New course: Selected Topics In Operations Research:

Semidefinite Programming

Course number: 16-711-613-01; **index:**47754
Instructor: Farid Alizadeh
Time & Place: Tentatively Mondays 1:10–3:40, RUTCOR Bldg. Rm 139

Semidefinite programming (SDP) is a new discipline in mathematical programming which includes linear programming as a special case, and itself is included in the general field of convex programming. In SDP one studies the problem of optimization of linear functions of symmetric matrices subject to linear equality and inequality constraints plus the additional constraint that some of the unknown matrices are positive semidefinite. In the course we will first review thoroughly the SDP problem and its duality theory, in particular analogs of Farkas lemma when extended to positive semidefinite matrices. Next we study algorithms for solving SDPs, in particular–since the only polynomial-time and possibly practical algorithms are based on extensions of Karmarkar's algorithm for linear programming–we emphasize interior point methods for SDP. Then we study various eigenvalue optimization problems that are special cases of SDP. Our focus then will shift towards applications of SDP in combinatorial optimization including approximation algorithms, polyhedral theory of graph problems, applications to general 0-1 integer programming, etc.

The course will be suitable for people with various interests. On the one hand there is an interesting interaction between matrix theory and convex optimization. On the other hand there are numerical stability issues for the interior point algorithms which present new challenges for students interested in numerical analysis. Finally, surprising and powerful applications in structural complexity theory and approximation algorithms for NP-hard problems should be attractive to theoretical computer science students. The course will deal with all these issues. Reading material will be based on research papers and chapters in books. Among them will be portions of the paper by the instructor on SDP [1], a chapter in the book by Grötschel, Lovász, and Schrijver [3], the famous paper of Goemans and Williamson [2], and the paper of Lovász and Schrijver [4].

Prerequisites: Except for the proverbial "mathematical maturity" no formal requirements is necessary in order to follow the lectures. The course, however, will demand a lot of background knowledge which will be covered in the lectures. Familiarity with matrix algebra and with theory of linear programming will make the course much easier, although all the necessary background will be developed in the course.

Either implementation projects, or research projects (culminating in a technical report for instance) may be required. In addition students will be expected to take turns in scribe duties: for each lecture one scribe will be designated who will be responsible for taking notes and transcribing them into LaTeX and making it available to all students. A more detailed curriculum will be available shortly and may be requested from the instructor by e-mail at `alizadeh@rutcor.rutgers.edu`.

References

[1] F. Alizadeh, *Interior point methods in semidefinite programming with applications to combinatorial optimization*, SIAM J. Optimization, 6 (1995).

[2] M. X. Goemans and D. P. Williamson, *Improved Approximation Algorithms for Maximum Cut and Satisfiability Problems Using Semidefinite Programming*, J. Assoc. Comput. Mach., (1994). (To appear). A preliminary version appeared in Proc. 26th Annual ACM Symposium on Theory of Computing.

[3] M. Grötschel, L. Lovász, and A. Schrijver, *Geometric Algorithms and Combinatorial Optimization*, Springer Verlag, 1988.

[4] L. Lovász and A. Schrijver, *Cones of Matrices and Setfunctions, and 0-1 Optimization*, SIAM J. Optimization, 1 (1991).