Final exam sample problems (4 problems)

Problem 1 (15 points).
Find maximum flow and minimum cut.

Problem 2 (35 points).
Solve minimum cost maximum flow problem with capacities from problem 1 and the following costs:

Problem 3 (30 points).
Caterer company must provide tablecloths. It can buy them for $3 each, it can also use laundry service for $2 each (it takes 1 day) or for $1 each (it takes 3 days). The demand during 10-day period is: 3, 1, 4, 1, 5, 9, 2, 6, 8, 1. The company does not have any tablecloths in the beginning, it does not have to wash them and does not have any profit from having them after the end of the period. We need to minimize expenses.

Formulate the problem as transshipment problem and solve it. Partial credit if formulated and solved as LP.
Problem 4 (20 points).
The LP
Maximize $x_1 + x_2 + x_3 + x_4 + x_5$ s.t.

\[
\begin{align*}
    x_1 + 3x_2 + 2x_3 - x_4 + 5x_5 & \leq 3 \\
    2x_1 + x_2 + 3x_4 + x_5 & \leq 5 \\
    -x_1 + 3x_2 + x_3 + x_4 + 2x_5 & \leq 4 \\
    x_1, \ldots, x_5 & \geq 0
\end{align*}
\]

has an optimal solution $x_1 = x_2 = x_5 = 0, x_3 = \frac{7}{3}, x_4 = \frac{5}{3}$. Write the dual problem and find its optimal solution.