

Homework 2, Due Thursday, March 6th

February 23, 2008

Reminder.

Midterm I on Thursday, March 13;

Midterm II on Tuesday, March 25;

Standard time and location: room 106, Scott Hall, 7.40 pm.

Problem 1 (30 points). For the linear constraints

$$x + y \geq 3$$

$$3y - 5x \leq 15$$

$$5x + 2y \geq 10$$

$$x \geq 0, y \geq 0$$

Draw the set of all feasible vectors and find:

a) All extreme points (vertices);

b) All degenerate extreme points;

c) All basic feasible pairs of inequalities and the corresponding basic feasible solutions;

d) All non-basic pairs of inequalities;

e) All basic infeasible pairs of inequalities and the corresponding vectors.

Find all optimal solutions for the following LP problems:

maximize x , minimize x ,

maximize y , minimize y ,

maximize $x + y$, minimize $x + y$,

maximize $2x + y$, minimize $2x + y$

maximize $5x - 3y$, minimize $5x - 3y$,

subject to the above constraints.

Problem 2 (35 points). For the linear constraints

$$x + y + z \leq 5$$

$$15x + 10y + 12z \leq 60$$

$$22x + 18y \leq 99$$

$$x \geq 0, y \geq 0, z \geq 0$$

Draw the set of all feasible vectors and find:

a) All extreme points (vertices);

b) All degenerate extreme points;

- c) All basic feasible triplets of inequalities and the corresponding basic feasible solutions;
- d) All non-basic triplets of inequalities;
- e) All basic infeasible triplets of inequalities and the corresponding vectors.

Find all optimal solutions for the following LP problems:

- maximize x , minimize x ,
 - maximize y , minimize y ,
 - maximize z , minimize z ,
 - maximize $x + 3y$, minimize $x + 3y$,
 - maximize $2x + z$, minimize $2x + z$
 - maximize $x + y + z$, minimize $x + y + z$,
 - maximize $x + 3y + 5z$, minimize $x + 3y + 5z$,
 - maximize $11x + 9y$, minimize $11x + 9y$,
- subject to the above constraints.

Problem 3 (25 points).

A polyhedron P is defined by a system S of m linear constraints in \mathbb{R}^n .

For $m = 6, n = 2$, for $m = 3, n = 3$, for $m = 4, n = 3$, and for $m = 5, n = 3$

- a) How many vertices can be in P ?
- b) How many degenerate vertices can be in P ?
- c) How many non-basic subsystems can be in S ?