LINEAR OPTIMIZATION HW 3

"The textbook" refers to the Kolman–Beck book.

1. Formulate an integer programming model for the *hitting set* problem, defined as follows: given a hypergraph H, find a subset of its vertices that intersects every edge.

2. In the problem of *bin packing*, items of different volumes must be packed into bins of given capacity in a way that minimizes the number of bins used. More precisely, suppose that we have n items, and item i has volume v_i ($i = 1, \ldots, n$), and we want to decide whether the items fit in K bins of volume V. Formulate this as an integer programming feasibility problem.

3. One may try to solve the aforementioned bin packing problem in the following, "greedy" fashion: take the items one by one, in an arbitrary order, and put each one in the first bin that it fits. For example, if $V = 6$, $n = 4$, and the items have volume $v_1 = 3$, $v_2 = 4$, $v_3 = 1, v_4 = 3$, then we will put item 1 to the first bin, item 2 in the second bin ($4 + 3 > 6$, so it doesn't fit in the first bin), item 3 goes to the first bin, and item four to the third, because it won't fit into any of the first two. This example clearly shows that this approach may not give the optimal solution, because we should be able to pack these items into two bins. However, the algorithm has a performance guarantee that shows that the solution it gives won't be very bad either: prove that the algorithm uses at most twice as many bins as necessary. (Hint: concentrate on bins which are less than half full.)

4. Consider the following LP relaxation of the knapsack problem:

$$\begin{array}{rl} \max & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.} & \mathbf{a}^\top \mathbf{x} \leq b \\ & 0 \leq x_i \leq 1 \ \ \forall i \end{array}$$

Find the optimal solution to this problem. More specifically, show that there is an optimal solution which has at most one fractional component.

5. Solve Problem 6 in Section 4.1 of the textbook.

Due on April 3, Thursday, in the beginning of the class.