Dual Heuristics on the Exact Solution of Large Steiner Problems

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1. Introduction

The Steiner Problem in Graphs (SPG) has very strong LP relaxations.

- Solving those relaxations is often the bottleneck of the best known exact algorithms.
  (Lucena and Beasley (1998), Koch and Martin (1998), Uchoa et al. (1999), Bahiense et al. (2000)).
- Dual heuristics for the SPG are combinatorial procedures designed to provide good dual solutions for those LPs.
- Good dual solutions can lead to good primal solutions.

We present very effective dual heuristics for the SPG.

- Many instances are solved to optimality by dual heuristics alone, which is much faster than other known methods.
- Even when optimality is not reached, dual heuristics lead to significant speedups in B&C algorithms, since they:
  - are often able to fix many variables by reduced costs;
  - provide an advanced start for the first node in a B&C;
  - can be integrated with the whole B&C procedure (under construction).

2. Wong's Dual Ascent

Directed cut formulation of SPG:

- create a directed graph $D = (V, A)$ by replacing each edge in $G = (V, E)$ by two opposite arcs;
- choose any terminal r to be the *root*;
- let $\mathcal{W}$ be the collection of all vertex-sets W containing some terminal but not the root;
- let $\delta^-(W)$ be the directed cut made up by the arcs entering W;
- let y_a be a binary variable where $y_a = 1$ iff arc a belongs to a Steiner arborescence rooted at r;
- any minimum cost Steiner arborescence in D maps to a minimum cost Steiner tree in G.

$$
(P)\begin{cases}
\text{Min} & \sum\limits_{a \in A} c_a y_a & & (1)\\
\text{s.t.} & \sum\limits_{a \in \delta^-(W)} y_a \geq 1 & \forall W \in \mathcal{W} & (2)\\
 & y_a \geq 0 & \forall a \in A & (3)
\end{cases}
$$

Let π_W be the dual variable associated to each constraint (2). The dual of (P) is:

$$(D)\begin{cases} \text{Max} & \sum\limits_{W \in \mathcal{W}} \pi_W & & (4) \\ \text{s.t.} & \sum\limits_{W \in \mathcal{W}: a \in \delta^-(W)} \pi_W \leq c_a & \forall a \in A & (5) \\ & \pi_W \geq 0 & \forall W \in \mathcal{W} & (6) \end{cases}$$

- Greedy dual heuristics are usually known as *dual ascent* procedures.
- Wong (1984) proposed a dual ascent for a related SPG multicommodity flow formulation.
    - The algorithm can be adapted for the directed cut formulation.

- Let π be a feasible solution of (D);
- arc a is *saturated* if the sum of the dual variables associated to cuts containing a is equal to its cost (i.e. its reduced cost is zero);
- let $D_\pi = (V, A')$ be the subgraph of D containing just the saturated arcs.
- a *root component* R is a strongly connected component of D_π such that:
  - R contains a terminal, but not the root;
  - there is no path in D_π from a terminal to R.
- Let $W = w(R)$ be the vertex-set formed by R and by the vertices that can reach R in D_π.

Wong's Dual Ascent

```
$\pi \longleftarrow \mathbf{0}$;
while there is a root component $R$ in $D_\pi$ {
    $W \longleftarrow w(R)$;
    Increase $\pi_W$ until some arc in $\delta^-(W)$ is saturated;
}
return $\pi$;
```

- The dual ascent ends when D_π contains a Steiner arborescence rooted at r.
- Wong obtained primal solutions by:
  1. computing a minimum cost spanning arborescence for D_π;
  2. pruning non-terminal leaves in this arborescence;
  3. repeating 1-2 until no pruning is possible.
- We get primal solutions by:
  1. using *reverse delete step* on D_π (Goemans and Williamson (1996)),
  2. then applying a fast local search on G.

3. Root Component Selection

Wong's original algorithm does not specify which root component should be selected in each iteration. We tested 9 criteria:

1. RANDOM
2. FIRST
3. CIRCULAR
4. MINSATURATED / MAXSATURATED
5. MINEDGES / MAXEDGES
6. MINVALUE / MAXVALUE

The algorithm runs in $O(|E|^2)$ time for criteria 1-3 and in $O(|T| \cdot |E|^2)$ for the others.

We tested each criterion on a set of 214 instances:

- 57 OR-Library instances, introduced by Beasley in 1990, divided into series C, D and E (preprocessed with Duin and Volgenant tests).
- 80 Incidence instances (also present in OR-Library), introduced by Duin (1994), divided into series 80, 160, 320 and 640.
- 77 VLSI instances from SteinLib, introduced by Koch and Martin (1998), divided into series *alue*, *alut*, *diw*, *dmxa*, *gap*, *msm* and *taq* (preprocessed with Uchoa et al. tests).

criterion	**time (s)**	**gap (%)**		**arcs left (%)**	**instances solved**
		avg	**stddev**		
minedges	13.56	1.80	2.50	53.93	64
circular	6.50	2.10	2.92	57.25	64
minsaturated	12.71	2.36	2.96	58.55	54
random	6.89	2.41	3.09	61.72	49
maxvalue	6.37	4.03	5.29	69.29	37
maxsaturated	6.93	5.00	6.11	70.07	35
first	6.29	5.18	6.38	70.44	37
minvalue	6.66	5.51	6.05	73.66	26
maxedges	6.46	6.13	6.90	74.01	31

Table 1: Results for all 214 instances

We adopted MINEDGES for the remaining experiments.

4. Dual Scaling

- Multiply current dual solution by a positive factor $\alpha < 1.0$, making all arcs unsaturated, then apply DUALASCENT again.
- Factor α is typically between 0.6 and 0.9.

Dual Scaling

```
$\pi^*, \pi \longleftarrow$ DUALASCENT($\mathbf{0}$);
for $it \longleftarrow 1$ to 5 do {
      $\pi \longleftarrow$ DUALASCENT($\alpha \cdot \pi$);
      if ($v(\pi) > v(\pi^*)$) $\pi^* \longleftarrow \pi$;
      else return $\pi^*$;
}
return $\pi^*$;
```

- Dual Scaling worked fine for OR-Lib instances. For example, with $\alpha = 0.875$, instance $e18$ dual bound increases from 552 to 559 (opt. 564).
- Not very effective for incidence and VLSI instances.
- However, dual scaling solutions are convenient to start the first node in a B&C. For example, in $e18$, solving an LP only with these cuts increases the bound to 563 in a few seconds.

5. Dual Adjustment

Local search procedure over current dual solution π^*.

- We try 2 kinds of movements:

1. Let y^* be the best known primal solution. For all $\pi_W > 0$ such that $\delta^-(W)$ crosses y^* more than once, make $\pi^*_W = 0$. Then apply DUALASCENT(π^*).
2. For each $\pi^*_W > 0$ (one at a time), make $\pi^*_W = 0$ and apply DUALASCENT(π^*).

- Whenever there is an improvement, π^* is updated and the procedure is repeated.

- Effective for instances of all classes.
- Dual adjustment increases the overall robustness. If a run of dual ascent yields a poor solution, dual adjustment is likely to work.
- When the dual ascent solution is almost optimal, dual adjustment alone seldom works.
- Applying dual adjustment after dual scaling almost always leads to some small improvement. This can very useful to close a gap of one unit.

6. Active Fixing by Reduced Costs

Let Z_P be the value of the best known primal solution and $\bar{c}_a$ be the reduced cost of arc a with respect to dual solution π. If $v(\pi) + \bar{c}_a \geq Z_P$, arc a can be fixed to 0.

- The dual heuristics can lead to gaps small enough to fix a significant number of arcs.
- *Active fixing* is a scheme designed to increase the number of arcs fixed.

Active Fixing

```
Let $\pi^*$ be the best known dual solution;
while "it's working" {
      select a "promising" arc $a$;
      $\pi' \longleftarrow \{\pi^* -$ the cuts containing $a\}$;
      Insert $a$ into $D_{\pi'}$;
      $\pi \longleftarrow$ DUALASCENT($\pi'$);
      Try to fix variables using $\pi$;
      if ($v(\pi) > v(\pi^*)$) update $\pi^*$;
}
```

- The reduced cost of a with respect to dual solution π will be maximum ($\bar{c}_a = c_a$).

- This really works for instances of all classes. The most remarkable results were achieved for complete Duin instances.
- Active Fixing can fix arcs that are saturated in π^*. When this happens, a new call to DUALASCENT may improve the dual solution.
- Active Fixing sometimes is a better adjustment procedure than Dual Adjustment itself.

7. Detailed Computational Results

We chose a different set of heuristics for each class of instances:

- OR-Lib - Scaling + Active Fixing
- VLSI - Active Fixing
- Duin - Dual Adjustment + Active Fixing

class	**time (s)**	**gap (%)**		**arcs left (%)**	**instances solved**
		avg	**stddev**		
OR-Library	0.34	1.25	2.69	24.56	34/57
VLSI	14.62	1.11	0.99	65.63	17/77
Incidence	21.97	2.84	2.98	63.60	13/80
Total	13.56	1.80	2.50	53.93	64/214

Table 2: Single Dual Ascent

class	**time (s)**	**gap (%)**		**arcs left (%)**	**instances solved**
		avg	**stddev**		
OR-Library	2.05	0.23	0.63	14.00	46/57
VLSI	18.52	0.71	0.88	46.71	33/77
Incidence	311.17	1.25	1.90	38.27	41/80
Total	123.54	0.78	1.38	34.80	120/214

Table 3: Complete Algorithm

8. Embedding Dual Heuristics within a B&C

Preliminary experiments have shown that just using the cuts provided by the dual heuristics to start the first node in a B&C is already effective.

- Speedups of 5 to 10 times were typical over VLSI instances.
- All OR-Lib instances are solved in a few seconds, except $e18$.
- No experiments over incidence instances yet.

We plan to fully embed the dual heuristics within the B&C:

- Whenever an LP is solved, its dual solution can be submitted to the dual heuristics and the next LP may receive an improved solution in terms of objective function or size.
- Once the gap becomes sufficiently small, Active Fixing dramatically reduces problem size. A few LP iterations may be enough to reach this point.

Computational Results

instance	**dimensions**			**first dual ascent**			**final results**			**arcs**	**gap**
	$\lvert V \rvert$	$\lvert E \rvert$	$\lvert T \rvert$	dual	primal	time (s)	dual	primal	time (s)	(%)	(%)
dv80-00	80	120	6	1592	1607	0.01	**1607**	**1607**	0.08	0	0
dv80-01	80	350	6	1471	1570	0.05	**1479**	**1479**	0.77	0	0
dv80-02	80	3160	6	**1175**	**1175**	0.20	**1175**	**1175**	0.22	0	0
dv80-03	80	160	6	**1570**	**1570**	0.02	**1570**	**1570**	0.02	0	0
dv80-04	80	632	6	1264	1296	0.10	**1276**	**1276**	0.16	0	0
dv80-10	80	120	8	**2608**	**2608**	0.02	**2608**	**2608**	0.02	0	0
dv80-11	80	350	8	1898	2064	0.06	1976	2056	11.48	37.10	4.04
dv80-12	80	3160	8	1559	1561	0.26	**1561**	**1561**	0.33	0	0
dv80-13	80	160	8	2186	2284	0.02	2283	2284	1.58	15.60	0.04
dv80-14	80	632	8	1725	1788	0.11	**1788**	**1788**	0.35	0	0
dv80-20	80	120	16	**4760**	**4760**	0.03	**4760**	**4760**	0.03	0	0
dv80-21	80	350	16	3577	3631	0.10	3631	3631	1.03	0	0
dv80-22	80	3160	16	3148	3168	0.48	**3158**	**3158**	0.98	0	0
dv80-23	80	160	16	4349	4437	0.05	**4354**	**4354**	2.98	0	0
dv80-24	80	632	16	3404	3593	0.21	3452	3550	24.66	83.00	2.83
dv80-30	80	120	20	5447	5519	0.03	**5519**	**5519**	0.09	0	0
dv80-31	80	350	20	4457	4737	0.10	4489	4737	3.13	99.90	5.52
dv80-32	80	3160	20	**3932**	**3932**	0.60	**3932**	**3932**	0.61	0	0
dv80-33	80	160	20	5082	5226	0.05	5147	5226	8.23	56.60	1.53
dv80-34	80	632	20	4237	4404	0.23	4252	4404	25.86	92.60	3.57
dv160-00	160	240	7	**2158**	**2158**	0.04	**2158**	**2158**	0.05	0	0
dv160-01	160	812	7	1610	1677	0.17	**1677**	**1677**	0.59	0	0
dv160-02	160	12720	7	**1352**	**1352**	1.15	**1352**	**1352**	1.22	0	0
dv160-03	160	320	7	**2170**	**2170**	0.07	**2170**	**2170**	0.09	0	0
dv160-04	160	2544	7	1488	1542	0.70	**1494**	**1494**	2.00	0	0
dv160-10	160	240	12	3770	3870	0.06	3783	3870	2.42	40.00	2.30
dv160-11	160	812	12	2808	2869	0.25	**2869**	**2869**	1.69	0	0
dv160-12	160	12720	12	2353	2369	1.65	2363	2364	6.66	5.40	0.04
dv160-13	160	320	12	3352	3356	0.15	**3356**	**3356**	1.14	0	0
dv160-14	160	2544	12	2517	2719	0.91	**2549**	**2549**	3.85	0	0
dv160-20	160	240	24	6843	6932	0.09	**6923**	**6923**	4.57	0	0
dv160-21	160	812	24	5439	5718	0.43	5454	5646	14.63	98.20	3.52
dv160-22	160	12720	24	4717	4738	4.09	**4729**	**4729**	5.33	0	0
dv160-23	160	320	24	6634	6721	0.16	6635	6721	10.46	55.50	1.29
dv160-24	160	2544	24	4991	5136	1.25	5003	5119	172.07	88.10	2.31
dv160-30	160	240	40	11810	11908	0.09	11816	11904	7.36	57.90	0.74
dv160-31	160	812	40	8870	9530	0.47	9026	9530	8.95	100.00	5.58
dv160-32	160	12720	40	7861	7876	6.83	**7876**	**7876**	47.24	0	0
dv160-33	160	320	40	10399	10483	0.18	**10414**	**10414**	11.91	0	0
dv160-34	160	2544	40	8212	8541	1.93	8227	8541	54.79	100.00	3.81

instance	**dimensions**			**first dual ascent**			**final results**			**arcs**	**gap**
	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert T\rvert$	dual	primal	time (s)	dual	primal	time (s)	(%)	(%)
dv320-00	320	480	8	**2847**	**2847**	0.08	**2847**	**2847**	0.08	0	0
dv320-01	320	1845	8	1993	2067	0.79	**2053**	**2053**	32.99	0	0
dv320-02	320	51040	8	1561	1577	7.13	**1565**	**1565**	11.50	0	0
dv320-03	320	640	8	2637	2697	0.29	**2673**	**2673**	3.87	0	0
dv320-04	320	10208	8	1690	1765	3.56	**1707**	**1707**	117.40	0	0
dv320-10	320	480	17	5459	5549	0.12	**5548**	**5548**	4.00	0	0
dv320-11	320	1845	17	4160	4273	1.15	4220	4273	90.69	28.20	1.25
dv320-12	320	51040	17	3319	3321	12.03	**3321**	**3321**	13.31	0	0
dv320-13	320	640	17	5159	5343	0.33	5174	5343	20.60	95.90	3.26
dv320-14	320	10208	17	3514	3716	6.17	3545	3696	1121.08	97.80	4.26
dv320-20	320	480	34	10000	10101	0.34	10021	10101	29.50	48.40	0.79
dv320-21	320	1845	34	7858	8391	1.86	7877	8391	40.06	100.00	6.52
dv320-22*	320	51040	34	6672	6688	34.84	**6686**	**6686**	61.94	0	0
dv320-23	320	640	34	9724	10040	0.66	9730	10040	4.86	100.00	3.18
dv320-24*	320	10208	34	6948	7153	10.73	6958	7137	61.73	99.70	2.57
dv320-30	320	480	80	23234	23280	0.59	23237	23279	134.53	59.30	0.18
dv320-31*	320	1845	80	17472	18397	3.53	17556	18397	284.54	100.00	4.79
dv320-32*	320	51040	80	15619	15668	91.34	15623	15668	3611.53	54.80	0.28
dv320-33	320	640	80	21226	21662	1.13	21278	21662	18.23	100.00	1.80
dv320-34*	320	10208	80	16124	16409	12.83	16130	16409	182.15	99.90	1.73
dv640-00	640	960	9	**4033**	**4033**	0.58	**4033**	**4033**	0.59	0	0
dv640-01	640	4135	9	**2392**	**2392**	2.38	**2392**	**2392**	2.41	0	0
dv640-02	640	204480	9	**1749**	**1749**	32.64	**1749**	**1749**	34.18	0	0
dv640-03	640	1280	9	**3278**	**3278**	0.83	**3278**	**3278**	0.83	0	0
dv640-04	640	40896	9	1892	1947	26.72	**1897**	**1897**	75.82	0	0
dv640-10	640	960	25	8599	8855	0.45	**8764**	**8764**	156.49	0	0
dv640-11	640	4135	25	5945	6348	4.66	5994	6348	105.59	100.00	5.90
dv640-12*	640	204480	25	4890	4910	83.09	**4906**	**4906**	179.45	0	0
dv640-13	640	1280	25	7966	8190	1.20	8035	8190	100.20	97.10	1.92
dv640-14*	640	40896	25	5131	5304	54.53	5136	5264	1955.94	98.30	2.49
dv640-20	640	960	50	15914	16143	0.75	16061	16143	117.67	48.80	0.51
dv640-21*	640	4135	50	11423	12466	7.64	11537	12466	328.61	100.00	8.05
dv640-22*	640	204480	50	9796	9835	313.41	9802	9821	3606.48	9.00	0.19
dv640-23	640	1280	50	14745	15173	3.17	14785	15173	78.24	100.00	2.62
dv640-24*	640	40896	50	10133	10252	68.08	10133	10252	1631.88	99.60	1.17
dv640-30	640	960	160	44858	45069	2.54	44863	45069	221.27	99.40	0.45
dv640-31*	640	4135	160	34654	36749	17.00	34859	36749	2816.29	100.00	5.42
dv640-32*	640	204480	160	30991	31097	703.38	30994	31097	5317.59	95.50	0.33
dv640-33	640	1280	160	42447	43386	4.80	42475	43386	106.23	100.00	2.14
dv640-34*	640	40896	160	31837	32190	149.62	31843	32190	1774.28	100.00	1.09

instance	**dimensions**			**first dual ascent**			**final results**			**arcs**	**opt**
	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert T\rvert$	dual	primal	time (s)	dual	primal	time (s)	(%)	
d1	234	433	5	**106**	**106**	0.04	**106**	**106**	0.04	0	**106**
d2	255	459	10	**220**	**220**	0.05	**220**	**220**	0.05	0	**220**
d3	31	48	18	**1565**	**1565**	0.00	**1565**	**1565**	0.00	0	**1565**
d4	24	36	14	**1935**	**1935**	0.00	**1935**	**1935**	0.01	0	**1935**
d5	24	37	16	**3250**	**3250**	0.01	**3250**	**3250**	0.01	0	**3250**
d6	746	1694	5	66	**67**	0.14	66	**67**	0.61	27.60	**67**
d7	725	1647	10	**103**	**103**	0.08	**103**	**103**	0.09	0	**103**
d8	287	515	84	**1072**	**1072**	0.18	**1072**	**1072**	0.19	0	**1072**
d9	69	116	34	**1448**	**1448**	0.02	**1448**	**1448**	0.03	0	**1448**
d10	47	78	30	**2110**	**2110**	0.01	**2110**	**2110**	0.02	0	**2110**
d11	975	3749	5	**29**	**29**	0.12	**29**	**29**	0.14	0	**29**
d12	956	3061	9	41	**42**	0.07	**42**	**42**	0.41	0	**42**
d13	423	828	84	**500**	**500**	0.27	**500**	**500**	0.28	0	**500**
d14	79	137	32	**667**	**667**	0.02	**667**	**667**	0.02	0	**667**
d15	23	37	14	**1116**	**1116**	0.01	**1116**	**1116**	0.01	0	**1116**
d16	1000	6725	5	**13**	14	0.21	**13**	**13**	0.42	0	**13**
d17	1000	6332	10	**23**	**23**	0.31	**23**	**23**	0.36	0	**23**
d18	813	2284	97	220	225	0.81	222	225	1.72	100.00	**223**
d19	700	1932	100	309	312	0.50	309	312	0.65	100.00	**310**
e1	657	1239	5	**111**	**111**	0.12	**111**	**111**	0.13	0	**111**
e2	678	1256	9	**214**	**214**	0.10	**214**	**214**	0.10	0	**214**
e3	120	195	64	**4013**	**4013**	0.05	**4013**	**4013**	0.05	0	**4013**
e4	50	82	31	5100	**5101**	0.01	**5101**	**5101**	0.04	0	**5101**
e5	16	25	11	**8128**	**8128**	0.00	**8128**	**8128**	0.01	0	**8128**
e6	1830	4277	5	**73**	**73**	0.25	**73**	**73**	0.28	0	**73**
e7	1876	4321	10	**145**	**145**	0.30	**145**	**145**	0.33	0	**145**
e8	894	1714	196	2639	2641	1.17	2639	2641	33.82	53.10	**2640**
e9	506	899	187	3602	**3604**	0.56	3603	**3604**	38.40	39.90	**3604**
e10	89	152	51	**5600**	**5600**	0.04	**5600**	**5600**	0.04	0	**5600**
e11	2487	10861	5	**34**	**34**	0.36	**34**	**34**	0.44	0	**34**
e12	2462	9836	10	66	**67**	0.59	66	**67**	6.58	12.70	**67**
e13	1418	2962	232	1274	1286	2.70	1275	1286	6.42	100.00	**1280**
e14	339	589	119	**1732**	1733	0.21	**1732**	1733	0.55	64.90	**1732**
e15	26	41	17	**2784**	**2784**	0.00	**2784**	**2784**	0.00	0	**2784**
e16	2500	19698	5	**15**	**15**	0.77	**15**	**15**	0.90	0	**15**
e17	2500	17045	10	**25**	**25**	0.63	**25**	**25**	0.76	0	**25**
e18	2119	6185	248	552	571	7.42	559	571	15.01	100.00	**564**
e19	1156	2757	174	754	763	1.44	755	763	2.11	100.00	**758**

instance	**dimensions**			**first dual ascent**			**final results**			**arcs**	**opt**
	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert T\rvert$	dual	primal	time (s)	dual	primal	time (s)	(%)	
c1	108	188	5	**85**	**85**	0.02	**85**	**85**	0.02	0	**85**
c2	82	144	8	**144**	**144**	0.02	**144**	**144**	0.02	0	**144**
c3	55	90	24	**754**	**754**	0.01	**754**	**754**	0.01	0	**754**
c4	51	81	24	**1079**	**1079**	0.02	**1079**	**1079**	0.02	0	**1079**
c5	15	24	10	**1579**	**1579**	0.01	**1579**	**1579**	0.01	0	**1579**
c6	353	795	5	**55**	**55**	0.05	**55**	**55**	0.05	0	**55**
c7	359	802	9	**102**	**102**	0.04	**102**	**102**	0.05	0	**102**
c8	128	238	30	**509**	511	0.03	**509**	**509**	0.08	0	**509**
c9	132	241	49	705	**707**	0.05	**707**	**707**	0.92	0	**707**
c10	15	21	10	**1093**	**1093**	0.01	**1093**	**1093**	0.01	0	**1093**
c11	488	1692	5	31	**32**	0.08	**32**	**32**	0.13	0	**32**
c12	454	1395	9	**46**	**46**	0.07	**46**	**46**	0.07	0	**46**
c13	177	332	37	256	**258**	0.05	**258**	**258**	3.62	0	**258**
c14	5	7	4	**323**	**323**	0.00	**323**	**323**	0.00	0	**323**
c15	4	5	3	**556**	**556**	0.00	**556**	**556**	0.00	0	**556**
c16	499	2715	5	**11**	**11**	0.09	**11**	**11**	0.11	0	**11**
c17	494	2294	8	**18**	19	0.07	**18**	**18**	0.15	0	**18**
c18	391	1044	50	109	114	0.12	111	114	0.40	100.00	**113**
c19	266	637	40	145	**146**	0.08	**146**	**146**	0.17	0	**146**

instance	**dimensions**			**first dual ascent**			**final results**			**arcs**	**opt**
	$\vert V\vert$	$\vert E\vert$	$\vert T\vert$	dual	primal	time (s)	dual	primal	time (s)	(%)	
alue2087	36	58	13	1044	1057	0.03	**1049**	**1049**	0.05	0	**1049**
alue2105	7	10	3	**1032**	**1032**	0.00	**1032**	**1032**	0.00	0	**1032**
alue3146	113	187	29	2218	2243	0.06	2226	**2240**	0.85	78.30	**2240**
alue5067	305	509	42	2564	**2586**	0.20	2564	**2586**	2.77	88.90	**2586**
alue5345	1191	2012	64	3421	3541	2.08	3423	3520	7.86	100.00	**3507**
alue5623	807	1377	58	3357	3421	1.46	3357	3421	4.26	99.90	**3413**
alue5901	1077	1847	58	3874	3922	2.17	3874	3922	11.07	99.30	**3912**
alue6179	480	817	51	2443	**2452**	0.26	2443	**2452**	2.70	52.90	**2452**
alue6457	849	1451	57	3022	3081	1.12	3024	3073	11.41	99.70	**3057**
alue6735	360	592	49	2675	**2696**	0.26	2678	**2696**	2.32	87.10	**2696**
alue6951	473	790	58	2362	2395	0.38	2374	2389	5.37	83.30	**2386**
alue7065*	13073	23017	445	23488	23970	174.80	23488	23970	186.61	100.00	23944
alue7066	1791	3151	9	2197	2266	3.77	2224	**2256**	54.21	97.50	**2256**
alue7080*	9272	16019	1402	61911	62632	402.09	61911	62632	412.36	100.00	62751
alut0787	9	13	5	**982**	**982**	0.00	**982**	**982**	0.00	0	**982**
alut0805	92	154	23	956	960	0.04	**958**	**958**	0.21	0	**958**
alut1181	234	412	42	2312	**2353**	0.16	2318	**2353**	1.61	96.80	**2353**
alut2010	435	742	45	3260	3322	0.43	3270	3312	3.98	98.70	**3307**
alut2288	1224	2195	59	3792	3853	2.24	3803	3845	13.93	99.50	**3843**
alut2566	715	1242	62	3012	3090	0.89	3017	3086	2.17	100.00	**3073**
alut2610*	10760	20185	182	11922	12398	73.75	11922	12398	80.84	100.00	12274
alut2625*	12196	22457	764	34650	35606	390.55	34650	35606	404.17	100.00	35540
diw0234	860	1599	21	1979	**1996**	0.49	1982	**1996**	5.92	74.80	**1996**
diw0445	31	49	11	1358	**1363**	0.01	1358	**1363**	0.06	59.20	**1363**
diw0459	16	25	7	1349	**1362**	0.01	1349	**1362**	0.03	64.00	**1362**
diw0473	14	22	6	1097	1102	0.00	**1098**	**1098**	0.00	0	**1098**
diw0487	4	5	3	**1424**	**1424**	0.00	**1424**	**1424**	0.00	0	**1424**
diw0559	27	45	9	**1570**	**1570**	0.01	**1570**	**1570**	0.01	0	**1570**
diw0778	178	318	15	2172	**2173**	0.09	2172	**2173**	0.18	53.50	**2173**
diw0779	2387	4508	37	4399	4471	4.74	4399	4471	19.25	100.00	**4440**
diw0795	290	529	9	1543	**1550**	0.16	**1550**	**1550**	1.20	0	**1550**
diw0801	393	719	10	1577	1593	0.27	1582	**1587**	3.20	45.70	**1587**
diw0819	1186	2214	22	3381	**3399**	1.86	3392	**3399**	40.88	53.30	**3399**
diw0820	1891	3563	32	4102	4194	3.32	4116	4182	20.43	99.80	**4167**
dmxa0368	47	76	9	1000	1019	0.01	1008	1019	0.10	64.50	**1017**
dmxa0454	15	22	5	**914**	**914**	0.00	**914**	**914**	0.00	0	**914**
dmxa0848	37	60	11	586	595	0.00	**594**	**594**	0.17	0	**594**
dmxa0903	58	99	8	578	**580**	0.02	578	**580**	0.07	60.10	**580**
dmxa1109	9	13	5	**454**	**454**	0.00	**454**	**454**	0.00	0	**454**
dmxa1200	32	48	13	**750**	**750**	0.01	**750**	**750**	0.01	0	**750**
dmxa1721	6	8	4	**780**	**780**	0.00	**780**	**780**	0.00	0	**780**
dmxa1801	354	641	17	1322	1375	0.21	1353	**1365**	4.13	72.20	**1365**

instance	**dimensions**			**first dual ascent**			**final results**			**arcs**	**opt**
	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert T\rvert$	dual	primal	time (s)	dual	primal	time (s)	(%)	
gap1904	50	84	11	**763**	**763**	0.01	**763**	**763**	0.01	0	**763**
gap2007	36	61	9	1084	**1104**	0.00	1085	**1104**	0.11	62.30	**1104**
gap2740	33	56	5	737	**745**	0.01	**745**	**745**	0.01	0	**745**
gap3036	28	42	9	**457**	**457**	0.01	**457**	**457**	0.01	0	**457**
gap3100	14	22	7	**640**	**640**	0.00	**640**	**640**	0.00	0	**640**
gap3128	2251	4091	62	4265	**4292**	2.72	4268	**4292**	46.75	89.60	**4292**
msm0580	12	17	6	462	475	0.00	**467**	**467**	0.00	0	**467**
msm0709	42	68	8	879	**884**	0.00	**884**	**884**	0.02	0	**884**
msm0920	13	18	7	**806**	**806**	0.00	**806**	**806**	0.00	0	**806**
msm1008	13	18	6	491	**494**	0.00	**494**	**494**	0.00	0	**494**
msm1234	7	9	4	545	**550**	0.01	545	**550**	0.01	61.10	**550**
msm1477	12	17	6	**1068**	**1068**	0.00	**1068**	**1068**	0.00	0	**1068**
msm1844	6	8	4	**188**	**188**	0.00	**188**	**188**	0.00	0	**188**
msm2152	176	309	23	1578	1600	0.08	**1590**	**1590**	0.91	0	**1590**
msm2326	9	12	5	**399**	**399**	0.00	**399**	**399**	0.00	0	**399**
msm2492	56	91	10	1454	**1459**	0.02	1454	**1459**	0.09	39.00	**1459**
msm2525	11	15	6	1280	**1290**	0.00	**1290**	**1290**	0.00	0	**1290**
msm2601	176	303	12	1401	1455	0.07	1426	**1440**	1.02	78.20	**1440**
msm2705	4	5	3	**714**	**714**	0.00	**714**	**714**	0.00	0	**714**
msm2802	9	14	5	**926**	**926**	0.00	**926**	**926**	0.00	0	**926**
msm2846	395	682	61	3101	3156	0.39	3113	3136	4.48	92.40	**3135**
msm3727	45	74	4	**1376**	**1376**	0.01	**1376**	**1376**	0.01	0	**1376**
msm3829	400	705	10	1545	**1571**	0.31	1553	**1571**	5.96	84.30	**1571**
msm4312	1342	2433	10	1962	**2016**	2.36	1962	**2016**	8.27	99.80	**2016**
msm4515	31	50	8	**630**	640	0.01	**630**	**630**	0.02	0	**630**
taq0014	1766	3155	86	5303	5335	3.87	5303	5335	27.02	99.40	**5326**
taq0023	48	80	8	618	623	0.00	**621**	**621**	0.02	0	**621**
taq0365	998	1797	21	1873	**1914**	0.96	1876	**1914**	4.65	99.60	**1914**
taq0377	2040	3603	118	6310	6395	6.76	6310	6395	15.75	100.00	**6393**
taq0431	123	216	10	892	**897**	0.03	892	**897**	0.31	37.00	**897**
taq0631	7	9	4	**581**	**581**	0.01	**581**	**581**	0.01	0	**581**
taq0739	73	121	12	830	**848**	0.03	833	**848**	0.16	71.50	**848**
taq0741	107	183	14	842	**847**	0.05	**847**	**847**	0.27	0	**847**
taq0751	136	233	15	924	**939**	0.05	929	**939**	0.64	54.10	**939**
taq0903	1851	3294	90	5007	5122	4.63	5007	5122	5.18	100.00	**5099**