How to convert an LP into Canonical Form / Standard Form:

\[
\begin{align*}
\text{min} & \quad 2x - y + 4z \\
\text{s.t.} & \quad -x + 7y + z \geq 3 \\
& \quad -3x + 5y \leq 15 \\
& \quad -5x + 2y + 4z = 10 \\
& \quad x, y \geq 0
\end{align*}
\]

\[
\begin{align*}
\text{max} & \quad 2x + y - 4z_1 + 4z_2 \\
\text{s.t.} & \quad -x - 7y - z_1 + z_2 \leq -3 \\
& \quad 3x + 5y \leq 15 \\
& \quad 5x + 2y + 4z_1 - 4z_2 \leq 10 \\
& \quad -5x - 2y - 4z_1 + 4z_2 \leq -10 \\
& \quad x \geq 0 \\
& \quad y \geq 0 \\
& \quad z_1 \geq 0 \\
& \quad z_2 \geq 0
\end{align*}
\]

\[
\begin{align*}
\text{max} & \quad 2x + y - 4z_1 + 4z_2 \\
\text{s.t.} & \quad -x - 7y - z_1 + z_2 + s_1 = -3 \\
& \quad 3x + 5y + s_2 = 15 \\
& \quad 5x + 2y + 4z_1 - 4z_2 = 10 \\
& \quad x \geq 0 \\
& \quad y \geq 0 \\
& \quad z_1 \geq 0 \\
& \quad z_2 \geq 0 \\
& \quad s_1 \geq 0 \\
& \quad s_2 \geq 0
\end{align*}
\]

What we needed to take care was \(x \leq 0\), \(z\) unconstrained (or unrestricted), converting \(\text{min}\) to \(\text{max}\), direction of inequalities. NOTE THAT we introduced as few variables and constraints as possible (this will be required in the exam too!).

- \(z = z_1 - z_2\) (because \(z\) is unrestricted), therefore \(z_1 \geq 0, z_2 \geq 0\).
- \(x = -x\) (because \(x \leq 0\)), therefore \(x \geq 0\).
Geometric solution method:

\[
\begin{align*}
\text{max} & \quad 3x_1 + 2x_2 + x_3 \\
\text{s.t.} & \quad x_1 + x_2 + x_3 \leq 5 \\
& \quad 3x_1 + 2x_2 \leq 12 \\
& \quad x_1 \geq 0 \\
& \quad x_2 \geq 0 \\
& \quad x_3 \geq 0
\end{align*}
\]

\[\begin{array}{cc}
3x & 2y = 12 \\
3x & y + z = 5
\end{array}\]

**Figure 1.** Geometrical solution

**WHAT ARE THE BASIC SOLUTIONS TO THIS SYSTEM?**
all of them, basic feasible and basic infeasible solutions. **WHAT IS A BASIC FEASIBLE SOLUTION!**

What is simplex table? What does it represent, what are its entries?

<table>
<thead>
<tr>
<th></th>
<th>$x_B$</th>
<th>$c_B^t B^{-1} A - c$</th>
<th>$c_B^t B^{-1} b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Proof of homework #2, question #2:

Let $B$ be the columns of the matrix $A$ corresponding to the positive components of extreme point $\bar{x}$ (we assume $B$ is not a basis, hence $\text{rank}(B) = t < m$. By rearranging the columns of $A$ we can rewrite $A = [B, N]$ (where $N$ is the rest of the columns from $A$). We apply Gaussian elimination to the columns of $B$. As the columns of $B$ are linearly independent (we proved in class), we arrive at the following matrix:

$$
\tilde{A} = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
0 & 0 & \cdots & 0 & N \\
0 & 0 & \cdots & 1 \\
0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & 0
\end{bmatrix}
$$

where we have the $t \times t$ identity matrix on the upper left and $N$ is the matrix after the Gauss elimination operator applied to matrix $\tilde{N}$. If the rows $t + 1$ to $m$ of $\tilde{A}$ is all-zeros, we derive the contradiction to the rank of matrix $A$ (as $\text{rank}(A) = m$). Else we select the column of $N$ with at least one non-zero from the rows $t + 1$ to $m$ and we can extend $\tilde{B}$ with this column to get a set of columns of rank $t + 1$. We iterate until we detect $\text{rank}(\tilde{B}) = m$. 