INTEGER PROGRAMMING HW 2

Hints are available for each problem, but first try each problem without any hints.

1. For each of the matrices below, decide whether it is TU or not. Justify your answers. Don't use the definition to prove a *yes* answer, it's quite useless (or tedious at least).

(a) $\begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & -1 & 0 & -1 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

2. Prove that any 0–1 matrix that satisfies the *consecutive ones property* (see the definition of this in the class notes) is TU.

3. Suppose that a matrix M has its entries from the interval $[0, 1]$, and that the row and column sums of M are all integers. Prove that the non-zero entries of M can be changed to zeros and ones without changing the row and column sums.

4. Let J be a set of jobs that need to be scheduled to be done on a single available machine. Each job j takes exactly 1 unit of time to finish, and has a deadline d_j. Consider the independence system $(J, \mathcal{F})$ where $\mathcal{F}$ consists of those subsets of jobs, which can be scheduled such that all of them are finished before their respective deadlines. Prove that $(J, \mathcal{F})$ is a matroid. (*You may give a direct proof of this claim, or show that this system belongs to one of those classes of matroids that we have seen in class.*)

5. Solve Wolsey 3.8.2.

6. Solve Wolsey 3.8.8.

7. Solve Wolsey 3.8.10.

Due on February 24, Tuesday, in class.