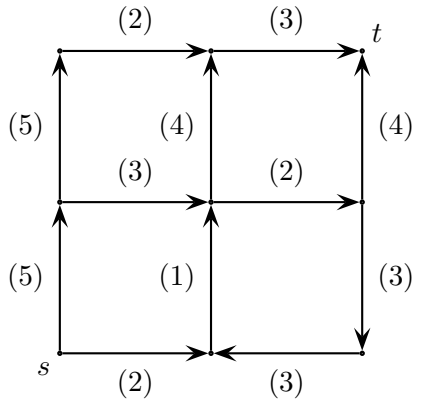
LINEAR OPTIMIZATION HW 4 – UNDERGRADUATE STUDENTS

1. Find a maximum flow in the network below using the augmenting path method of Ford and Fulkerson. List each augmenting path, and show the flow after each iteration of the algorithm. Also find a minimum s–t cut, and verify that the max flow–min cut theorem holds for this network.



2. Scarlet Airlines operates a cargo plane that can hold up to 30000 pounds of cargo occupying up to 20000 cubic feet. We have contracted to transport the following items.

Item type	Weight	Volume	Number	Cost if *not* carried
1	4000	1000	3	\$800
2	800	1200	10	\$150
3	2000	2200	4	\$300
4	1500	500	5	\$500

For example, we have contracted 10 items of type 2, each of which weighs 800 pounds and takes up to 1200 cubic feet of space. The last column refers to the cost of subcontracting shipment to another carrier.

For each pound we carry, the cost of flying the plane increases by 5 cents. Which items should we put in the plane, and which should we ship via other carrier, in order to have the lowest shipping cost? Formulate this problem as an IP.

3. Scarlet Airlines runs flights between n cities. The distance between city i and city j is given by the number d_{ij}. The company needs a hub within distance R of each of the n cities, and they want to determine the smallest number of hubs needed to meet this requirement. Formulate their problem as an IP.

4. (Fixed Charge problem.) Suppose that in an IP we have (besides a bunch of other variables) a nonnegative variable x and a binary variable y. We need to add the

following constraint: *if* $x > 0$*, then* $y = 1$. (If $x = 0$, then y can be either 0 or 1.) Assuming that you are explicitly given a number M such that $x \leq M$ in any optimal solution, formulate this constraint as a linear inequality, or a system of linear inequalities. (Hint: use the bound M, it can't be done otherwise.)

5. A subset of the edges in an undirected graph is called an *edge cover* if every vertex is an endpoint of at least one of them. The *edge cover problem* in a graph is defined as follows: given an undirected graph $G = (V, E)$, in which each vertex is adjacent to at least one edge, find an edge cover of G of minimum cardinality. Formulate this problem as an IP.

Due on November 18, Tuesday, in class.