

HOMEWORKS 3: April 8 - May 1, 2008.

There are many problems in this homework.

Do not postpone for the last moment.

Problem 1 (25 points). Write the following linear programming problems in the standard and canonical forms. Keep the numbers of variables and constraints as small as possible

minimize $2x - y + 4z$ subject to

$$-x + y + z \geq 3$$

$$3y - 5x \leq 15$$

$$-5x + 2y + 4z = 10$$

$$x \leq 0, y \geq 0$$

minimize $2z - 2y + 7x$ subject to

$$-3x + 5y + 2z \geq 3$$

$$y - 7x \leq 30$$

$$y + 8z - 3x = 10$$

$$x \leq 2, y \geq 1$$

Problem 2 (20 points). For the following linear programming problem write the dual one and transform it to the standard and canonical forms.

minimize $2x - y + 4z$ subject to

$$-x + y + z \geq 3$$

$$3y - 5x \leq 15$$

$$-5x + 2y + 4z = 10$$

$$x \leq 0, y \geq 0$$

Problem 3 (50 points). Given two matrices

$$1 \quad 4 \quad 9 \quad 6 \quad 8$$

$$2 \quad 3 \quad 8 \quad 7 \quad 6$$

$$3 \quad 2 \quad 7 \quad 8 \quad 4$$

$$4 \quad 1 \quad 6 \quad 9 \quad 2$$

$$1 \quad 4 \quad 1 \quad 0 \quad 9$$

$$2 \quad 3 \quad 0 \quad 1 \quad 8$$

$$3 \quad 2 \quad 0 \quad 1 \quad 7$$

$$4 \quad 1 \quad 1 \quad 0 \quad 6$$

for each one:

- list all minimal linearly dependent sets of columns;
- list all maximal linearly independent sets of columns;
- list all minimal sets of columns which span all columns;
- a', b', c') The same for rows;
- compute the rank

Problem 4 (30 points)

Maximize $-x_1 + 3x_2 + x_3$ subject to

$$-x_1 + 2x_2 - 7x_3 \leq 6$$

$$x_1 + x_2 - 3x_3 \leq 15$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

Solve by simplex method and geometrically, indicate the order in which the extreme points are examined by simplex method.

Problem 5 (25 points)

Maximize $x_1 + 2x_2 + x_3 + x_4$ subject to

$$2x_1 + x_2 + 3x_3 + x_4 \leq 8$$

$$2x_1 + 3x_2 + 4x_4 \leq 12$$

$$3x_1 + x_2 + 2x_3 \leq 18$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$$

Problem 6 (25 points)

Maximize $5x_1 + 3x_2$ subject to

$$2x_1 + x_2 \leq 6$$

$$2x_1 - x_2 \geq 0$$

$$x_1 - x_2 \leq 0$$

$$x_1 \geq 0, x_2 \geq 0$$

Solve by simplex method and geometrically, indicate the order in which the extreme points are examined by simplex method, indicate degenerate points.

Problem 7 (25 points)

Maximize $5x_1 + 8x_2 + x_3$ subject to

$$x_1 + x_2 + x_3 \leq 7$$

$$2x_1 + 3x_2 + 3x_3 \leq 12$$

$$3x_1 + 6x_2 + 5x_3 \leq 24$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

Solve by simplex method and geometrically, indicate the order in which the extreme points are examined by simplex method, indicate degenerate points

Problem 8 (40 points)

Maximize $2x_1 + 5x_2 - x_3$ subject to

$$-4x_1 + 2x_2 + 6x_3 = 4$$

$$6x_1 + 9x_2 + 12x_3 = 3$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

Solve by 2-phase simplex method

Problem 9 (45 points)

a) Show that cycling occurs when solving the following problem by simplex method (if the most negative entry in the objective line is chosen).

b) Use Bland's rule to terminate cycling and obtain an optimal solution if one exists.

Minimize $-x_1 + 7x_2 + x_3 + 2x_4$ subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$

$$(1/2)x_1 - (11/2)x_2 - (5/2)x_3 + 9x_4 + x_6 = 0$$

$$(1/2)x_1 - (3/2)x_2 - (1/2)x_3 + x_4 + x_7 = 0$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0, x_7 \geq 0$$

Problem 10 (35 points)

Maximize $2x_1 - x_2 + x_3 - x_4 + x_5$ subject to

$$x_1 + x_2 - x_3 + x_4 + x_5 = 3$$

$$2x_1 - x_2 + x_3 - 2x_4 = 2$$

$$3x_1 - x_3 + 3x_4 \geq 2$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0$$

Problem 11 (80 points).

For problems 4 - 9 write the dual problems and solve them.

Hint: Solve the primal problem first and then make use of its last simplex-tableau to get the solution of the dual problem.

Problem 12 (20 points)

Carry out Phase I for the following problem:

Maximize $x_1 + x_2 + 2x_4$ subject to

$$3x_1 + x_2 + 3x_3 + 2x_4 = 10$$

$$x_1 - 3x_2 + 2x_3 \leq 7$$

$$x_1 + 2x_2 + 3x_3 + x_4 \geq 4$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$$