MATH 354:03 LINEAR OPTIMIZATION, SPRING 2012
HANDOUT #2

How to convert an LP into Canonical Form / Standard Form:

$$\left\{\begin{array}{lllllll} \min & 2x & -y & +4z & & \\ \text{s.t.} & -x & +7y & +z & \geq & 3 \\ & -3x & +5y & & \leq & 15 \\ & -5x & +2y & +4z & = & 10 \\ & x & & & \leq & 0 \\ & & y & & \geq & 0 \end{array}\right\} \xrightarrow{C.F.} \left\{\begin{array}{lllllll} \max & 2x & +y & -4z_1 & +4z_2 & & \\ \text{s.t.} & -x & -7y & -z_1 & +z_2 & \leq & -3 \\ & 3x & +5y & & & \leq & 15 \\ & 5x & +2y & +4z_1 & -4z_2 & \leq & 10 \\ & -5x & -2y & -4z_1 & +4z_2 & \leq & -10 \\ & x & & & & \geq & 0 \\ & & y & & & \geq & 0 \\ & & & z_1 & & \geq & 0 \\ & & & & z_2 & \geq & 0 \end{array}\right\}$$

$$\xrightarrow{S.F.} \left\{\begin{array}{lllllllll} \max & 2x & +y & -4z_1 & +4z_2 & & & & \\ \text{s.t.} & -x & -7y & -z_1 & +z_2 & +s_1 & & = & -3 \\ & 3x & +5y & & & & +s_2 & = & 15 \\ & 5x & +2y & +4z_1 & -4z_2 & & & = & 10 \\ & x & & & & & & \geq & 0 \\ & & y & & & & & \geq & 0 \\ & & & z_1 & & & & \geq & 0 \\ & & & & z_2 & & & \geq & 0 \\ & & & & & s_1 & & \geq & 0 \\ & & & & & & s_2 & \geq & 0 \end{array}\right\}$$

What we needed to take care was $x \leq 0$, z unconstrained (or unrestricted), converting min to max, direction of inequalities. NOTE THAT **we introduced as few variables and constraints as possible** (this will be required in the exam too!).

- $z = z_1 - z_2$ (because z is unrestricted), therefore $z_1 \geq 0,\ z_2 \geq 0$.
- $x = -x$ (because $x \leq 0$), therefore $x \geq 0$.

Geometric solution method:

$$
\begin{array}{lrrrcl}
\max & 3x_1 & +2x_2 & +x_3 & & \\
\text{s.t.} & x_1 & +x_2 & +x_3 & \leq & 5 \\
 & 3x_1 & +2x_2 & & \leq & 12 \\
 & x_1 & & & \geq & 0 \\
 & & x_2 & & \geq & 0 \\
 & & & x_3 & \geq & 0
\end{array}
$$

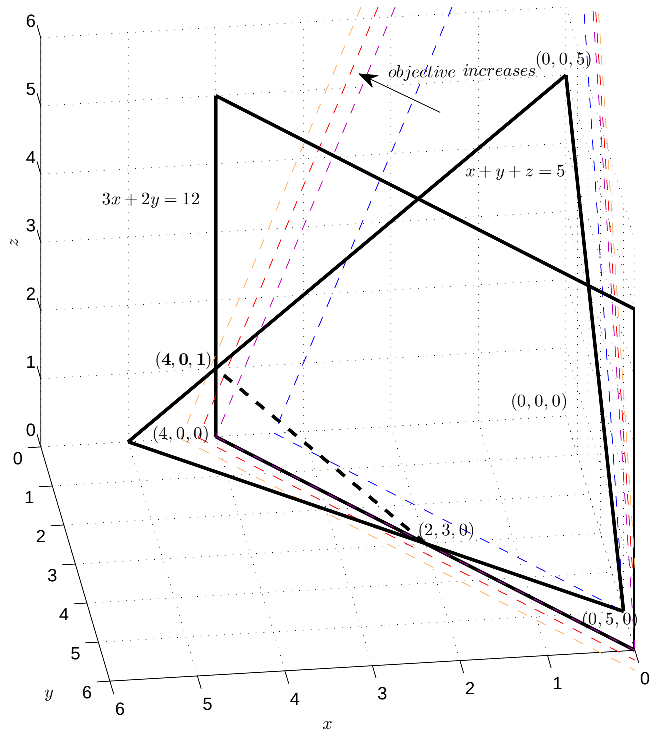

FIGURE 1. Geometrical solution

WHAT ARE THE BASIC SOLUTIONS TO THIS SYSTEM? all of them, basic feasible and basic infeasible solutions. **WHAT IS A BASIC FEASIBLE SOLUTION!**

What is simplex table? What does it represent, what are it's entries?

	x	
	$c_B^T B^{-1} A - c$	$c_B^T B^{-1} b$
x_B	$B^{-1} A$	$B^{-1} b$

Proof of homework #2, question #2:

Let $\bar{B}$ be the columns of the matrix A corresponding to the positive components of extreme point $\bar{x}$ (we assume $\bar{B}$ is not a basis, hence $\text{rank}(\bar{B}) = t < m$. By rearranging the columns of A we can rewrite $A = \left[\bar{B}, \bar{N}\right]$ (where $\bar{N}$ is the rest of the columns from A). We apply Gaussian elimination to the columns of $\bar{B}$. As the columns of $\bar{B}$ are linearly independent (we proved in class), we arrive at the following matrix:

$$\bar{A} = \begin{bmatrix} 1 & 0 & \ldots & 0 & \\ 0 & 1 & \ldots & 0 & \\ 0 & 0 & \ldots & 0 & N \\ 0 & 0 & \ldots & 1 & \\ 0 & 0 & \ldots & 0 & \\ \vdots & \vdots & \vdots & \vdots & \\ 0 & 0 & \ldots & 0 & \end{bmatrix} \tag{1}$$

where we have the $t \times t$ identity matrix on the upper left and N is the matrix after the Gauss elimination operator applied to matrix $\bar{N}$. If the rows $t+1$ to m of $\bar{A}$ is all-zeros, we derive the contradiction to the rank of matrix A (as $\text{rank}(A) = m$). Else we select the column of $\bar{N}$ with at least one non-zero from the rows $t+1$ to m and we can extend $\bar{B}$ with this column to get a set of columns of rank $t+1$. We iterate until we detect $\text{rank}(\bar{B}) = m$.