Hereditary efficiently dominatable graphs

Martin Milanič

University of Primorska
FAMNIT and PINT
Koper, Slovenia

7th Slovenian International Conference on Graph Theory
Bled, June 19-25, 2011
Efficient dominating sets

\[ G = (V, E) \]: finite, simple, undirected graph

A vertex \( v \in V \) dominates itself and all its neighbors

A set \( D \subseteq V \) is an efficient dominating set in \( G \) if every vertex in \( V \) is dominated by exactly one vertex in \( D \):

\[ |N[v] \cap D| = 1 \]

for all \( v \in V \).

(1-)perfect code / perfect (independent) dominating set
Efficient dominating sets

\[ G = (V, E) \]: finite, simple, undirected graph

A vertex \( v \in V \) dominates itself and all its neighbors.

A set \( D \subseteq V \) is an efficient dominating set in \( G \) if every vertex in \( V \) is dominated by exactly one vertex in \( D \):

\[ |N[v] \cap D| = 1 \]

for all \( v \in V \).

(1-)perfect code / perfect (independent) dominating set

Martin Milanič, University of Primorska

Hereditary efficiently dominatable graphs
Efficient dominating sets

\[ G = (V, E) \]: finite, simple, undirected graph

A vertex \( v \in V \) dominates itself and all its neighbors

A set \( D \subseteq V \) is an efficient dominating set in \( G \) if every vertex in \( V \) is dominated by exactly one vertex in \( D \):

\[ |N[v] \cap D| = 1 \]

for all \( v \in V \).

(1-)perfect code / perfect (independent) dominating set
Efficient dominating sets

Equivalently:

- $D$ is an independent set of vertices such that
- every vertex outside $D$ has a unique neighbor in $D$.  

Efficient dominating sets

Equivalently:
- \( D \) is an independent set of vertices such that
- every vertex outside \( D \) has a unique neighbor in \( D \).

Equivalently:
\[ \{ N[v] \mid v \in D \} \]
forms a partition of \( V \).
Some small graphs do not contain any efficient dominating sets:

- bull
- fork
- $C_4$
All paths contain efficient dominating sets:

\[ P_k \]

- \( k \equiv 0 \mod 3 \)
- \( k \equiv 1 \mod 3 \)
- \( k \equiv 2 \mod 3 \)

\[ C_k \] contains an efficient dominating set \( \iff k \equiv 0 \mod 3. \)
G is **efficiently dominatable** if it contains an efficient dominating set.

All efficient dominating sets of G are of the same size:
- every efficient dominating set is a minimum dominating set.

Determining whether G is efficiently dominatable is **NP-complete**. Even for:
- planar cubic graphs,
- planar bipartite graphs,
- chordal bipartite graphs,
- chordal graphs,
- line graphs of planar bipartite graphs of max degree three.
G is **efficiently dominatable** if it contains an efficient dominating set.

All efficient dominating sets of G are of the same size:
- every efficient dominating set is a minimum dominating set.

Determining whether G is efficiently dominatable is **NP-complete**. even for:
- planar cubic graphs,
- planar bipartite graphs,
- chordal bipartite graphs,
- chordal graphs,
- line graphs of planar bipartite graphs of max degree three.
$G$ is efficiently dominatable if it contains an efficient dominating set.

All efficient dominating sets of $G$ are of the same size:
- every efficient dominating set is a minimum dominating set.

Determining whether $G$ is efficiently dominatable is NP-complete. even for:
- planar cubic graphs,
- planar bipartite graphs,
- chordal bipartite graphs,
- chordal graphs,
- line graphs of planar bipartite graphs of max degree three.
... but polynomially solvable for:

- trees, interval graphs, series-parallel graphs,
- split graphs, block graphs, circular-arc graphs,
- permutation graphs, trapezoid graphs,
- cocomparability graphs, distance-hereditary graphs,
- AT-free graphs,
- graphs of bounded treewidth or clique-width.
The efficiently dominatable graphs do not form a hereditary class:

- not ED
- ED
$G$ is hereditary efficiently dominatable (HED) if every induced subgraph of $G$ is efficiently dominatable.

We are interested in:

- characterizations,
- algorithmic aspects.
$G$ is **hereditary efficiently dominatable (HED)** if every induced subgraph of $G$ is efficiently dominatable.

We are interested in:

- characterizations,
- algorithmic aspects.
Proposition

Every HED graph is \((\text{bull, fork, } C_{3k+1}, C_{3k+2})\)-free.

The converse holds as well.

To prove this, we first study the structure of \((\text{bull, fork, } C_4)\)-free graphs.
Hereditary efficiently dominatable graphs

**Proposition**

Every HED graph is \((\text{bull, fork, } C_{3k+1}, C_{3k+2})\)-free.

The converse holds as well.

To prove this, we first study the structure of 
\((\text{bull, fork, } C_4)\)-free graphs.
Theorem

Let $G$ be a \textit{(bull, fork, $C_4$)-free} graph. Then, $G$ can be built from paths and cycles by applying a sequence of the following operations:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.
Theorem

Let $G$ be a \textit{(bull, fork, $C_4$)-free} graph. Then, $G$ can be built from \textit{paths and cycles} by applying a sequence of the following operations:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.
A decomposition theorem

Theorem

Let $G$ be a $(\text{bull, fork, } C_4)$-free graph. Then, $G$ can be built from paths and cycles by applying a sequence of the following operations:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.
A decomposition theorem

**Theorem**

Let $G$ be a *(bull, fork, $C_4$)-free* graph. Then, $G$ can be built from *paths and cycles* by applying a sequence of the following operations:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.
A decomposition theorem

Theorem

Let $G$ be a $(bull, fork, C_4)$-free graph. Then, $G$ can be built from paths and cycles

by applying a sequence of the following operations:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.
Theorem

Let $G$ be a \((bull, fork, C_4)\)-free graph. Then, $G$ can be built from paths and cycles by applying a sequence of the following operations:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.
A decomposition theorem

Let $G$ be a $(\text{bull, fork, } C_4)$-free graph. Then, $G$ can be built from paths and cycles by applying a sequence of the following operations:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.
Theorem

Let $G$ be a (bull, fork, $C_4$)-free graph. Then, $G$ can be built from paths and cycles by applying a sequence of the following operations:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.
Rafts and semi-rafts

Rafts of order 2, 3 and 4:

$R_2$

$R_3$

$R_4$
Rafts and semi-rafts

Rafts of order 2, 3 and 4:

$R_2$

$R_3$

$R_4$

Semi-rafts of order 2, 3 and 4:

$S_2$

$S_3$

$S_4$
Raft expansion

non-adjacent vertices

a raft
Semi-raft expansion

adjacent vertices

a semi-raft
Theorem

Let $G$ be a $(\text{bull}, \text{fork}, C_4)$-free graph. Then, $G$ can be built from paths and cycles by applying a sequence of the following operations:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.
A decomposition theorem

**Theorem**

Let $G$ be a \((\text{bull, fork, } C_{3k+1}, C_{3k+2})\)-free graph. Then, $G$ can be built from paths and \{cycles $C_{3k}$ ; $k \in \mathbb{N}$\} by applying a sequence of the following operations:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.
A decomposition theorem

Theorem

Let $G$ be a $(bull, fork, C_{3k+1}, C_{3k+2})$-free graph. Then, $G$ can be built from

paths and $\{cycles C_{3k} ; k \in \mathbb{N}\}$

by applying a sequence of the following operations:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.

Martin Milanič, University of Primorska
Hereditary efficiently dominatable graphs
The set of efficiently dominatable graphs is closed under each of the operations used in the theorem:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.

**Corollary**

Every \((bull, fork, C_{3k+1}, C_{3k+2})\)-free graph is efficiently dominatable.

**Theorem**

The class of hereditary efficiently dominatable graphs equals the class of \((bull, fork, C_{3k+1}, C_{3k+2})\)-free graphs.
Characterization of HED graphs

The set of efficiently dominatable graphs is closed under each of the operations used in the theorem:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.

**Corollary**

Every (bull, fork, $C_{3k+1}$, $C_{3k+2}$)-free graph is efficiently dominatable.

**Theorem**

The class of hereditary efficiently dominatable graphs equals the class of (bull, fork, $C_{3k+1}$, $C_{3k+2}$)-free graphs.
Characterization of HED graphs

The set of efficiently dominatable graphs is closed under each of the operations used in the theorem:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.

Corollary

Every (bull, fork, $C_{3k+1}$, $C_{3k+2}$)-free graph is efficiently dominatable.

Theorem

The class of hereditary efficiently dominatable graphs equals the class of (bull, fork, $C_{3k+1}$, $C_{3k+2}$)-free graphs.
Characterization of HED graphs

The set of efficiently dominatable graphs is closed under each of the operations used in the theorem:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.

**Corollary**

Every \((bull, fork, C_{3k+1}, C_{3k+2})\)-free graph is efficiently dominatable.

**Theorem**

The class of hereditary efficiently dominatable graphs equals the class of \((bull, fork, C_{3k+1}, C_{3k+2})\)-free graphs.
The set of efficiently dominatable graphs is closed under each of the operations used in the theorem:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.

**Corollary**

Every (bull, fork, $C_{3k+1}$, $C_{3k+2}$)-free graph is efficiently dominatable.

**Theorem**

The class of hereditary efficiently dominatable graphs equals the class of (bull, fork, $C_{3k+1}$, $C_{3k+2}$)-free graphs.
The set of efficiently dominatable graphs is closed under each of the operations used in the theorem:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.

**Corollary**

*Every (bull, fork, $C_{3k+1}, C_{3k+2}$)-free graph is efficiently dominatable.*

**Theorem**

*The class of hereditary efficiently dominatable graphs equals the class of (bull, fork, $C_{3k+1}, C_{3k+2}$)-free graphs.*
Characterization of HED graphs

The set of efficiently dominatable graphs is closed under each of the operations used in the theorem:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.

**Corollary**

Every \((bull, fork, C_{3k+1}, C_{3k+2})\)-free graph is efficiently dominatable.

**Theorem**

The class of hereditary efficiently dominatable graphs equals the class of \((bull, fork, C_{3k+1}, C_{3k+2})\)-free graphs.
The set of efficiently dominatable graphs is closed under each of the operations used in the theorem:

- disjoint union of two graphs,
- duplicating a vertex,
- adding a dominating vertex,
- raft expansion,
- semi-raft expansion.

**Corollary**

*Every* (bull, fork, $C_{3k+1}$, $C_{3k+2}$)-free graph *is efficiently dominatable.*

**Theorem**

*The class of hereditary efficiently dominatable graphs equals the class of* (bull, fork, $C_{3k+1}$, $C_{3k+2}$)-free graphs.*
Finding efficient dominating sets efficiently?

Is there an efficient algorithm for finding an efficient dominating set in a given efficiently dominatable graph?

No (unless $P = NP$).
Is there an efficient algorithm for finding an efficient dominating set in a given efficiently dominatable graph?

No (unless P = NP).
Finding efficient dominating sets efficiently?

Is there an efficient algorithm for finding an efficient dominating set in a given efficiently dominatable graph?

No (unless \( P = NP \)).
Finding efficient dominating sets efficiently?

Is there an efficient algorithm for finding an efficient dominating set in a given efficiently dominatable graph?

No (unless \( P = NP \)).
Is there an efficient algorithm for finding an efficient dominating set in a given hereditary efficiently dominatable graph?

Yes! We will see two approaches.
Is there an efficient algorithm for finding an efficient dominating set in a given hereditary efficiently dominatable graph?

Yes! We will see two approaches.
Is there an efficient algorithm for finding an efficient dominating set in a given hereditary efficiently dominatable graph?

Yes! We will see two approaches.
A polynomial-time robust algorithm

**Input:** a graph $G$

**Output:** either an efficient dominating set in $G$, or a proof that $G$ is not hereditary efficiently dominatable.

Algorithm:
- if $G$ contains an induced bull, fork, or $C_4 \rightarrow G$ is not HED
- while $G$ is decomposable, decompose $\rightarrow$ a set $\mathcal{H}$ of indecomposable graphs
- if there exists an $H \in \mathcal{H}$ such that $H = C_{3k+1}$ or $C_{3k+2} \rightarrow G$ is not HED
- otherwise, each $H \in \mathcal{H}$ is either $P_k$ or $C_{3k} \rightarrow$ we can find an ED set in every $H$; these sets can be mapped to an ED set in $G$. 

Martin Milanič, University of Primorska

Hereditary efficiently dominatable graphs
A polynomial-time robust algorithm

**Input:** a graph $G$

**Output:** either an efficient dominating set in $G$, or a proof that $G$ is not hereditary efficiently dominatable.

**Algorithm:**

- if $G$ contains an induced bull, fork, or $C_4 \rightarrow G$ is not HED
- while $G$ is decomposable, decompose $\rightarrow$ a set $\mathcal{H}$ of indecomposable graphs
- if there exists an $H \in \mathcal{H}$ such that $H = C_{3k+1}$ or $C_{3k+2} \rightarrow G$ is not HED
- otherwise, each $H \in \mathcal{H}$ is either $P_k$ or $C_{3k} \rightarrow$ we can find an ED set in every $H$; these sets can be mapped to an ED set in $G$. 

Martin Milanič, University of Primorska

Hereditary efficiently dominatable graphs
A polynomial-time robust algorithm

**Input:** a graph $G$

**Output:** either an efficient dominating set in $G$, or a proof that $G$ is not hereditary efficiently dominatable.

**Algorithm:**

- if $G$ contains an induced bull, fork, or $C_4$ $\rightarrow G$ is not HED
- while $G$ is decomposable, decompose $\rightarrow$ a set $\mathcal{H}$ of indecomposable graphs
  - if there exists an $H \in \mathcal{H}$ such that $H = C_{3k+1}$ or $C_{3k+2}$ $\rightarrow G$ is not HED
  - otherwise, each $H \in \mathcal{H}$ is either $P_k$ or $C_{3k}$ $\rightarrow$ we can find an ED set in every $H$; these sets can be mapped to an ED set in $G$. 
A polynomial-time robust algorithm

**Input:** a graph $G$

**Output:** either an efficient dominating set in $G$, or a proof that $G$ is not hereditary efficiently dominatable.

**Algorithm:**

- if $G$ contains an induced bull, fork, or $C_4 \rightarrow G$ is not HED
- while $G$ is decomposable, decompose $\rightarrow$ a set $\mathcal{H}$ of indecomposable graphs
- if there exists an $H \in \mathcal{H}$ such that $H = C_{3k+1}$ or $C_{3k+2} \rightarrow G$ is not HED
- otherwise, each $H \in \mathcal{H}$ is either $P_k$ or $C_{3k} \rightarrow$ we can find an ED set in every $H$; these sets can be mapped to an ED set in $G$. 
A polynomial-time robust algorithm

**Input:** a graph $G$

**Output:** either an efficient dominating set in $G$, or a proof that $G$ is not hereditary efficiently dominatable.

Algorithm:

- if $G$ contains an induced bull, fork, or $C_4 \rightarrow G$ is not HED
- while $G$ is decomposable, decompose $\rightarrow$ a set $\mathcal{H}$ of indecomposable graphs
  - if there exists an $H \in \mathcal{H}$ such that $H = C_{3k+1}$ or $C_{3k+2} \rightarrow G$ is not HED
  - otherwise, each $H \in \mathcal{H}$ is either $P_k$ or $C_{3k} \rightarrow$ we can find an ED set in every $H$; these sets can be mapped to an ED set in $G$. 

Martin Milanič, University of Primorska

Hereditary efficiently dominatable graphs
**Input:** a graph $G$

**Output:** either an efficient dominating set in $G$, or a proof that $G$ is not hereditary efficiently dominatable.

**Algorithm:**

- if $G$ contains an induced bull, fork, or $C_4 \rightarrow G$ is not HED
- while $G$ is decomposable, decompose $\rightarrow$ a set $\mathcal{H}$ of indecomposable graphs
  - if there exists an $H \in \mathcal{H}$ such that $H = C_{3k+1}$ or $C_{3k+2} \rightarrow G$ is not HED
  - otherwise, each $H \in \mathcal{H}$ is either $P_k$ or $C_{3k} \rightarrow$ we can find an ED set in every $H$; these sets can be mapped to an ED set in $G$. 
**Input:** a graph $G$

**Output:** either an efficient dominating set in $G$, or a proof that $G$ is not hereditary efficiently dominatable.

**Algorithm:**
- if $G$ contains an induced bull, fork, or $C_4 \rightarrow G$ is not HED
- while $G$ is decomposable, decompose $\rightarrow$ a set $\mathcal{H}$ of indecomposable graphs
- if there exists an $H \in \mathcal{H}$ such that $H = C_{3k+1}$ or $C_{3k+2} \rightarrow G$ is not HED
- otherwise, each $H \in \mathcal{H}$ is either $P_k$ or $C_{3k} \rightarrow$ we can find an ED set in every $H$; these sets can be mapped to an ED set in $G$. 
A polynomial-time robust algorithm

Input: a graph $G$
Output: either an efficient dominating set in $G$, or a proof that $G$ is not hereditary efficiently dominatable.

Algorithm:

- if $G$ contains an induced bull, fork, or $C_4 \rightarrow G$ is not HED
- while $G$ is decomposable, decompose $\rightarrow$ a set $\mathcal{H}$ of indecomposable graphs
- if there exists an $H \in \mathcal{H}$ such that $H = C_{3k+1}$ or $C_{3k+2} \rightarrow G$ is not HED
- otherwise, each $H \in \mathcal{H}$ is either $P_k$ or $C_{3k} \rightarrow$ we can find an ED set in every $H$; these sets can be mapped to an ED set in $G$. 

Martin Milanič, University of Primorska

Hereditary efficiently dominatable graphs
A polynomial-time robust algorithm

Input: a graph $G$
Output: either an efficient dominating set in $G$, or a proof that $G$ is not hereditary efficiently dominatable.

Algorithm:
- if $G$ contains an induced bull, fork, or $C_4 \rightarrow G$ is not HED
- while $G$ is decomposable, decompose $\rightarrow$ a set $\mathcal{H}$ of indecomposable graphs
- if there exists an $H \in \mathcal{H}$ such that $H = C_{3k+1}$ or $C_{3k+2} \rightarrow G$ is not HED
- otherwise, each $H \in \mathcal{H}$ is either $P_k$ or $C_{3k} \rightarrow$ we can find an ED set in every $H$; these sets can be mapped to an ED set in $G$. 
A polynomial-time robust algorithm

Input: a graph $G$
Output: either an efficient dominating set in $G$, or a proof that $G$ is not hereditary efficiently dominatable.

Algorithm:
- if $G$ contains an induced bull, fork, or $C_4 \rightarrow G$ is not HED
- while $G$ is decomposable, decompose $\rightarrow$ a set $\mathcal{H}$ of indecomposable graphs
- if there exists an $H \in \mathcal{H}$ such that $H = C_{3k+1}$ or $C_{3k+2} \rightarrow G$ is not HED
- otherwise, each $H \in \mathcal{H}$ is either $P_k$ or $C_{3k} \rightarrow$ we can find an ED set in every $H$; these sets can be mapped to an ED set in $G$. 
A polynomial-time robust algorithm

**Input:** a graph $G$

**Output:** either an efficient dominating set in $G$, or a proof that $G$ is not hereditary efficiently dominatable.

**Algorithm:**

- if $G$ contains an induced bull, fork, or $C_4 \rightarrow G$ is not HED
- while $G$ is decomposable, decompose $\rightarrow$ a set $\mathcal{H}$ of indecomposable graphs
- if there exists an $H \in \mathcal{H}$ such that $H = C_{3k+1}$ or $C_{3k+2} \rightarrow G$ is not HED
- otherwise, each $H \in \mathcal{H}$ is either $P_k$ or $C_{3k} \rightarrow$ we can find an ED set in every $H$; these sets can be mapped to an ED set in $G$. 

Another approach

**efficient domination number**

= maximum number of vertices that can be efficiently dominated

= \( \max \{|D \cup N(D)| \mid D \subseteq V \) independent, every \( v \in V \setminus D \) has at most one neighbor in \( D \} \)

---

**The efficient domination problem:**

Given a graph \( G \), compute the efficient domination number of \( G \).
Another approach

efficient domination number
= maximum number of vertices that can be efficiently dominated
= \( \max\{ |D \cup N(D)| \mid D \subseteq V \text{ independent, every } v \in V \setminus D \text{ has at most one neighbor in } D \} \)

The efficient domination problem:
Given a graph \( G \), compute the efficient domination number of \( G \).
Another approach

efficient domination number
= maximum number of vertices that can be efficiently dominated
= \( \max\{ |D \cup N(D)| \mid D \subseteq V \text{ independent, every } v \in V \setminus D \text{ has at most one neighbor in } D \} \)

The efficient domination problem:
Given a graph \( G \), compute the efficient domination number of \( G \).
Reduction to the MWIS problem

$G^2$ – square of a graph $G$:

- $V(G^2) = V(G)$,
- $uv \in E(G^2) \iff d_G(u, v) \leq 2$.

What are the independent sets in $G^2$?

Observation

Efficient domination number of $G = \text{maximum weight of an independent set in } G^2$ where

$$w(x) = |N[x]|$$

for all $x \in V(G)$. 

Martin Milanič, University of Primorska

Hereditary efficiently dominatable graphs
Reduction to the MWIS problem

$G^2$ – square of a graph $G$:

- $V(G^2) = V(G)$,
- $uv \in E(G^2) \iff d_G(u, v) \leq 2$.

What are the independent sets in $G^2$?

**Observation**

Efficient domination number of $G = \text{maximum weight of an independent set in } G^2 \text{ where}$

$$w(x) = |N[x]|$$

for all $x \in V(G)$.
Reduction to the MWIS problem

The efficient domination problem is polynomially solvable in every class of graphs $X$ such that

the maximum-weight independent set (MWIS) problem is polynomially solvable in the class

$$\{ G^2 \mid G \in X \}.$$  

Theorem

The MWIS problem is polynomially solvable for claw-free graphs.

Oriolo–Pietropaoli–Stauffer 2008
Nobili–Sassano 2010
Faenza–Oriolo–Stauffer 2011
The efficient domination problem is polynomially solvable in every class of graphs $X$ such that the maximum-weight independent set (MWIS) problem is polynomially solvable in the class

$$\{ G^2 \mid G \in X \}.$$
The efficient domination problem is polynomially solvable in every class of graphs $X$ such that the maximum-weight independent set (MWIS) problem is polynomially solvable in the class

$$\{ G^2 \mid G \in X \}.$$ 

**Theorem**

*The MWIS problem is polynomially solvable for claw-free graphs.*

- Oriolo–Pietropaoli–Stauffer 2008
- Nobili–Sassano 2010
- Faenza–Oriolo–Stauffer 2011
Proposition

If $G$ is $(E, \text{net})$-free then $G^2$ is claw-free.

Corollary

The ED number can be computed in polynomial time for $(E, \text{net})$-free graphs.
(E, net)-free graphs

**Proposition**

If $G$ is $(E, \text{net})$-free then $G^2$ is claw-free.

**Corollary**

The ED number can be computed in polynomial time for $(E, \text{net})$-free graphs.
The same approach can be used to show that the efficient domination problem is polynomial for:

- cocomparability graphs,
- interval graphs,
- circular-arc graphs,
- trapezoid graphs,
- strongly chordal graphs,
- AT-free graphs.

All these graph classes are closed under taking squares, and the MWIS problem is polynomial on each of them.
Characterizations of hereditary efficiently dominatable graphs.

HED graphs can be recognized in polynomial time by:
(1) expressing their defining property in MSOL,
(2) using the fact that they are of bounded clique-width,
(3) applying the theorem of Courcelle-Makowsky-Rotics (2000).

Is there a combinatorial polynomial-time algorithm for recognizing hereditary efficiently dominatable graphs?

What is the complexity of recognizing $(C_{3k+1}, C_{3k+2})$-free graphs?
Characterizations of hereditary efficiently dominatable graphs.

HED graphs can be recognized in polynomial time by:
(1) expressing their defining property in MSOL,
(2) using the fact that they are of bounded clique-width,
(3) applying the theorem of Courcelle-Makowsky-Rotics (2000).

Is there a combinatorial polynomial-time algorithm for recognizing hereditary efficiently dominatable graphs?

What is the complexity of recognizing $(C_{3k+1}, C_{3k+2})$-free graphs?
Characterizations of hereditary efficiently dominatable graphs.

HED graphs can be recognized in polynomial time by:
(1) expressing their defining property in MSOL,
(2) using the fact that they are of bounded clique-width,
(3) applying the theorem of Courcelle-Makowsky-Rotics (2000).

Is there a combinatorial polynomial-time algorithm for recognizing hereditary efficiently dominatable graphs?

What is the complexity of recognizing $(C_{3k+1}, C_{3k+2})$-free graphs?
Characterizations of hereditary efficiently dominatable graphs.

HED graphs can be recognized in polynomial time by:
1. expressing their defining property in MSOL,
2. using the fact that they are of bounded clique-width,

Is there a combinatorial polynomial-time algorithm for recognizing hereditary efficiently dominatable graphs?

What is the complexity of recognizing \((C_{3k+1}, C_{3k+2})\)-free graphs?

Martin Milanič, University of Primorska
Characterizations of hereditary efficiently dominatable graphs.

HED graphs can be recognized in polynomial time by:
(1) expressing their defining property in MSOL,
(2) using the fact that they are of bounded clique-width,
(3) applying the theorem of Courcelle-Makowsky-Rotics (2000).

Is there a combinatorial polynomial-time algorithm for recognizing hereditary efficiently dominatable graphs?

What is the complexity of recognizing \((C_{3k+1}, C_{3k+2})\)-free graphs?
Characterizations of hereditary efficiently dominatable graphs.

HED graphs can be recognized in polynomial time by:
(1) expressing their defining property in MSOL,
(2) using the fact that they are of bounded clique-width,
(3) applying the theorem of Courcelle-Makowsky-Rotics (2000).

Is there a combinatorial polynomial-time algorithm for recognizing hereditary efficiently dominatable graphs?

What is the complexity of recognizing $(C_{3k+1}, C_{3k+2})$-free graphs?
Characterizations of hereditary efficiently dominatable graphs.

HED graphs can be recognized in polynomial time by:
(1) expressing their defining property in MSOL,
(2) using the fact that they are of bounded clique-width,
(3) applying the theorem of Courcelle-Makowsky-Rotics (2000).

Is there a combinatorial polynomial-time algorithm for recognizing hereditary efficiently dominatable graphs?

What is the complexity of recognizing $(C_{3k+1}, C_{3k+2})$-free graphs?
Thank you!