MATH 354:03 LINEAR OPTIMIZATION, SPRING 2013
HANDOUT #3

1. Optimality Conditions

We consider the following problem:

$$\begin{array}{lrcl} \max & c^T x & & \\ \text{s.t.} & Ax & \leq & b \\ & x & \geq & 0 \end{array} \tag{1}$$

We stated the optimality conditions for this problem is:

i. $Ax \leq b$, $x \geq 0$ (Primal feasibility).
ii. $A^T w - Iv = c$, $w \geq 0$, $v \geq 0$ (Dual feasibility).
iii. $w^T(b - Ax) = 0$, $x^T v = 0$ (Complementary slackness).

Therefore to check if $\bar{x} \in \{x | Ax \leq b, x \geq 0\}$ is optimal, we can use the following implications:

- if $\bar{x}_i > 0$ then $v_i = 0$.
- if $a_i^T \bar{x} < b_i$ then $w_i = 0$.

since otherwise this would violate *complementary slackness* (iii). Then we can try to find a solution to the reduced system in *dual feasibility* using the first phase of 2-phase simplex method. If we find a solution, $\bar{x}$ is optimal.

2. Duality

Let us again consider (1). Using optimality conditions we derived a dual problem:

$$\begin{array}{lrcl} \min & b^T w & & \\ \text{s.t.} & A^T w & \geq & c \\ & w & \geq & 0 \end{array} \tag{2}$$

3. Numerical Examples

3.1. **Optimality Conditions.**

$$\begin{array}{lrrcl} \max & x_1 & +2x_2 & & \\ \text{s.t.} & x_1 & +x_2 & \leq & 6 \\ & x_1 & & \geq & 3 \\ & & x_2 & \geq & 2 \\ & x_1 & & \geq & 0 \\ & & x_2 & \geq & 0 \end{array} \tag{3}$$

Verify optimality conditions for the following vectors $\{\begin{pmatrix} 6 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \end{pmatrix}\}$.

3.2. **Forms of LP.** Convert Problem 3 to its canonical form first and then to its standard form.

3.3. **Duality.** What is the dual to Problem 3? (Use both approaches, first take dual using canonical form, say D_1, then take dual using the **Duality Table**, say D_2, compare the differences, reduce D_1 into D_2 using standard variable and constraint manipulations!)