

LINEAR OPTIMIZATION HW 4

1. The IP below is an IP formulation of the maximum matching problem in a graph $G = (V, E)$.

$$\begin{aligned} \max \quad & \sum_{i=1}^{|E|} x_i \\ \text{s.t.} \quad & \sum_{k: \{j,k\} \in E} x_k \leq 1 \quad \forall j \in V \\ & x_i \in \{0, 1\} \quad \forall i \in E \end{aligned}$$

Its LP relaxation can be written as follows:

$$\begin{aligned} \max \quad & \sum_{i=1}^{|E|} x_i \\ \text{s.t.} \quad & \sum_{k: \{j,k\} \in E} x_k \leq 1 \quad \forall j \in V \\ & x_i \geq 0 \quad \forall i \in E \\ & x_i \leq 1 \quad \forall i \in E \end{aligned}$$

- (a) Prove that the IP is indeed a formulation of the maximum matching problem.
(b) Prove that the constraints $x_i \leq 1$ can be deleted from the LP relaxation without changing the optimal solution of the LP. (Let's call this the simplified LP relaxation of the problem.)
2. The IP below is an IP formulation of the minimum vertex cover problem in a graph $G = (V, E)$.

$$\begin{aligned} \min \quad & \sum_{i=1}^{|V|} x_i \\ \text{s.t.} \quad & x_j + x_k \geq 1 \quad \forall \{j, k\} \in E \\ & x_i \in \{0, 1\} \quad \forall i \in V \end{aligned}$$

Its LP relaxation can be written as follows:

$$\begin{aligned} \min \quad & \sum_{i=1}^{|V|} x_i \\ \text{s.t.} \quad & x_j + x_k \geq 1 \quad \forall \{j, k\} \in E \\ & x_i \geq 0 \quad \forall i \in V \\ & x_i \leq 1 \quad \forall i \in V \end{aligned}$$

- (a) Prove that the IP is indeed a formulation of the minimum vertex cover problem.
 - (b) Prove that the constraints $x_i \leq 1$ can be deleted from the LP relaxation without changing the optimal solution of the LP. (Let's call this the simplified LP relaxation of the problem.)
3. What's the dual of the simplified LP relaxation of the maximum matching problem? What's the dual of the simplified LP relaxation of the minimum vertex cover problem? (You should obtain something nice here.)
4. Prove that for bipartite graphs the size of the maximum matching equals to the size of the minimum vertex cover. Show by example that the the size of the maximum matching may not be equal to the size of the minimum vertex cover in a non-bipartite graph.

Due on April 17, Thursday, in the beginning of the class.