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RRR 1-2015 **EXTENDED COMPLEMENTARY NIM**, Endre Boros, Vladimir Gurvich, Nhan Bao Ho, and Kazuhisa Makino

In the standard Nim with n heaps, a player by one move can reduce (by a positive amount) exactly one heap of his choice. In this paper we consider the game of *complementary* Nim (Co-Nim), in which a player by one move can reduce at least one and at most $n-1$ heaps, of his choice. An explicit formula for the Sprague-Grundy (SG) function of Co-Nim was obtained by Jenkyns and Mayberry in 1980. We consider a further generalization, called *extended complementary* Nim (Exco-Nim). In this game there is one extra heap and a player by one move can reduce at least one and at most $n-1$ of the first n heaps, as in Co-Nim, and (s)he can also reduce the extra heap, whenever it is not empty. The $\mathcal{P}$-positions of Exco-Nim are easily characterized for any n. For $n \geq 3$ the SG function of Exco-Nim is a simple generalization of the SG function of Co-Nim. Somewhat surprisingly, for $n = 2$ the SG function of Exco-Nim looks much more complicated and "behaves in a chaotic way". For this case we provide only some partial results and some conjectures. (Note that for $n = 2$, Co-Nim and the standard two-heap Nim coincide.)

Key words: Moore's k-Nim, extended complementary Nim, impartial games, $\mathcal{P}$-positions, Sprague-Grundy function.

RRR 2-2015 **ON THE SPRAGUE-GRUNDY FUNCTION OF EXACT K-NIM**, Endre Boros, Vladimir Gurvich, Nhan Bao Ho, Kazuhisa Makino, and Peter Mursic

Moore's generalization of the game of Nim is played as follows. Given two integer parameters n, k such that $1 \leq k \leq n$, and n piles of tokens. Two players take turns. By one move a player reduces at least one and at most k piles. The player who makes the last move wins. The $\mathcal{P}$-positions of this game were characterized by Moore in 1910 and an explicit formula for its Sprague-Grundy function was given by Jenkyns and Mayberry in 1980, for the case $n = k + 1$ only. We modify Moore's game and introduce Exact k-Nim in which each move reduces exactly k piles. We give a simple polynomial algorithm computing the Sprague-Grundy function of Exact k-Nim in case $2k > n$ and an explicit formula for it in case $2k = n$. The last case shows a surprising similarity to Jenkyns and Mayberry's solution even though the meaning of some of the expressions are quite different.

Key words: Moore's Nim, exact Nim, impartial games, $\mathcal{P}$-position, Sprague-Grundy function.

RRR 3-2015 SLOW K-NIM, Vladimir Gurvich and Nhan Bao Ho

Given n piles of tokens and a positive integer $k \leq n$, we study the following two impartial combinatorial games $\text{Nim}^1_{n,\leq k}$ and $\text{Nim}^1_{n,=k}$. In the first (resp. second) game, a player, by one move, chooses at least 1 and at most (resp. exactly) k non-empty piles and removes one token from each of these piles. For the normal and misère version of each game we compute the Sprague-Grundy function for the cases $n = k = 2$ and $n = k + 1 = 3$.

For game $\text{Nim}^1_{n,\leq k}$ we also characterize its $\mathcal{P}$-positions for the cases $n \leq k+2$ and $n = k+3 \leq 6$.

Key words: impartial games, Nim, Moore's Nim, k-Nim, normal and misère versions, $\mathcal{P}$-positions, Sprague-Grundy function.

RRR 4-2015 **ON TAME, PET, DOMESTIC, AND MISERABLE IMPARTIAL GAMES**,

Vladimir Gurvich and Nhan Bao Ho

We study impartial games whose normal and misère plays are closely related. The first such class, so-called *tame* games, was introduced by Conway in 1976. Here we will consider a proper subclass, called *pet* games, and introduce a proper superclass, called *domestic* games. For each of three classes we provide several equivalent characterizations; some of which can be efficiently verified. These characterizations are based on an important subfamily of the tame games introduced in 2007 by the first author and called *miserable* games. We show that "slight modifications" turn these concepts into tame, pet, and domestic games. We also show that the sum of miserable games is miserable and find several other classes that respect sums. The obtained techniques allow us to prove that very many well-known impartial games fall into the considered classes. Such examples include all subtraction games which are pet, game Euclid which is miserable and hence tame, as well as many versions of the Wythoff game and Nim that may be miserable, pet, or domestic.

Keywords: impartial games, normal and misère play, sums of impartial games, Sprague-Grundy function, tame, pet, domestic, miserable.