Instructor: Vladimir Gurvich.

Course title: Games, Graphs, and Boolean functions.

Schedule: March 17, Tuesday, 14 - 18; March 24, Tuesday, 14 - 18; March 27, Friday, 9 - 12; March 31, Tuesday, 14 - 18; April 3, Friday, 9 - 12; April 7, Tuesday, 14 - 18; April 10, Friday, 9.00, final exam.

Course description:

1. **Game forms**.

1.0. Game forms as discrete functions of several discrete variables.

1.1. Main classes: Nash-solvable (NS), dominance solvable (DS), acyclic (AC), assignable (AS), tight (T), and totally tight (TT) game forms.

1.2. Main diagram for two-person game forms:

AS $\Leftarrow$ TT $\Leftrightarrow$ AC $\Rightarrow$ DS $\Rightarrow$ NS $\Leftrightarrow$ T.

1.3. Tightness and Nash-solvability are equivalent for two-person game forms; tightness is not necessary or sufficient for Nash-solvability of three-person game forms.

1.4. Characterizing totally tight two-person game forms; dominance-solvability, assignability, and acyclicity result from total tightness.

2. **Effectivity functions (EFFs) in game theory**.

2.0. EFFs as Boolean functions whose set of variables is the mixture of all players and outcomes;

2.1. Main classes: monotone, self-dual (maximal), convex, sub- and superadditive EFFs.

2.2. EFFs of game forms; characterizing playing and playing-minor EFFs; Moulin and Peleg's Theorem.

2.3. The core of a cooperative game; stable EFFs; criteria of stablity.

2.4. Balanced EFFs are stable; Danilov and Sotskov's interpretation of Scarf's Theorem.

2.5. Convex EFFs are stable; Peleg's Theorem.

2.6. Characterizing stable and self-dual EFFs; Abdou's Theorem. Not every stable EFF is majorized by a stable and self-dual one; a counter-example to Moulin, Danilov, and Sotskov conjecture.

2.7. EFFs and veto-voting.

2.8. Self-dual EFFs and Nash-solvable two-person game forms.

3. **Effectivity functions in graph theory**.

3.1. EFFs of graphs; the EFF of a graph is playing-minor.

3.2. Perfect graphs and balanced EFFs; Lovasz's (1972) proof of Berge's weak perfect graph conjecture.

3.3. Kernel-solvable graphs and stable EFFs; Keiding's (1985) criterion for EFF's stability.

3.4. The Berge and Duchet conjecture on perfect and kernel-solvable graphs; a proof based on Scarf's Theorem; cf. sections 3.2, 3.3, and 2.4.

3.5. Kernels in circulants; a counterexample to Duchet's kernel conjecture.

4. **Gallai's d-graphs and positional game forms**.

4.1. Complementary connected d-graphs. Gallai d-graphs Π and Δ are the only locally minimal complementary connected d-graphs.

4.2. Decomposing Π- and Δ-free d-graphs; decomposition trees and positional game forms are in one-to-one correspondence.

4.3. Characterizing normal forms of d-person positional games in terms of d-graphs and Boolean functions.

5. **Extending Cameron, Edmonds, and Lovasz's Theorem to non-hereditary classes of graphs.**

5.1. Substitution and modular decomposition; Gallai's decomposition of Δ-free d-graphs; Cameron, Edmonds, and Lovasz's Theorem.

5.2 On graphs whose maximal cliques and stable sets intersect; CIS-graphs and CIS-d-graphs; non-hereditary but substitution-closed classes.

5.3. Berge and Chvatal's sufficient conditions for the CIS-property to hold; Deng, Li, and Zang's Theorem.

5.4. Conjecture: CIS-d-graphs are Δ-free, partial results.

5.5 On graphs whose maximal cliques and stable sets intersect, except a unique pair; almost CIS-graphs and split graphs; Wu, Zhang, and Zang's Theorem.

No prerequisites.

Literature: references for each lecture will be given in class; see also 8 extended abstracts and bibliography on the web.