MATH 354:03 LINEAR OPTIMIZATION, SPRING 2013
THEORETICAL PROJECT
DUE ON 11:59PM 5/11/2012 BY EMAIL

WARNING: Anything past the deadline won't be considered.

(1) Let us consider the polyhedron $P = \{x \in \mathbb{R}^n : Ax \leq b\}$. Prove that, an inequality $\alpha^T x \leq \beta$ is redundant for P (or a *linear consequence* of P) if and only if $\max\{\alpha^T x : Ax \leq b\} \leq \beta$.

(2) What is the alternative system to $\{(x, y) : Ax + By = b, f \leq x \leq g, y \geq 0\} \neq \emptyset$? (*Hint*: use Farkas' Lemma)

(3) Assume you have a black box that gives you a solution of a system of linear equalities, or tells you that it is empty. Show how to use this tool to solve LP problem.

(4) Assume either $\{x \in \mathbb{R}^n : Ax \leq b\} \neq \emptyset$ or $\{w \in \mathbb{R}^m : A^T w = c, w \geq 0\} \neq \emptyset$. Prove the following equality holds:

$$\sup\{c^T x : Ax \leq b\} = \inf\{b^T w : A^T w = c, w \geq 0\}$$

(5) Consider the problem $\min\{w^T x : Ax \geq -w, x \geq 0\}$ where $A^T = -A$. Prove any optimal solution, x^*, satisfies $w^T x^* = 0$. (*Hint*: use Duality)