

HOMEWORK 1,
FEB. 12, DUE FEB. 28, 2008

Problem 1 (35 points)

Local distances between six vertices $V = \{1, 2, 3, 4, 5, 6\}$ are given by the following vertex-vertex incidence matrix:

0 1 x 7 x x
x 0 2 x 8 x
x x 0 3 x 9
8 x x 0 4 x
x 7 x x 0 3
2 x 6 x x 0

where symbol x stands for $+\infty$.

a) Find the vertex-arc incidence matrix and set up an LP model to find the distance from 1 to 6 and from 6 to 1. Do not solve the obtained LP problem.

b) By the Dijkstra algorithm find the tree of the shortest paths and distance $D_{i,j}$ from vertex i to all other vertices; do for $i = 1, 3, 5$.

c) Prove that $D_{a,b} + D_{b,c} \geq D_{a,c} \geq 0$ for every triplet $a, b, c \in V$.

d) Is it true that $D_{a,b} = D_{b,a}$ for every pair $a, b \in V$?

If yes, prove it; if not give an example.

Problem 2 (35 points) Local distances between vertices $V = \{1, 2, 3, 4, 5\}$ are

$d_{1,2} = 1, d_{1,3} = 3, d_{2,3} = 1, d_{2,4} = 4, d_{3,2} = 2,$
 $d_{3,4} = 1, d_{4,2} = -3, d_{4,3} = 2, d_{5,2} = -1, d_{5,4} = 3,$

a) Find the vertex-vertex incidence matrix A_{VV} .

b) Find the vertex-arc incidence matrix A_{VE} and set up LP models to find the distance from 1 to 3 and from 3 to 5. Do not solve the obtained LP problems.

c) Find the distance $D_{i,j}$ for all pairs $i, j \in V$.

Problem 3 (40 points) Given a pipeline with six nodes $V = \{1, 2, 3, 4, 5, 6\}$ and 10 arcs:

$(1, 2), (1, 3), (2, 3), (2, 4), (3, 4), (3, 5), (4, 3), (4, 5), (4, 6),$ and $(5, 6)$.

a) Write the vertex-vertex and vertex-arc incidence matrices of this graph and set up an LP model to find a maximum flow from 1 to 6 for a given capacity vector c . Do not solve the obtained LP problem.

b) Consider the following four capacity vectors:

$(1, 5, 4, 3, 2, 4, 3, 2, 1, 5); (5, 4, 3, 2, 1, 1, 2, 3, 4, 5);$
 $(9, 1, 3, 5, 7, 8, 6, 4, 2, 10); (9, 7, 5, 3, 1, 2, 4, 6, 8, 10).$

In each case find a minimum cut and maximum flow between nodes 1 and 6.

Problem 4 (30 points). For problem 2 on page 57 from the book:

- a) set up a LP model and solve the obtained LP problem.
- b) Find the feasible set and all extreme points (vertices), feasible and infeasible.
- c) Find the value of objective function in each extreme point.

Problem 5 (10 points). For problem 11 on page 60 from the book, set up a LP model (but do not solve the obtained LP problem) and consider the following two modifications:

- a) There are upper limits: the manufacturer can sell at most 110 kg of Chewy, 70 kg of Crunchy, and 100 kg of nutty.
- b) Nutty must contain exactly 5 % of raisins.