DIMACS

Center for Discrete Mathematics & Theoretical Computer Science
Founded as a National Science Foundation Science and Technology Center

CCICADA

Command, Control, and Interoperability Center for Advanced Data Analysis
A Department of Homeland Security Center of Excellence

DIMACS/CCICADA Workshop on Stochastic Networks: Reliability, Resiliency, and Optimization

Heterogeneous models for nonlinear flows on networks

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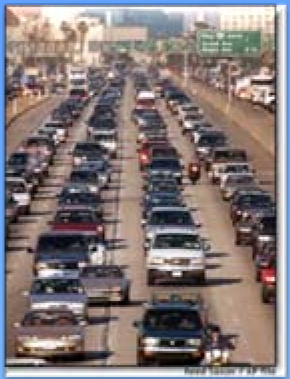
Lighthill-Witham-Richards model:

$$\rho_t + (\rho\, v(\rho))_x = 0, \quad \rho \in [0, \rho_{max}] \text{ density of cars}$$



Vehicular Traffic

Irrigation Channels

De Saint-Venant equation:

$$\begin{cases} H_t + (H v)_x = 0 \\ v_t + \left[\frac{1}{2} v^2 + g\, H\right]_x = 0 \end{cases}$$

H water height, v velocity, g gravity

the processing rate

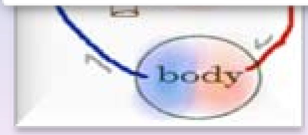
Gas pipelines

Isothermal Euler with friction:

$$\begin{cases} \rho_t + (\rho\, u)_x = 0 \\ (\rho\, u)_t + (\rho\, u^2 + a^2 \rho)_x = -f_g\, \frac{\rho\, u |\rho\, u|}{2 D \rho}. \end{cases}$$

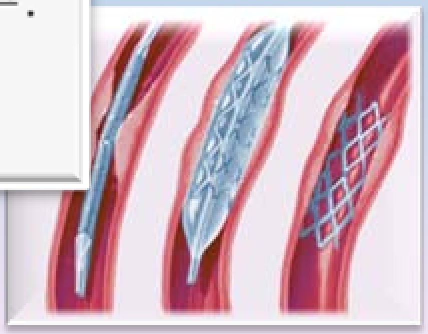
ρ density, u velocity

Real fluids



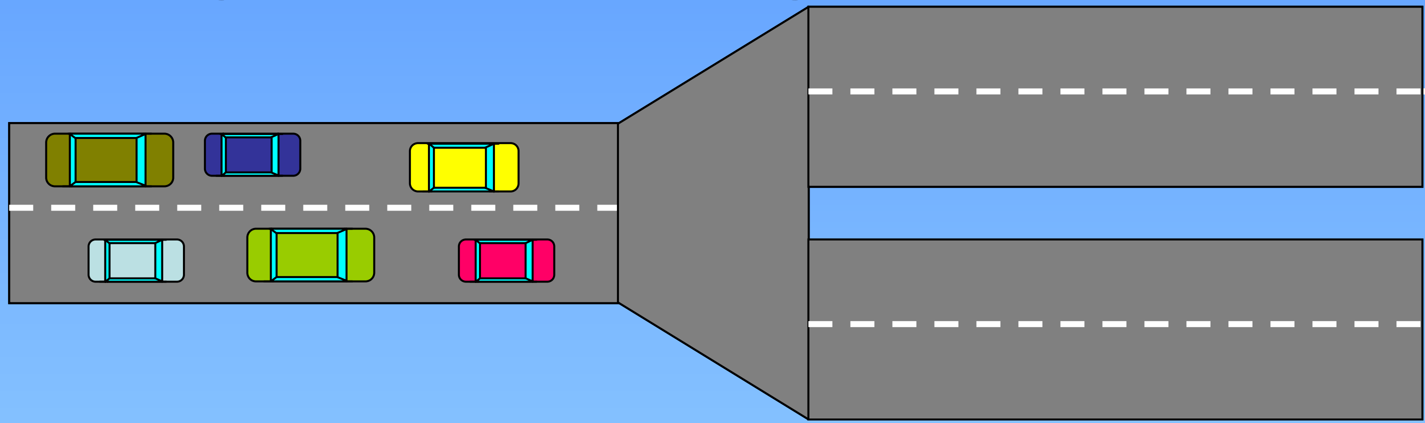
Blood circulation

Bio-Medical



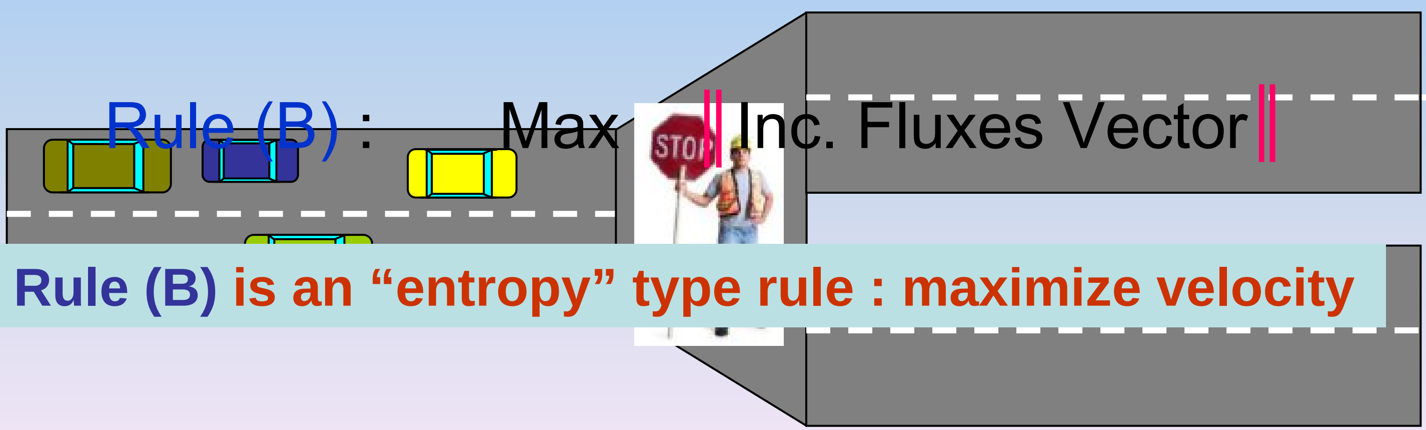
Vascular stents

Dynamics at junctions



Rule (A) : Out. Fluxes Vector = $A \cdot$ Inc. Fluxes Vector

Traffic distribution matrix $A = (\alpha_{ji})$, $0<\alpha_{ji}<1$, $\Sigma_j \alpha_{ji} = 1$



Rule (B) : Max $\|$Inc. Fluxes Vector$\|$

Rule (B) is an "entropy" type rule : maximize velocity

Integration of models and scales

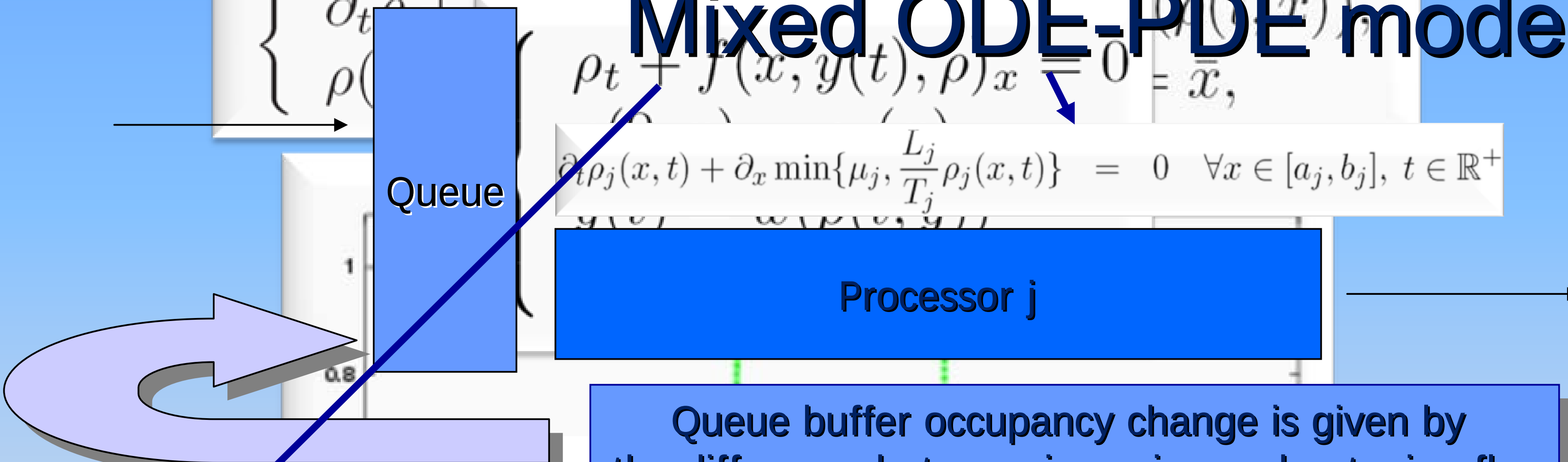
Car trajectories and moving bottlenecks

Goettlich-Herty-Klar supply chain model

Mixed ODE-PDE model

$$\rho_t + f(x, y(t), \rho)_x = 0$$

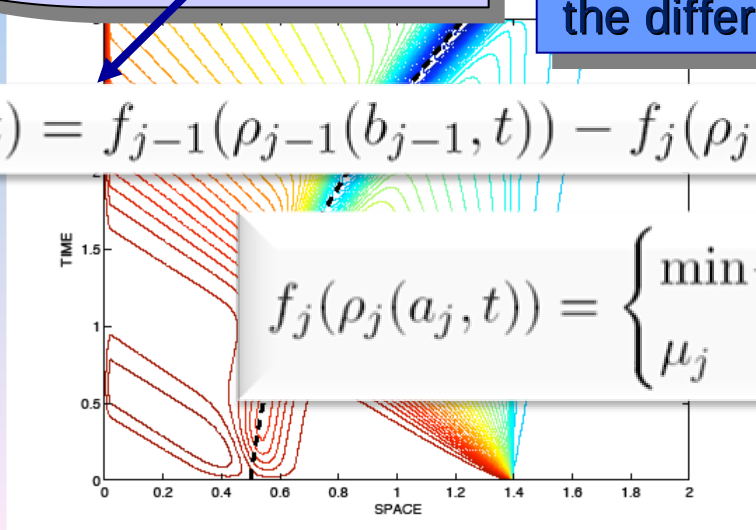
$$\partial_t \rho_j(x,t) + \partial_x \min\{\mu_j, \frac{L_j}{T_j}\rho_j(x,t)\} = 0 \quad \forall x \in [a_j, b_j],\ t \in \mathbb{R}^+$$

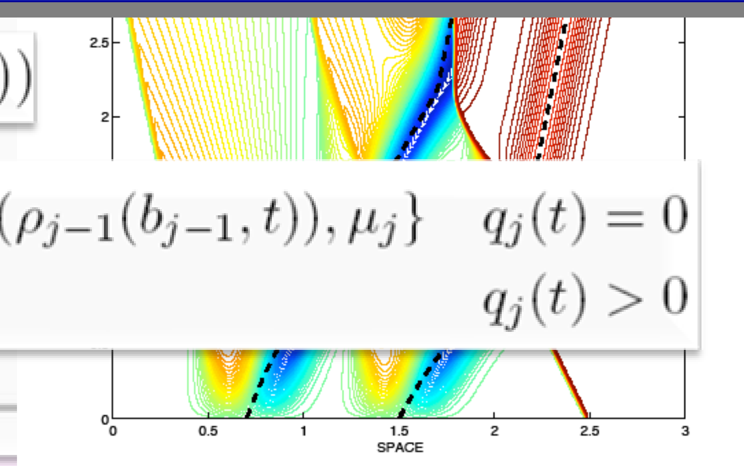


Queue buffer occupancy change is given by the difference between incoming and outgoing flux

$$\partial_t q_j(t) = f_{j-1}(\rho_{j-1}(b_{j-1}, t)) - f_j(\rho_j(a_j, t))$$

$$f_j(\rho_j(a_j,t)) = \begin{cases} \min\{f_{j-1}(\rho_{j-1}(b_{j-1},t)), \mu_j\} & q_j(t) = 0 \\ \mu_j & q_j(t) > 0 \end{cases}$$



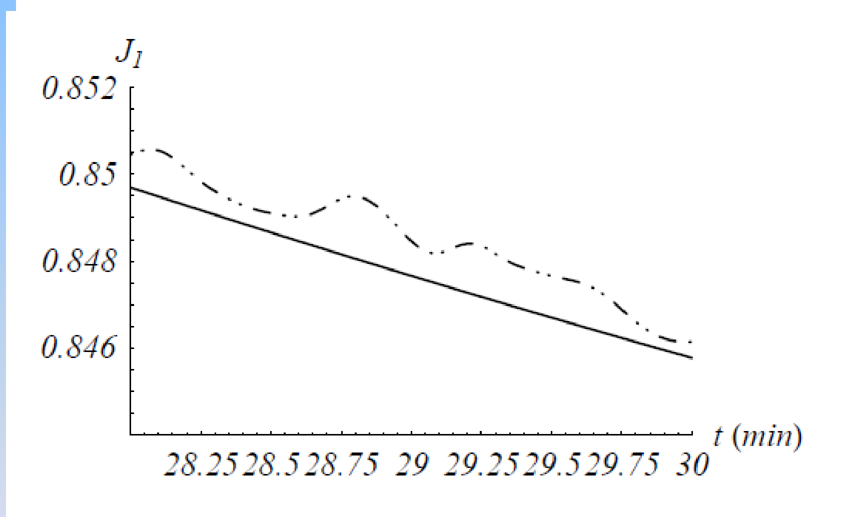


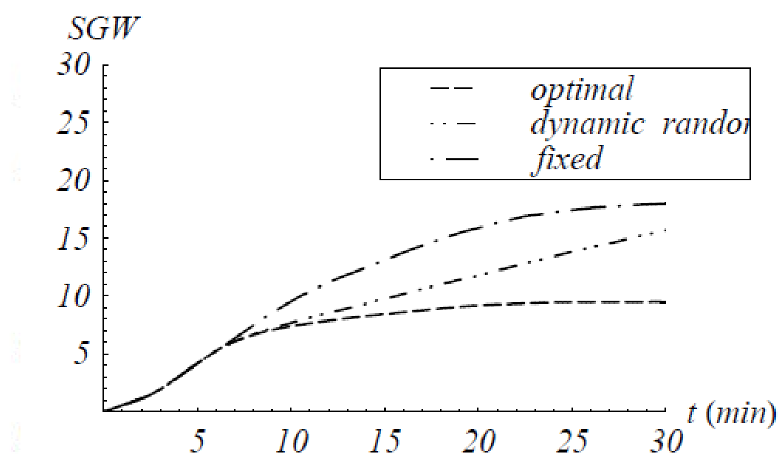
Optimization of vehicular traffic

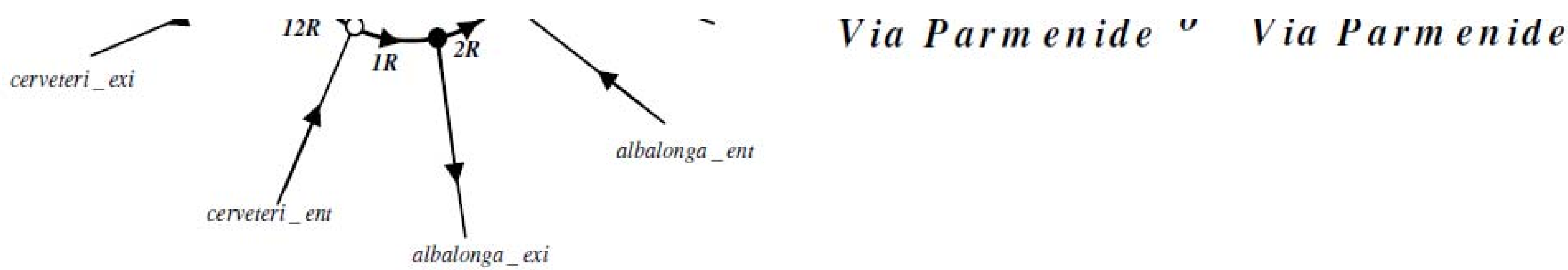
$$J_1(t) = \sum_i \int_{I_i} v(\rho_i(t,x))\,dx,$$

$$J_2(t) = \sum_i \int_{I_i} \frac{1}{v(\rho_i(t,x))}\,dx.$$

$$SGW = \int_0^T \int_{\cup I_i} |Dv(\rho)|\,dt dx.$$







Optimal control for supply chains

$$\begin{cases} \partial_t \rho_j(x,t) + \partial_x \min\{\mu_j, \upsilon_j \rho_j(x,t)\} = 0 & j=1,\ldots,N, \\ \dot{q}_j(t) = f_{j-1}(\rho_{j-1}(b_{j-1},t)) - f_j^{inc} & j=2,\ldots,N, \\ \rho_1(a_1,t) = u(t) & \\ \rho_j(x,0) = \rho_{j,0}(x) & j=1,\ldots,N, \\ q_j(x,0) = q_{j,0} & j=2,\ldots,N, \end{cases}$$

$$J(u) = \sum_{j=1}^{n} \int_0^T q_j(t)dt + \int_0^T [\upsilon_N \cdot \rho_N(b_N,t)) - \psi(t)]^2 \, dt \doteq J_1(u) + J_2(u),$$

Existence of solutions

Take minimizing sequence: compactness by Helly and Ascoli Arzela' Theorem.

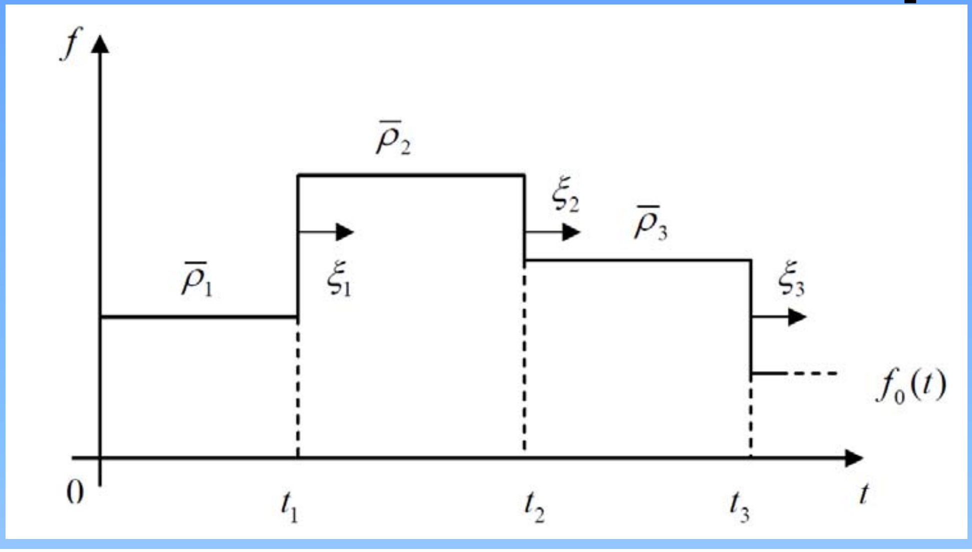
$$q_n \to q \text{ in } C^0, \text{ thus } J_1(u_n) \to J_1(u)$$

$$\int_0^T \left((\upsilon_N \cdot \rho_N^n(b_N,t)) - \psi(t))^2 - (\upsilon_N \cdot \rho_N(b_N,t)) - \psi(t))^2 \right) dt =$$

$$\int_0^T \left((\upsilon_N)^2 \left((\rho_N^n(b_N,t))^2 - (\rho_N(b_N,t))^2 \right) + 2\psi(t)\upsilon_N \left(\rho_N^n(b_N,t)\right) - \rho_N(b_N,t) \right) dt \leq$$

$$||2\psi(t) + \upsilon_N(\rho_N^n(b_N,t) + \rho_N(b_N,t))||_\infty \cdot ||\upsilon_N(\rho_N^n(b_N,t) - \rho_N(b_N,t))||_{L^1}.$$

Tangent vectors for numerical optimization



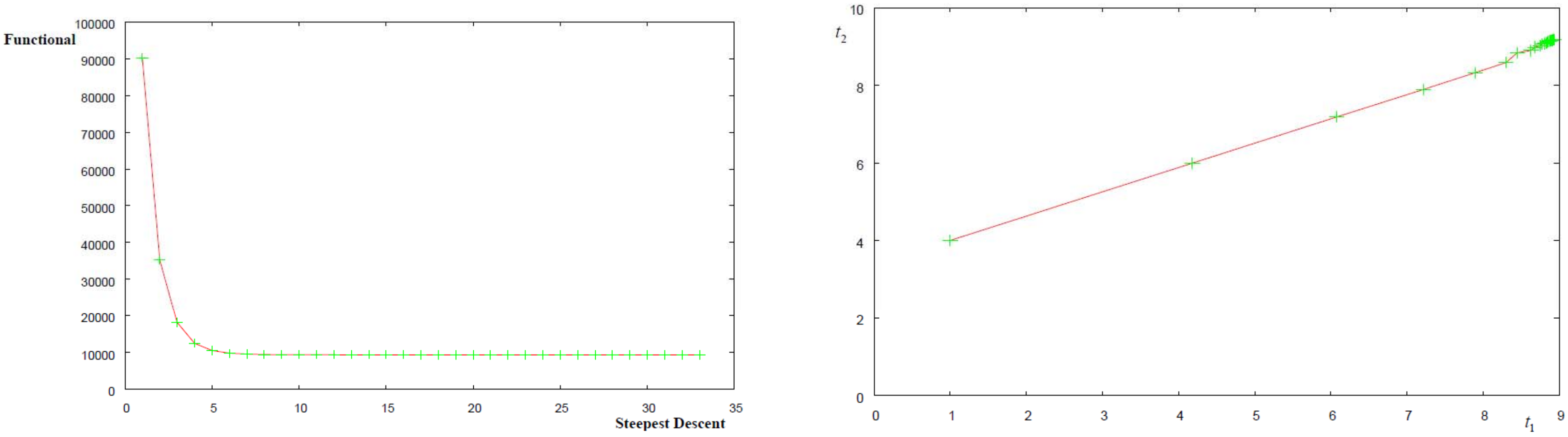
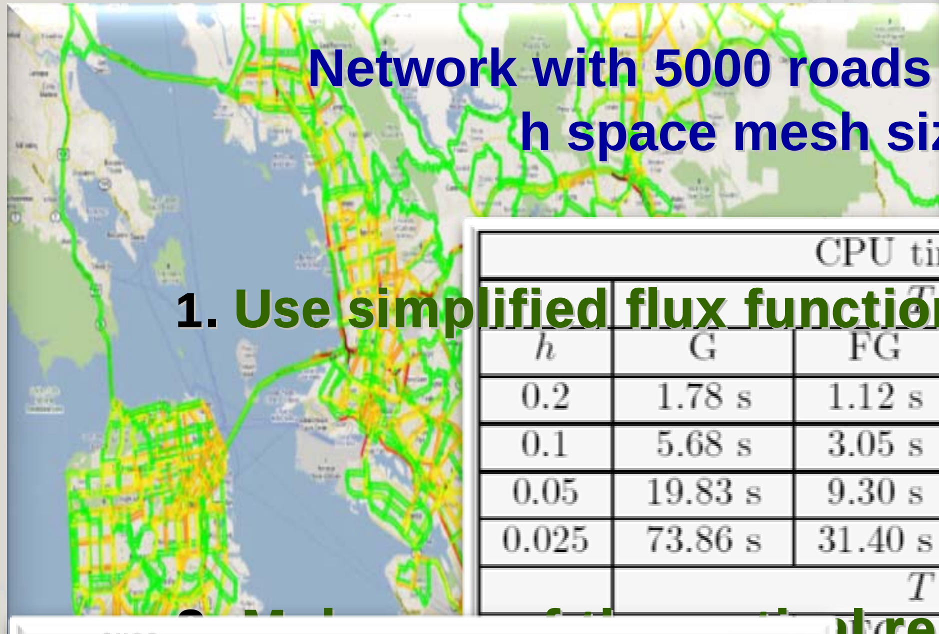
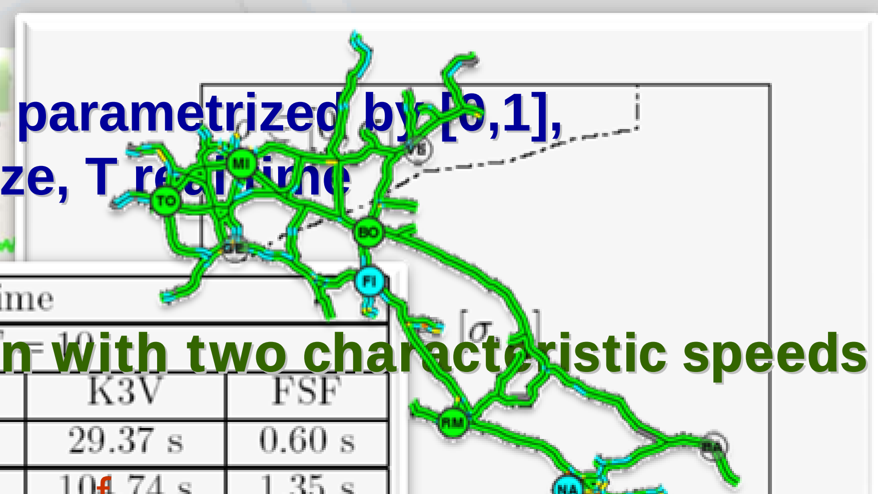


Figure 7: Supply chain with 11 arcs, case a. Left: J_1 versus iteration steps; right: "path" followed by the steepest descent algorithm in the plane (t_1, t_2).

Cyber-infrastructures for info-mobility



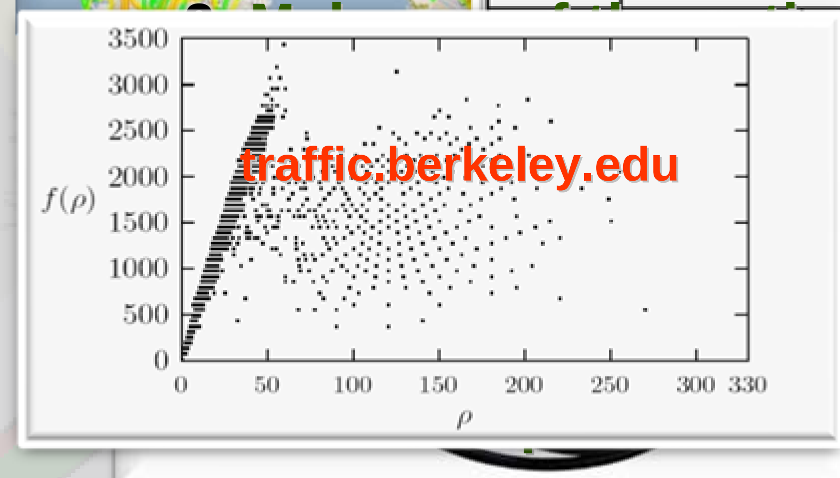


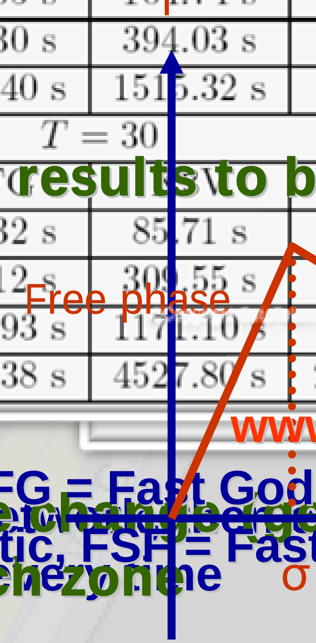
Network with 5000 roads parametrized by [0,1], h space mesh size, T real time

CPU time				
	$T = 10$			
h	G	FG	K3V	FSF
0.2	1.78 s	1.12 s	29.37 s	0.60 s
0.1	5.68 s	3.05 s	104.74 s	1.35 s
0.05	19.83 s	9.30 s	394.03 s	[illegible]
0.025	73.86 s	31.40 s	1515.32 s	[illegible]
	$T = 30$			
[illegible]	[illegible]	[illegible]	85.71 s	[illegible]
[illegible]	[illegible]	[illegible]	309.55 s	[illegible]
[illegible]	[illegible]	[illegible]	1171.10 s	[illegible]
[illegible]	[illegible]	[illegible]	4527.80 s	[illegible]

1. Use simplified flux function with two characteristic speeds

2. Make use of theoretical results to b... of





FG = Fast Godu... etic, FSF = Fast ... ch zone ... time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Paolo Frasca
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Rinaldo Colombo
Giuseppe Coclite
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Axel Klar
ANIMAL GROUPS

Thank you for your attention!

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Real data

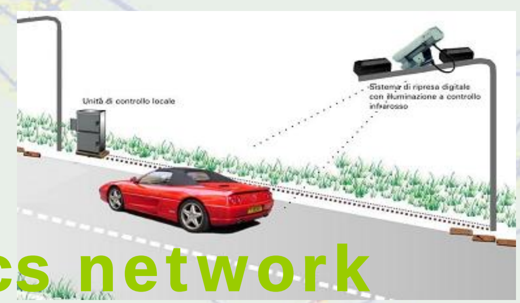
Problems :

1. Dimensionality: big networks
2. Data: measurements and elaboration

Manual counting

Satellite data

Videocameras



Plates reading

1500 arcs network

NETWORK of SALERNO

Lighthill-Whitham-Richards model

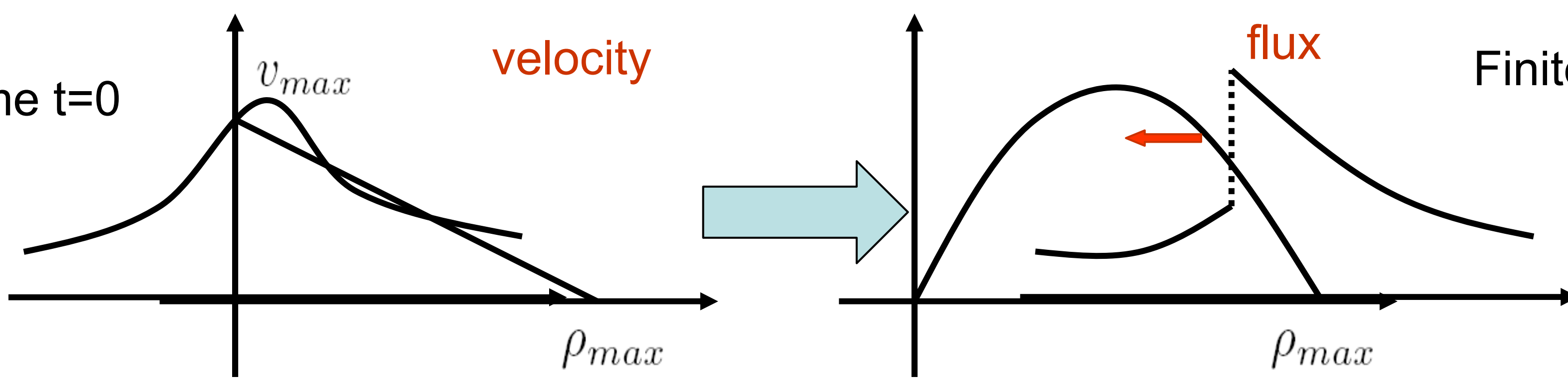
The flux is given by the density times the average velocity $f(t,x) = \rho(t,x) \cdot v(t,x)$

If we assume that the average velocity depends only on density $v(t,x) = v(\rho(t,x))$

Lighthill-Witham-Richards model:

$$\rho_t + (\rho\, v(\rho))_x = 0, \quad \rho \in [0, \rho_{max}]$$

Time t=0



Finite time

Non unique (weak) solutions

Entropy (gas dynamics)

(disorder is increasing, stable shocks)

Networks and Re di Roma square

More incoming than exiting road → Priority parameters

Bifurcations, merging, complicate junctions, traffic circles

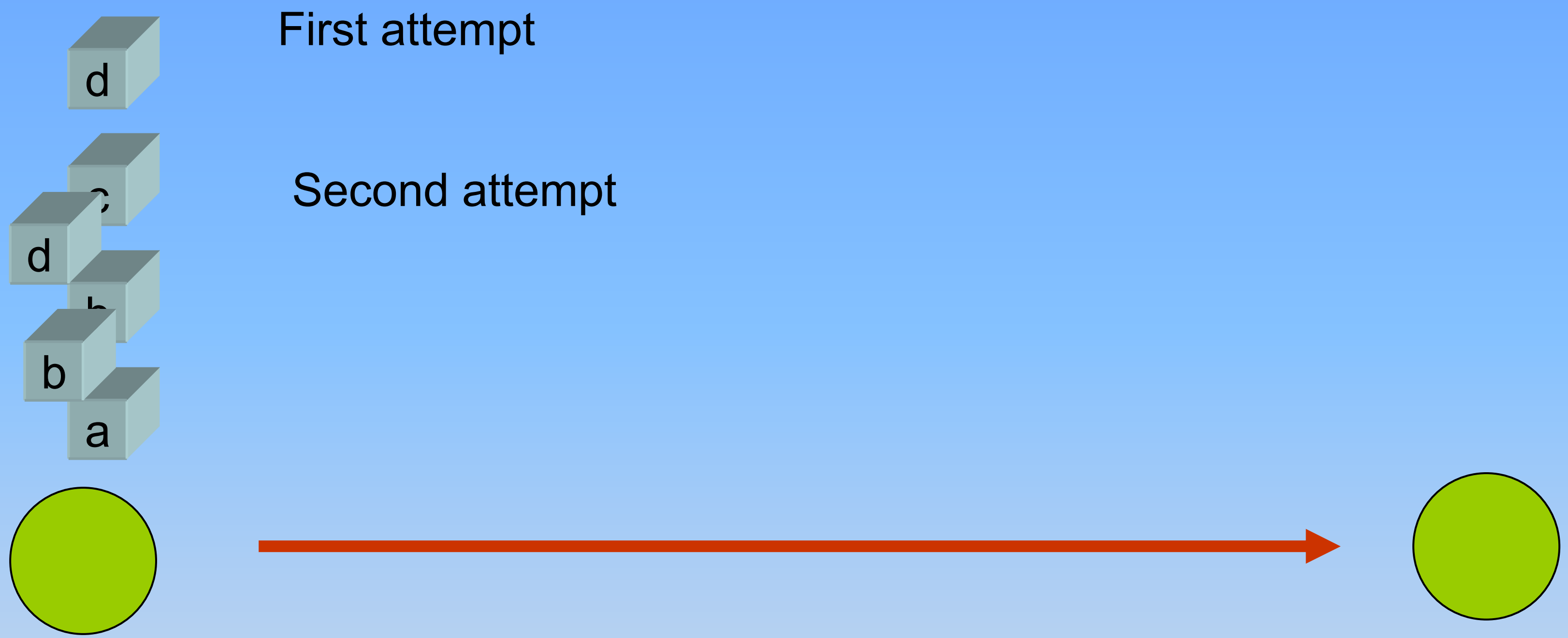
Theory : existence of solutions on networks for BV initial data.

Road flux total variation in x ~ Junctions flux total variation in t

MOVIE

ZOOM

Model for data networks



We essentially invert Rules (A) and (B), giving more importance to through flux than traffic distribution.

In the nth attempt $(1-\mathcal{P}(R))\mathcal{P}(R)^{n-1}$ packets are sent and $\mathcal{P}(R)^n$ are lost.

Maximal fluxes on each line γ_i^{max} and γ_j^{max}

The through flux is $\Gamma = \min\{\sum_i \gamma_i^{max}\ ,\ \sum_j \gamma_j^{max}\}$.