New course: Selected Topics In Operations Research: **Semidefinite Programming**
Course number: 16-711-613-01; **index:**47754
Instructor: Time & Place: Mondays 1:10–3:40, RUTCOR Bldg. Rm 139
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Introduction

Semidefinite programming (SDP) is a new discipline in mathematical programming which includes linear programming as a special case, and itself is included in the general field of convex programming. In SDP one studies the problem of optimization of linear functions of symmetric matrices subject to linear equality and inequality constraints plus the additional constraint that some of the unknown matrices are positive semidefinite.

Prerequisites:

Except for the proverbial "mathematical maturity" no formal requirements is necessary in order to follow the lectures. The course, however, will demand a lot of background knowledge which will be covered in the lectures. Familiarity with matrix algebra and with theory of linear programming will make the course much easier, although all the necessary background will be developed in the course.

WorldWide Web access

A semidefinite programming page is partially created and will be expanded during the semester. The address is

`http://new-rutcor.rutgers.edu/ãlizadeh`

You should be able to find pointers to all references and reading material needed for this course. The page contains pointers to publicly available software, papers, and a list of references in bibtex format. A special pointer to the course is also available, in which lecture notes will be posted.

Required work

Each student is required to take charge of at least one of the lectures and carefully prepare lecture notes. Then within a week, he/she should transcribe the notes into LaTeX and make it available to all students. The Notes will be posted on the course home page in www.

In addition either implementation projects, or research projects –hopefully culminating in a research paper–may be required. A programming project may be for instance an efficient and numerically stable implementation of one of the many interior point methods for SDP. Another project may be design and implementation of an SDP relaxation method for a specific class of NP-hard combinatorial optimization problem. For instance, one could try to use the Lov'asz ϑ function (which can be computed by solving an SDP) and try to find maximum cliques (or independent sets) or minimum coloring of graphs, hopefully in a reasonable time, most of the time.

For research papers, I will try to outline several open problems in each lecture. While some of these problems are long standing well-known open problem and thus unlikely to be resolved during the course, others may be within grasp. Since semidefinite programming has only recently attracted some attention, there may be many problems that are not all that difficult, but nevertheless remain unresolved, simply because many people are not aware of them and not enough people have worked on them. Hopefully, some of these problems will be settled by you, by the end of the semester.

In each lecture I'll also give several homework problems. Though solving them are not mandatory, your encouraged to study them, and try to solve them.

Topics

1. **Topic 1: Duality theory.**(3 lectures) We will study duality theory in the general context of cones. Weak duality, Farkas type lemmas and strong duality are among topics. In the special context of SDP we will also study complementary slackness theorem. Next we will study degeneracy in SDP.

2. **Topic 2: Eigenvalue optimization and other examples of SDP.** (1-2 lectures) Next we will study various forms that SDP may arise in optimization problems. In most cases semidefinite programs arise as a result of attempting to minimize the largest, or sum of the few largest eigenvalues of matrix subject to constraints. We will study these kinds of problems specially in the context of graph theory and show how they can be turned into semidefinite programs.

3. **Topic 3: polynomial time algorithms.** (2 lectures) Semidefinite programming would not have attracted much attention had it not been for the fact that polynomial time algorithms exist for their solution. Ellipsoid method, for instance can be applied, but possibly more practical is interior point algorithms. We will cover one of these algorithms, the emphasis will be on ease of understanding. We will also discuss numerical stability problems that are specific to semidefinite programming.

4. **Topic 4: Combinatorial Optimization.** (5 lectures) This will be the central topic of our course, and we will spend at least 1/3 of the course on this subject. We will examine the Lovász ϑ function in detail. In particular, its use as approximation to the Shannon capacity of graphs, and its connection to the polynomiality of clique and coloring problem in perfect graphs. We will also study $\mathrm{TH}(G)$, a convex set approximating clique polytopes of graph, which as it turns out is the feasible region of a semidefinite program. Next we will study the recent exciting results regarding approximation of NP-hard problems by semidefinite programming. In particular we will study Goemans–Williamson result for MAXCUT and MAXSAT problems, and Frieze and Jerrum's extension to k-cut problem.We will also study the vector-coloring technique of Karger et al and approximate max clique method of Alon and Kahale which is based on the ϑ function. studied.

References

For references and reading material refer to the SDP home page in www. Four main reading material are listed below.

References

[1] F. Alizadeh, *Interior point methods in semidefinite programming with applications to combinatorial optimization*, SIAM J. Optimization, 6 (1995).

[2] M. X. Goemans and D. P. Williamson, *Improved Approximation Algorithms for Maximum Cut and Satisfiability Problems Using Semidefinite Programming*, J. Assoc. Comput. Mach., (1994). (To appear). A preliminary version appeared in Proc. 26th Annual ACM Symposium on Theory of Computing.

[3] M. Grötschel, L. Lovász, and A. Schrijver, *Geometric Algorithms and Combinatorial Optimization*, Springer Verlag, 1988.

[4] L. Lovász and A. Schrijver, *Cones of Matrices and Setfunctions, and 0-1 Optimization*, SIAM J. Optimization, 1 (1991).