases, when the opponent has no legal move, that is, after the move from 2 to 1.

An integer $k \in Z_{\geq 0}$ is called *vile* if the maximum power of 2 that is still a divisor of k is even, or in other words, if the binary representation of k ends with an even number of zeros; otherwise, if this number is odd, k is called *dopey*.

In this short note, we will solve (both the normal and misère versions of) the game when k is vile and obtain partial results when k is dopey.

In the normal version, the set of P-positions is an arithmetic progression ak, where $a = 0, 1, \ldots$, if k is vile. When k is dopey, for the P-positions $n_0, n_1, \ldots$, we have $n_0 = 0, n_1 = k+1$, and $n_{i+1} - n_i$ is either k or $k+1$ for all $i \in \mathbf{Z}_{\geq 0}$. Yet, in the latter case, it seems not easy to choose among k and $k+1$. We conjecture that the differences $n_{i+1} - n_i$ between the successive P-positions form a periodical sequence.

In the misère version of the game, the P-positions are shifted by $+1$ with respect to the P-positions of the corresponding normal version.

Keywords: vile and dopey numbers, impartial games, subtraction games, games of memory one, normal and misère versions

RRR 2-2012 **SCENARIO DECOMPOSITION OF RISK-AVERSE TWO STAGE STOCHASTIC PROGRAMMING PROBLEMS**, Ricardo A. Collado, Dávid Papp, Andrzej Ruszczyński

We develop methods that exploit the dual representation of coherent risk measures to produce efficient algorithms that can solve nonlinear risk-averse stochastic problems by solving a series of linear sub problems. Our main theoretical tool is the development of duality theory for risk-averse two stage stochastic problems. The basic model that we consider is:

$$\min_{x \in X} \ \rho_1 \left(c^\top x + \rho_2 \left[\mathcal{Q}(x, \xi) \right] \right), \tag{1}$$

where ρ_1, ρ_2 are coherent risk measures and $\mathcal{Q}(x, \xi)$ is the optimal value of a second stage linear problem with a random vector $\xi \in \Omega$. We show methods that solve (1) when the underlying probability space Ω is finite.

We also develop a new type of bundle method, called the *truncated bundle method*, which exploits the topological properties of the domain of the functions to obtain better running time than the classical bundle method. This algorithm solves general two stage stochastic programs and has value and applicability on its own. As a testbed for our methods we consider a problem in manufacturing and transportation and implement in

AMPL all of our methods for this problem. The numerical results from problems with hundreds of first stage scenarios as well as hundreds of second stage scenarios are compared across all the methods developed.

RRR 3-2012 **EUCLIDEAN TRAVELING SALESMAN ON ORIENTABLE SURFACES**, Ricardo A. Collado

Following closely Arora's technique in [1], we obtain a randomized PTAS to the traveling salesman problem on Euclidean graphs embedded on orientable surfaces.

RRR 4-2012 **NETWORK INTERDICTION – MODELS, APPLICATIONS, UNEXPLORED DIRECTIONS**, Ricardo A. Collado, Dávid Papp

A large variety of models have been proposed for different interdiction problems. These include combinatorial optimization, stochastic programming, and game theoretic approaches. In this note we attempt to collect the most researched models, match them with applications, and summarize the latest algorithmic and complexity results. In Section 1 we introduce the basic ideas and define the necessary terms. Section 2 is concerned with various models that have been proposed in the literature, as well as with algorithms and complexity bounds. In Section 3 we examine which models would be appropriate for which applications. Finally, in Section 4 we outline two promising research directions for the future.

RRR 5-2012 **STOCHASTIC NETWORK INTERDICTION**, Ricardo A. Collado

We introduce the network interdiction problem and show a reformulation of it in the form of a two-stage stochastic problem. We also use the theory of coherent risk measures to develop a risk-averse variant of the network interdiction problem.

RRR 6-2012 **SEPARABLE CONCAVE OPTIMIZATION APPROXIMATELY EQUALS PIECEWISE-LINEAR OPTIMIZATION**, Thomas L. Magnanti, Dan Stratila

We study the problem of minimizing a nonnegative separable concave function over a compact feasible set. We approximate this problem to within a factor of $1 + \epsilon$ by a piecewise-linear minimization problem over the same feasible set. Our main result is that when the feasible set is a polyhedron, the number of resulting pieces is polynomial in the input size of the polyhedron and linear in $1/\epsilon$. For many practical concave cost problems, the resulting piecewise-linear cost problem can be formulated as a well-studied discrete optimization problem. As a result, a variety of polynomial-time exact algorithms, approximation algorithms, and polynomial-time heuristics for discrete optimization problems immediately yield fully polynomial-time approximation schemes, approximation algorithms, and polynomial-time heuristics for the corresponding concave cost problems.

We illustrate our approach on two problems. For the concave cost multicommodity flow problem, we devise a new heuristic and study its performance using computational experiments. We are able to approximately solve significantly larger test instances than previously possible, and obtain solutions on average within 4.27% of optimality. For the concave cost facility location problem, we obtain a new $1.4991 + \epsilon$ approximation algorithm.

RRR 7-2012 **USING BOXES AND PROXIMITY TO CLASSIFY DATA INTO SEVERAL CATEGORIES**, Martin Anthony, Joel Ratsaby

The use of boxes for pattern classification has been widespread and is a fairly natural way in which to partition data into different classes or categories. In this paper we consider multi-category classifiers which are based on unions of boxes. The classification method studied may be described as follows: find boxes such that all points in the region enclosed by each box are assumed to belong to the same category, and then classify remaining points by considering their distances to these boxes, assigning to a point the category of the nearest box. This extends the simple method of classifying by unions of boxes by incorporating a natural way (based on proximity) of classifying points outside the boxes. We analyse the generalization accuracy of such classifiers and we obtain generalization error bounds that depend on a measure of how definitive is the classification of training points.

RRR 8-2012 **FLOW-BASED CAPACITY ALLOCATION IN THE CEE REGION: SENSITIVITY ANALYSIS, MULTIPLE OPTIMA, REAL INCOME**, Ácos Füzi, Gergely Mádi-Nagy

The paper introduces the mechanism of the Flow-based Capacity Allocation (FBA) method of the Central-Eastern Europe (CEE) Region. It reveals the properties of the underlying linear programming problem and discusses their practical consequences. A non-standard sensitivity analysis method of the market spread auction is developed. Finally, the objective and the real overall income of the auction are compared by the aid of a global optimization problem. Several numerical examples and results of practical test problems are presented.

Keywords: Linear programming, Sensitivity analysis, Global optimization

RRR 9-2012 **SINGLE COMMODITY STOCHASTIC NETWORK DESIGN UNDER PROBABILISTIC CONSTRAINT WITH DISCRETE RANDOM VARIABLES**, András Prékopa, Merve Unuvar

Single commodity networks are considered, where demands at the nodes are random. The problem is to find minimum cost optimal capacities at the nodes and arcs subject to the constraint that all demands should be met on a prescribed probability level (reliability constraint) and some constraints on the capacities should be satisfied. The reliability constraint is formulated in terms of the Gale–Hoffman feasibility inequalities but their number is reduced by elimination technique. The concept of a p-efficient point is used in a smart way to convert and then relax the problem into an LP. Two solution techniques are presented depending on if all p-efficient points are known or are simultaneously generated with the solution of the LP. The joint distribution of the demands is used to obtain the p-efficient points for all non-eliminated stochastic inequalities and the solution of a multiple choice knapsack problem is used to generate new p-efficient points. The model can be applied to planning in interconnected power systems, flood control networks, design of shelter and road capacitates in evacuation, parking lot capacities, financial networks, etc. Numerical examples are presented.

RRR 10-2012 **STRONGLY POLYNOMIAL PRIMAL-DUAL ALGORITHMS FOR CONCAVE COST COMBINATORIAL OPTIMIZATION PROBLEMS**, Thomas L. Magnanti, Dan Stratila

We introduce an algorithm design technique for a class of combinatorial optimization problems with concave costs. This technique yields a strongly polynomial primal-dual algorithm for a concave cost problem whenever such an algorithm exists for the fixed-charge counterpart of the problem. For many practical concave cost problems, the fixed-charge counterpart is a well-studied combinatorial optimization problem. Our technique preserves constant factor approximation ratios, as well as ratios that depend only on certain problem parameters, and exact algorithms yield exact algorithms.

Using our technique, we obtain a new 1.61-approximation algorithm for the concave cost facility location problem. For inventory problems, we obtain a new exact algorithm for the economic lot-sizing problem with general concave ordering costs, and a 4-approximation algorithm for the joint replenishment problem with general concave individual ordering costs.

RRR 11-2012 **CHESS-LIKE GAMES MAY HAVE NO UNIFORM NASH EQUILIBRIA EVEN IN MIXED STRATEGIES**, Endre Boros, Vladimir Gurvich, Svetlana Solovieva, Emre Yamangil

Recently, it was shown that Chess-like games may have no uniform (subgame perfect) Nash equilibria in pure positional strategies. Moreover, Nash equilibria may fail to exist already in two-person games in which all infinite plays are equivalent and ranked as the worst outcome by both players. In this paper, we extend this negative result further, providing examples that are uniform Nash equilibria free even in the (independently) mixed strategies. Given (independently) mixed strategies of all players, we consider two definitions of the corresponding mixed play and effective payoff: given by the Markov or a priori realization.

Keywords: pure, mixed, and independently mixed strategies; uniform (or subgame perfect) Nash equilibrium; Markov and a priori realizations.

RRR 12-2012 **AN IMPROVED CUTTING PLANE METHOD FOR THE SOLUTION OF PROBABILISTIC CONSTRAINED PROBLEM WITH DISCRETE RANDOM VARIABLES**, Emre Yamangil, András Prékopa

We consider a probabilistic constrained stochastic programming problem with discrete random variables. Two methods, a cutting plane and a column generation method have already been developed for the solution of the problem. In this paper we blend them together and obtain a method that is faster than the earlier ones. We also present a refined algorithm for the generation of p-level efficient points of a discrete distribution.

RRR 13-2012 **A POTENTIAL REDUCTION ALGORITHM FOR TWO-PERSON ZERO-SUM LIMITING AVERAGE PAYOFF STOCHASTIC GAMES**, Endre Boros, Khaled Elbassioni, Vladimir Gurvich, Kazuhisa Makino

We suggest a new algorithm for two-person zero-sum undiscounted stochastic games focusing on stationary strategies. Given a positive real ϵ, let us call a stochastic game ϵ-ergodic, if its values from any two initial points differ by at most ϵ. The proposed new

algorithm outputs for every $\epsilon > 0$ in finite time either a pair of stationary strategies for the two players guaranteeing that the values from any initial points are within an ϵ range, or identifies two initial points u and v and corresponding stationary strategies for the players proving that the game values starting from u and v are at least $\epsilon/24$ apart. In particular, the above result shows that if a stochastic game is ϵ-ergodic, then there are stationary strategies for the players proving 24ϵ-ergodicity. This result strengthens a result by Vrieze (1980) claiming that if a stochastic game is 0-ergodic, then there are ϵ-optimal stationary strategies for every $\epsilon > 0$.

The suggested algorithm extends the approach recently introduced for stochastic games with perfect information, and is based on the classical potential transformation technique that changes the range of local values at all positions without changing the normal form of the game.

RRR 14-2012 **SOLUTION OF AN OPTIMAL RESERVOIR CAPACITY PROBLEM UNDER PROBABILISTIC CONSTRAINTS**, Merve Unuvar, Eren Erman Ozguven, András Prékopa

We formulate and solve probabilistic constrained stochastic programming problems, where we prescribe lower and upper bounds for k-out-of-n and consecutive-k-out-of-n reliabilities in the form of probabilistic constraints. The practical problem we are dealing with is mentioned in a paper by Prékopa, Szántai, Zsuffa (2010), where four optimization problems are formulated in connection with water resource problem. However, solutions are offered for three of them and it is the fourth one which is the starting point of our paper. The problem is to determine the optimal capacity of a water release, or pump station, to satisfy the demand for irrigation, i.e., a reliability constraint where the reliability is one of the above-mentioned type. For the non-consecutive type reliability problem, normal and gamma distributions are used for inflow and demand values, respectively. By using the property of standard gamma distribution, reliability constraint is written up as an equation which can then be solved by simulation. For the k-consecutive case, different probability bounds are used in order to solve the reliability equation. To create lower and upper bounds for the reliability constraint, the discrete binomial moment problem is used, which are indeed LP's are constructed. S_1, S_2, S_3 sharp lower bounds, Hunter's upper bound and Cherry tree upper bound are calculated to obtain desired probability level for the reliability constraint. Bi-section algorithm is later applied to find the optimal water reservoir capacity level.

Keywords: Probabilistic modeling, Optimization, Bounding, Bi-section algorithm

RRR 15-2012 **ON CANONICAL FORMS FOR TWO-PERSON ZERO-SUM LIMITING AVERAGE PAYOFF STOCHASTIC GAMES**, Endre Boros, Khaled Elbassioni, Vladimir Gurvich, Kazuhisa Makino

We consider two-person zero-sum mean payoff undiscounted stochastic games. We give a sufficient condition for the existence of a saddle point in uniformly optimal stationary strategies. Namely, we obtain sufficient conditions that enable us to bring the game, by applying *potential transformations* to a *canonical form* in which *locally* optimal strategies are *globally* optimal, and hence the value for every initial position and the optimal strategies of both players can be obtained by playing the local game at each state. We show that this condition is satisfied by the class of *additive transition games*, that is, the special case when the transitions at each state can be decomposed into two parts, each

controlled completely by one of the two players.

An important special case of additive games is the so-called *BWR-games* which are played by two players on a directed graph with positions of three types: Black, White and Random. We give an independent proof for the existence of canonical form in such games, and use this to derive the existence of canonical form (and hence of a saddle point in uniformly optimal stationary strategies) in a wide class of games, which includes *stochastic games with perfect information*, *switching controller games* and *additive rewards, additive transition games*.

RRR 16-2012 **ON RANK-PROFILES OF STABLE MATCHINGS**, Endre Boros, Liliya Fedzhora, Vladimir Gurvich, Steven Jaslar

We study the quality of stable matchings from the individuals' viewpoint. To each matchings we associate its *rank-profile* describing the individuals' satisfaction with the matching. We provide a complete and computationally efficient characterization of the rank-profiles that can arise from men-optimal, women-optimal, and arbitrary stable matchings. We also study *uniquely stable* rank-profiles, that is, for which there exists a stable matching problem that has only one stable matching and this matching has this particular rank-profile. We give some necessary and some sufficient conditions for unique stability and show that characterizations of men-optimal and women-optimal rank-profiles is reduced to the characterization of uniquely stable rank-profiles. Our characterizations imply that the set of all stable rank-profiles is monotone, unlike the sets of men-optimal, women-optimal, and uniquely stable rank-profiles. We also show that both stable and uniquely stable matchings may be highly disadvantageous for all participating individuals, simultaneously. Namely, we show that there are stable and even uniquely stable rank-profiles in which no individual gets a better partner than his/her middle choice. Furthermore, this result is sharp, since a stable matching in which all individuals get a partner ranked below their middle choice cannot be stable. Finally, we demonstrate an "instability of stable matchings" from a quality point of view.

Key Words: stable matching, preference list, rank-profile, Gale-Shapley algorithm and theorem

RRR 17-2012 **COMPLEXITY OF BILEVEL COHERENT RISK PROGRAMMING**, Jonathan Eckstein

This paper considers a bilevel programming approach to applying coherent risk measures to extended two-stage stochastic programming problems. This formulation technique avoids the time-inconsistency issues plaguing naive models and the incomposability issues which cause time-consistent formulations to have complicated, hard-to-explain objective functions. Unfortunately, the analysis here shows that such bilevel formulations, when using the standard mean-semideviation and average-value-at-risk measures, are $\mathcal{NP}$-hard. While not necessarily indicating that solution of such models is impractical, these results suggest that it may prove difficult and will likely require some kind of implicit enumeration method.

RRR 18-2012 **ON TAME, PET, MISERABLE, AND STRONGLY MISERABLE IMPARTIAL GAMES**, Vladimir Gurvich

We consider tame impartial games and develop the Sprague-Grundy theory for misère

playing the sum of such games that looks simpler than the classical theory suggested by Conway in 1976, which is based on the concept of genus.

An impartial game is called *pet* if the sets of P-positions of its normal and misère versions are disjoint. We provide several equivalent characterizations and show that the pet games form a proper subfamily of the tame games.

For example, NIM, Wythoff's NIM, and game Euclid are tame but not pet, while all subtraction games, the Fraenkel extension NIM(a) of Wythoff's NIM(1), as well as its further extension NIM(a, b) recently suggested by the author are pet games whenever $a > 1$. Thus, very many important impartial games are tame or pet.

Keywords: combinatorial, impartial, tame, pet, miserable, and strongly miserable games; normal and misère play, Sprague-Grundy function, NIM, Wythoff's NIM, Fraenkel's NIM, game Euclid, game Mark, subtraction games; swap, tame, and critical positions.

RRR 19-2012 **LEARNING ON FINITE METRIC SPACES**, Martin Anthony, Joel Ratsaby

In "M. Anthony and J. Ratsaby. Maximal width lerning of binary functions. *Theoretical Computer Science*, 411:138–147, 2010", the notion of *sample width* for binary classifiers mapping from the real line was introduced, and it was shown that the performance of such classifiers could be quantified in terms of this quantity. This paper considers how to generalize the notion of sample width so that we can apply it where the classifiers map from some finite metric space. By relating the learning problem to one involving the domination numbers of certain graphs, we obtain generalization error bounds that depend on the sample width and on certain measures of 'density' of the underlying metric space. We also discuss how to employ a greedy set-covering heuristic to bound generalization error.

RRR 20-2012 **TRIANGLE INEQUALITY FOR RESISTANCES**, Vladimir Gurvich

Given an electrical circuit each edge e of which is an isotropic conductor with a monomial conductivity function $y_e^* = y_e^r/\mu_e^s$. In this formula, y_e is the potential difference and y_e^* current in e, while μ_e is the resistance of e, while r and s are two strictly positive real parameters common for all edges.

In 1987, Gvishiani and Gurvich proved that, for every two nodes a, b of the circuit, the effective resistance $\mu_{a,b}$ is well-defined and for every three ordered nodes a, b, c the inequality $\mu_{a,b}^{s/r} \leq \mu_{a,c}^{s/r} + \mu_{c,b}^{s/r}$ holds. It obviously implies the standard triangle inequality $\mu_{a,b} \leq \mu_{a,c} + \mu_{c,b}$ when $s \geq r$. In 1992, the same authors showed that the equality holds if and only if c belongs to every path between a and b.

Recently, Pavel Chebotarev has found several earlier works of 1967 by Gerald Subak-Sharpe in which the inequality was shown for the case $s = r = 1$. Furthermore, it was rediscovered in 1993 by Douglas J. Klein and Milan Randić.

In this report we provide the story of the considered inequality with more details.

Key words: distance, metric, ultrametric; potential, voltage, current, Ohm law, Joule-Lenz heat, Maxwell's minimum energy dissipation principle, pressure, flow, maximum flow, minimum cut, shortest path, bottleneck path.

RRR 21-2012 **CLOSED LOOP LAYOUT**, Sadegh Niroomand, Béla Vizvári

In layout problem of manufacturing cells, rectangular cells to be positioned without overlapping. The objective is to minimize the total transportation cost. The types of layouts are categorized according to the shape of the transportation system's track. In the case of a closed loop layout, the track has a rectangular shape. A common difficulty of all layout problems is the manner in which distances are measured. A frequently used approximation is the Manhattan distance. However, it is significantly shorter than the real distance in many cases. Both meta-heuristics and exact models suggested by earlier studies use the Manhattan distance. In this paper, a new mathematical model is suggested for the closed loop layout with exact distances. Many feasible solutions are generated for benchmark problems that are competitive with the solutions provided by meta-heuristics.

RRR 22-2012 **ON THE GENERALIZATION OF THE MDS METHOD**, Sadegh Niroomand, Szabolcs Takács, Béla Vizvári

The Multi-Dimensional Scaling (MDS) method is used in statistics to detect hidden interrelations among multi-dimensional data and it has a wide range of applications. The method's input is a matrix that describes the similarity/dissimilarity among objects of unknown dimension. The objects are generally reconstructed as points of a lower dimensional space to reveal the geometric configuration of the objects. The original MDS method uses Euclidean distance, for measuring both the distance of the reconstructed points and the bias of the reconstructed distances from the original similarity values. In this paper, these distances are distinguished, and distances other than Euclidean are also used, generalizing the MDS method. Two different distances may be used for the two different purposes. Therefore the instances of the generalized MDS model are denoted as (l_p, l_q) model, where the first distance is the type of distance of the reconstructed points and the second one measures the bias of the reconstructed distances and the similarity values. In the case of l_1 and l_∞ distances mixed-integer programming models are provided. The computational experiences show that the generalized model can catch the key properties of the original configuration, if any exist.
Keywords: Multidimensional Scaling, Mixed Integer Linear Programming, Optimization

RRR 23-2012 **IMPROVED BOUNDS ON THE EXISTENCE OF A FEASIBLE FLOW IN A STOCHASTIC TRANSPORTATION NETWORK**, Merve Unuvar, Ozlem Cavus, András Prékopa

A. Prékopa and E. Boros published a paper "On the Existence of a Feasible Flow in a Stochastic Transportation Network" that appeared in Operations Research 39(1991), 119-129. In that paper the authors gave a powerful technique to evaluate transportation system reliability. At the nodes production takes place and the products are shipped on the arcs. Interconnected power systems are the primary examples for the application of the method. Reliability means that in case of efficiency in the electricity production, the power system can assist each other so that all demand can be met. The mentioned reliability calculation method uses probability bounding techniques developed by the same authors. However, there are some other bounds based on individual probability calculations which can be used to compute more efficient bounds. The proposed project aims to improve on the reliability calculation method, by using other and more recent

probability bounding techniques. These improved bounds; hunter bound and cherry tree bounds are calculated based on individual probabilities.

RRR 24-2012 **DISCOUNTED APPROXIMATIONS OF UNDISCOUNTED STOCHASTIC GAMES AND MARKOV DECISION PROCESSES ARE ALREADY POOR IN THE ALMOST DETERMINISTIC CASE**, Endre Boros, Khaled Elbassioni, Vladimir Gurvich, Kazuhisa Makino

It is shown that the discount factor needed to solve an undiscounted mean payoff stochastic game to optimality is exponentially close to 1, even in one-player games with a single random node and polynomially bounded rewards and transition probabilities. On the other hand, for the class of the so-called irreducible games with perfect information and a constant number of random nodes, we obtain a pseudo polynomial algorithm using discounts.

Keywords: discounted and undiscounted stochastic games and Markov decision processes, Black, White, and Random positions, Cesaro and Abel sums

RRR 25-2012 **PROPERTIES AND CALCULATION OF MULTIVARIATE RISK MEASURES: MVAR AND MCVAR**, Jinwook Lee, András Prékopa

A recent paper by Prékopa (2012) presented results in connection with Multivariate Value-at-Risk (MVaR) that has been known for some time under the name of p-quantile or p-Level Efficient Point (pLEP) and introduced a new multivariate risk measure, called Multivariate Conditional Value-at-Risk (MCVaR). The purpose of this paper is to further develop the theory and methodology of MVaR and MCVaR. This includes new methods to numerically calculate MCVaR, for both continuous and discrete distributions. Numerical examples with recent financial market data are presented.

Keywords: Multivariate risk measure; Multivariate Value-at-Risk (MVaR); Multivariate Conditional Value-at-Risk (MCVaR); Multivariate quantile function; Multivariate stochastic order; Projection of MVaR; Stochastically dependent structure; Corporate M&A (Mergers and Acquisitions) deals; Risk of correlated assets; Low-correlation investment

RRR 26-2012 **ON THE EXISTENCE OF NASH EQUILIBRIA IN PURE STATIONARY STRATEGIES FOR THE N-PERSON POSITIONAL GAMES WITH PERFECT INFORMATION, NO MOVES OF CHANCE, AND MEAN OR TOTAL EFFECTIVE COST**, Vladimir Gurvich, Vladimir Oudalov

We study existence of Nash equilibria (NE) in pure stationary strategies in n-person positional games with no moves of chance, with perfect information, and with the mean or total effective cost function.

We construct a NE-free three-person game with positive local costs, disproving the conjecture suggested by Boros and Gurvich in Math. Soc. Sci. 46 (2003) 207-241.

Still, the following four problems remain open:

Whether NE exist in all *two*-person games with total effective costs such that (I) all local costs are strictly positive or (II) without directed cycles of the cost zero?

If NE exist in all n-person games with the terminal (transition-free) cost functions, provided all directed cycles form a unique outcome c and (III) assuming that c is worse

than any terminal outcome or (IV) without this assumption?

For $n = 3$ cases (I) and (II) are answered in the negative, while for $n = 2$ cases (III) and (IV) are proven. We briefly survey other negative and positive results on Nash-solvability in pure stationary strategies for the games under consideration.

Keywords: Stochastic games, Chess-like games, local cost, mean and total effective cost, pure and stationary strategies, Nash equilibrium

RRR 27-2012 **THE GAMES SEKI AND D-SEKI**, Andrey Gol'berg, Vladimir Gurvich, Diogo Andrade, Konrad Borys, Gabor Rudolf

Let $A : I \times J \to \mathbf{Z}_+$, be a non-negative integer $m \times n$ matrix each row and column of which contain a strictly positive entry. The game *SEKI* is defined as follows. Two players R and C alternate turns and it is specified who begins.

By one move, a player can either reduce a strictly positive entry of A by 1 (an active move) or pass. The game results in a draw after any two successive passes, one of R and one of C. Player R (respectively, C) wins when a row (respectively, a column) appears each entry of which is 0. However, such zero row and column may appear simultaneously, after a move. In this case, we assume that the player who made this last move is the winner. Yet, we will also study another version of the game, called D-SEKI, in which the above case is defined as a draw.

A matrix A is called a *seki* or a *d-seki* if it is a draw in the corresponding game, does not matter R or C begins. Furthermore, a seki or d-seki is called *complete* if each player *must* pass, that is, if (s)he makes an active move, the opponent wins.

Both games SEKI and D-SEKI are difficult. We present their complete analysis only for the 2×2 matrices, while in general we obtain only some sufficient conditions for a player to win and also some partial results and conjectures mostly related to the complete seki. These results for D-SEKI look simpler but, in return, SEKI, unlike D-SEKI, is closely connected to the so-called shared life in the classical game of GO.

Both SEKI and D-SEKI are of independent interest as combinatorial games.

Key words: combinatorial games with positive incentive, draw, pass, GO, shared life, seki, complete seki, integer doubly stochastic matrix.

RRR 28-2012 **MODELING SEQUENCE SCRAMBLING AND RELATED PHENOMENA IN MIXED-MODEL ASSEMBLY LINES**, Authors: Gábor Rudolf, Nilay Noyan, Vincent Giard

In this paper we examine the various effects that workstations and repair loops with identical parallel processors and stochastic processing times have on the performance of a mixed-model assembly line. Of particular interest are issues related to sequence scrambling. In many production systems (especially those operating on just-in-time or in-line vehicle sequencing principles), the sequence of orders is selected carefully to optimize line efficiency while taking into account various line balancing and product spacing constraints. However, this sequence is often altered due to stochastic factors during production. This leads to significant economic consequences, due to either the degraded performance of the assembly line, or the added cost of restoring the sequence (via the use of systems such as mix banks or automated storage and retrieval systems). We develop analytical formulas to quantify both the extent of sequence scrambling caused by a station of the assembly line, and the effects of this scrambling on downstream performance.

We also develop a detailed Markov chain model to analyze related issues regarding line stoppages and throughput. We demonstrate the usefulness of our methods on a range of illustrative numerical examples, and discuss the implications from a managerial point of view.

RRR 29-2012 **SAMPLE WIDTH FOR MULTI-CATEGORY CLASSIFIERS**, Martin Anthony, Joel Ratsaby

In a recent paper, the authors introduced the notion of *sample width* for binary classifiers defined on the set of real numbers. It was shown that the performance of such classifiers could be quantified in terms of this sample width. This paper considers how to adapt the idea of sample width so that it can be applied in cases where the classifiers are multi-category and are defined on some arbitrary metric space.

RRR 30-2012 **CONVEXITY AND SOLUTIONS OF STOCHASTIC MULTIDIMENSIONAL KNAPSACK PROBLEMS WITH PROBABILISTIC CONSTRAINTS**, Kunikazu Yoda, András Prékopa

In the multidimensional knapsack problem a set of items, each with a value and a multi-dimensional size, is given and we want to select a subset of them in such a way that the total value of the selected items is maximized while the total size satisfies some capacity constraint for each dimension. In this paper we assume that the sizes are independent random variables such that each size follows the same type of probability distribution, not necessarily with the same parameter. A joint probabilistic constraint is imposed on the capacity constraints and the objective function is the same as that of the underlying deterministic problem. We showed that the problem is convex, under some condition on the parameters, for special continuous and discrete distributions: gamma, normal, Poisson, and binomial, where the latter two discrete distribution functions are approximated by logconcave continuous distribution functions.

Keywords: stochastic programming; probabilistic constraints; multidimensional knapsack problem; convexity

RRR 31-2012 **A NEW APPROACH TO SELECT SIGNIFICANT PATTERNS IN LOGICAL ANALYSIS OF DATA**, Juan Felix Avila Herrera, Munevver Mine Subasi

Logical Analysis of Data (LAD) is a supervised learning algorithm which integrates principles of combinatorics, optimization and the theory of Boolean functions. Current implementations of LAD use greedy-type heuristics to select patterns to form an LAD model. In this paper we present a new approach based on integer programming and network flows to identify significant patterns to generate an LAD model. Our approach allows the user-specified significance requirements such as statistical significance, Hamming distance, homogeneity, coverage, and/or prevalence of patterns. We present experiments on benchmark datasets to demonstrate the utility of our integer programming and network flow based pattern selection method.

RRR 32-2012 **AUGMENTED LAGRANGIAN AND ALTERNATING DIRECTION METHODS FOR CONVEX OPTIMIZATION: A TUTORIAL AND SOME ILLUSTRATIVE COMPUTATIONAL RESULTS**, Jonathan Eckstein

The alternating direction of multipliers (ADMM) is a form of augmented Lagrangian algorithm that has experienced a renaissance in recent years due to its applicability to optimization problems arising from "big data" and image processing applications, and the relative ease with which it may be implemented in parallel and distributed computational environments. This chapter aims to provide an accessible introduction to the analytical underpinnings of the method, which are often obscured in treatments that do not assume knowledge of convex and set-valued analysis. In particular, it is tempting to view the method as an approximate version of the classical augmented Lagrangian algorithm, using one pass of block coordinate minimization to approximately minimize the augmented Lagrangian at each iteration. This chapter, assuming as little prior knowledge of convex analysis as possible, shows that the actual convergence mechanism of the algorithm is quite different, and then underscores this observations with some new computational results in which we compare the ADMM to algorithms that do indeed work by approximately minimizing the augmented Lagrangian.

RRR 33-2012 **KUSUOKA REPRESENTATIONS OF COHERENT RISK MEASURES IN FINITE PROBABILITY SPACES**, Nilay Noyan, Gábor Rudolf

Kusuoka representations provide an important and useful characterization of law invariant coherent risk measures in atomless probability spaces. However, the applicability of these results is limited by the fact that such representations do not always exist in probability spaces with atoms, such as finite probability spaces. We introduce the class of functionally coherent risk measures, which allow us to use Kusuoka representations in any probability space. We show that this class contains every law invariant risk measure that can be coherently extended to a family containing all finite discrete distributions. Thus, it is possible to preserve the desirable properties of law invariant coherent risk measures on atomless spaces without sacrificing generality. We also specialize our results to risk measures on finite probability spaces, and prove that in such spaces the family of risk measures with finite Kusuoka representations is dense among all law invariant coherent risk measures over a bounded class of random variables.